THE EFFECT OF FLUID VISCOSITIES ON COUNTER-CURRENT SPONTANEOUS IMBIBITION

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ABSTRACT

It is now well known that counter current spontaneous imbibition can be scaled in time to account for the effects of permeability, porosity, interfacial tension, and geometry. This paper reports on a theoretical study to investigate the dependence of the time-scale factor on the viscosities of the two fluids. The current best correlations predict that the time-scale factor depends on the square-root of the product of these two viscosities. Although this correlation agrees well with much published data, the quality of the predictions deteriorates as the values of the viscosities diverge. Furthermore, there is no mechanistic explanation for the correlation. Using a newly developed approximate analytical solution for counter-current spontaneous imbibition into a linear sample (a right cylinder with the radial face and one end face sealed), a detailed examination of the influence of the two viscosities was performed and compared with the square-root model. The mechanism of spontaneous imbibition was then analyzed assuming plug flow of the invading phase to obtain a new form for the viscosity correlation. Although this correlation has the correct shape for the viscosity dependence, it does not agree with the data over the range of viscosities studied. However, a simple heuristic change to the model produced a third correlation equation that gives excellent agreement with all the data. There is no mechanistic explanation for this third correlation; however, it is based on the limiting forms of the analytical solution as the mobility ratio becomes much larger than unity and much smaller than unity.

INTRODUCTION

Several authors [1-3], have shown both experimentally and analytically that linear, counter-current, spontaneous displacement of a non-wetting fluid by a wetting fluid in a porous media results in a series of saturation profiles that are self-similar. Furthermore, several authors have shown that the profiles scale with the square-root of time (see [4] for a recent review). Recent work [5] has shown that these two conditions lead to an approximate, analytical solution of this problem until the penetrating wetting fluid contacts the no-flow boundary. This solution is general in nature, allowing for any shape of capillary pressure and relative permeability functions. In the present paper, this solution has been used to determine the dependence of spontaneous imbibition on the

viscosities of the two fluids as well as the shapes and magnitudes of the capillary pressure and relative permeability curves. This work leads to a suggested function to correlate experimental results.

THEORY

The approximate solution derived in [5] is valid for counter-current, spontaneous imbibition of a wetting phase against a non-wetting phase in a homogeneous porous medium for the case in which the fluids are incompressible (including compressible fluids with pressure gradients significantly less than the absolute pressure) and immiscible. Gravity effects are neglected. The process was developed for the case of linear flow parallel to the axis of a columnar sample of infinite length with the columnar surface sealed. For a finite length sample, the solution is valid until the wetting phase front reaches the downstream end of the sample.

An outline of the solution follows. The basic equations governing counter-current, spontaneous imbibition flow are the generalized Darcy law, the capillary pressure definition, and the continuity equations. Combining these equations results in an equation for the wetting phase:

$$q_w = M_w A \frac{\partial S_w}{\partial x} \tag{1}$$

where q_w is the flow rate of the wetting phase A is the cross-sectional area, S_w is the saturation of the wetting phase, x is the coordinate along the axis of the sample, and M_w is the function

$$M_w = \frac{K k_{rw} k_{rnw}}{\mu_w k_{rnw} + \mu_{nw} k_{rw}} \frac{\partial P_c}{\partial S_w} \tag{2}$$

where K is the permeability, k_{rw} and k_{rnw} are the relative permeabilities (w for the wetting phase and nw for the non-wetting phase, μ_w and μ_{nw} are the viscosities, and P_c is the capillary pressure.

The classic frontal advance theory leads to the equation

$$\left(\frac{\partial x}{\partial t}\right)_{S_w} = \frac{1}{\phi A}\left(\frac{\partial q_w}{\partial S_w}\right)_t \tag{3}$$

where the left-hand side is the speed at which a point with a given saturation proceeds and the derivative on the right-hand side defines the gradient of velocity with saturation. Here t is time and ϕ is porosity.

The basis of the approximate solution is the fact that, if the solution is self-similar, then the saturation profile must simply stretch along the *x*-direction with time. We approximate the condition for such stretching by

$$\left(\frac{\partial x}{\partial t}\right)_{S_w} = a(t)\left(S_{wo} - S_w\right) \tag{4}$$

where $a(t)$ is a function of time and S_{wo} is the saturation of the wetting phase at the open face. If S_{wo} is assumed as constant throughout the displacement it may be shown that

$$q_w = \frac{a(t)\,\phi\, A}{2}\left[\left(2S_{wo} - S_w\right)S_w - \left(2S_{wo} - S_{wi}\right)S_{wi}\right] \tag{5}$$

For a self-similarity solution, the functional dependence of the position of the front ($x_f(t)$) on t is $\sqrt{t}$. Therefore, in order for a solution to exist for the problem, $a(t)$ must have the form

$$a(t) = \frac{a_c}{\sqrt{t}} \tag{6}$$

This leads to the following equation for the constant a_c in terms of the saturation at the front, S_{wf}

$$a_c = \sqrt{\frac{1}{\phi}\,\frac{\bar{S}_w - S_{wi}}{\left(S_{wo} - S_{wi}\right)^2}\int_{S_{wo}}^{S_{wf}} \frac{2\,M_w}{\left(2\,S_{wo} - S_w\right)S_w - \left(2\,S_{wo} - S_{wi}\right)S_{wi}}\,dS_w} \tag{7}$$

where the mean saturation is given by

$$\bar{S}_w = \frac{\int_{S_{wo}}^{S_{wf}} \frac{2\,M_w S_w}{\left(2\,S_{wo} - S_w\right)S_w - \left(2\,S_{wo} - S_{wi}\right)S_{wi}}\,dS_w}{\int_{S_{wo}}^{S_{wf}} \frac{2\,M_w}{\left(2\,S_{wo} - S_w\right)S_w - \left(2\,S_{wo} - S_{wi}\right)S_{wi}}\,dS_w} \tag{8}$$

In the present paper, the relative permeability functions will be assumed to be Corey curves

$$k_{rnw} = k_{rnwe}\,k_{rnws} \tag{9}$$

where

$$k_{rnws} = \left(\frac{S_{wo} - S_w}{1 - S_{wr} - S_{nwr}}\right)^{n_{nw}} \tag{10}$$

and

$$k_{rw} = k_{rwe}k_{rws} \tag{11}$$

where

$$k_{rws} = \left(\frac{S_w}{1 - S_{wr} - S_{nwr}}\right)^{n_w} \tag{12}$$

where n_w and n_{nw} are the Corey exponents, k_{rwe} and k_{rnwe} are the relative permeability endpoints, and S_{wr} and S_{nwr} are the residual saturations. The capillary pressure function will be assumed to be of the form

$$P_c = P_{ct} + P_{cp} \ln\left(\frac{S_w}{1 - S_{wr} - S_{nwr}}\right) \tag{13}$$

where P_{ct} and P_{cp} are constants. The capillary pressure function has the derivative

$$\frac{dP_c}{dS_w} = \frac{P_{cp}}{S_w} \tag{14}$$

Defining the mobility ratio as

$$R_m = \frac{\mu_{nw} k_{rwe}}{\mu_w k_{rnwe}} \tag{15}$$

and making the appropriate substitutions, the saturation function becomes

$$M_w = \frac{P_{cp} K k_{rwe}}{\mu_w} \frac{k_{rws} k_{rnws}}{S_w \left(k_{rnws} + R_m k_{rws}\right)} \tag{16}$$

For the case where the initial saturation of the wetting phase is zero, the mean saturation becomes

$$\overline{S}_w = \frac{\int_{S_{wo}}^{S_{wf}} \frac{2 k_{rws} k_{rnws}}{\left(k_{rnws} + R_m k_{rws}\right)\left(2S_{wo} - S_w\right)} dS_w}{\int_{S_{wo}}^{S_{wf}} \frac{2 k_{rws} k_{rnws}}{S_w \left(k_{rnws} + R_m k_{rws}\right)\left(2S_{wo} - S_w\right)} dS_w} \tag{17}$$

and the time scale factor becomes

$$a_c = \sqrt{\frac{P_{cp} K k_{rwe}}{\phi \mu_w S_{wo}^2} \int_{S_{wo}}^{S_{wf}} \frac{2 k_{rws} k_{rnws}}{\left(k_{rnws} + R_m k_{rws}\right)\left(2S_{wo} - S_w\right)} dS_w} \tag{18}$$

This equation has two limits. When the mobility ratio becomes much less than unity

$$a_c \cong \sqrt{\frac{P_{cp} K k_{rwe}}{\phi \mu_w S_{wo}^2} \int_{S_{wo}}^{S_{wf}} \frac{2 k_{rws}}{\left(2S_{wo} - S_w\right)} dS_w} \tag{19}$$

In this case, the viscous resistance is entirely due to the wetting phase; this is the air/liquid case. When the mobility ratio becomes much greater than unity

$$a_c \cong \sqrt{\frac{P_{cp} K k_{rwe}}{\phi \mu_w S_{wo}^2 R_m} \int_{S_{wo}}^{S_{wf}} \frac{2 k_{rnws}}{\left(2S_{wo} - S_w\right)} dS_w} \tag{20}$$

In this case the viscous resistance is entirely due to the non-wetting phase; this is the water/viscous oil case.

SCALING WITH VISCOSITY

Figure 1 shows the scale factor given by Eq. 18 for a range of mobility ratios between 0.0000001 and 1000000, and for Corey exponents of 1,2,3,4 and 5. It is noted that only five points appear at the two limits, while up to 25 points appear for mid-range values of

mobility ratio. The reason for this is that the two limits meet the criteria for the limiting equations described above. Hence, in the low mobility ratio range, the results are insensitive to the Corey exponent for the non-wetting phase, while in the high mobility range, the results are insensitive to the Corey exponent for the wetting phase. In the mid-range, the results are sensitive to both exponents.

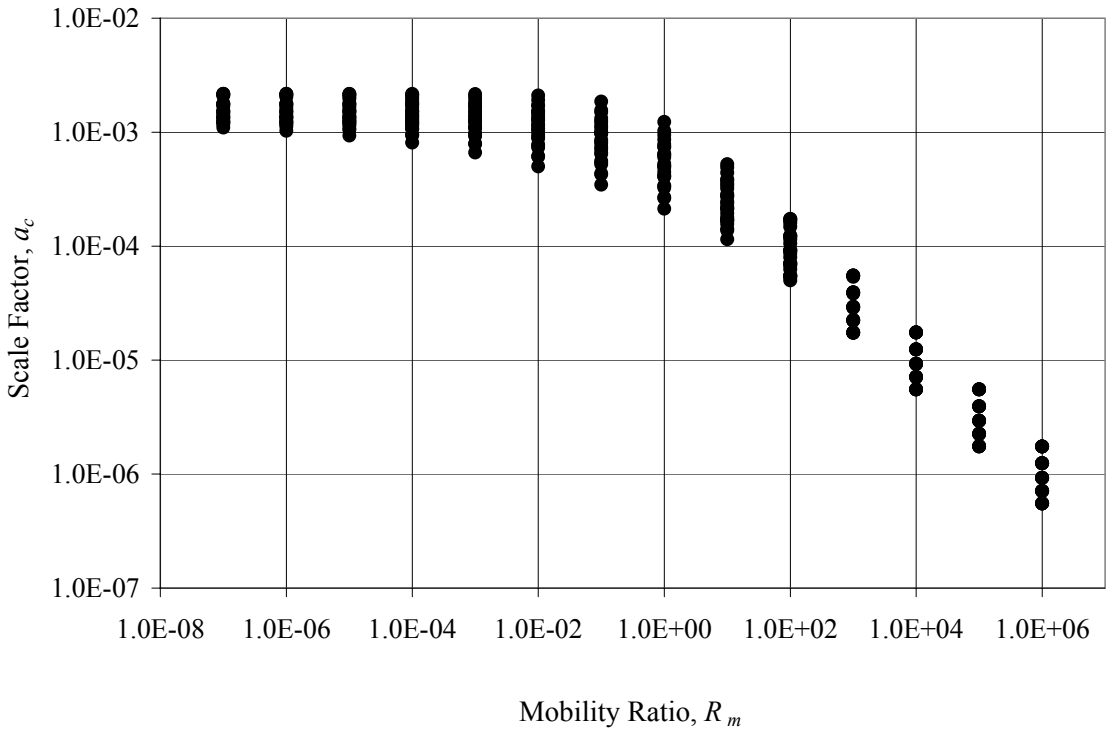

Figure 1 The variation of the scaling factor with the mobility ratio for relative permeability curves having Corey exponents between 1 and 5. The uppermost points are for n_w=1, n_{nw}=1; the lowermost points are for n_w=5, n_{nw}=5.

In order to explore the sensitivity of this viscosity scale factor, results typical for a Berea sandstone core will be used (see Li, Ruth, Mason and Morrow [6]). In this case, 3.0 and 2.6 will be assumed for the non-wetting and wetting Corey exponents respectively. The end-point relative permeabilities are assumed as 0.6 and 0.04 for the non-wetting and wetting phases respectively. The value of P_{cp} will be taken as 2007 kPa. Correlations will be studied for wetting phase viscosities of 1 cp and 10 cp, and varying values of the non-wetting phase viscosity from 0.001 cp to 10000 cp. Therefore, variations in mobility ratio will be achieved by varying both viscosities. We will assume an initial water saturation of zero, a porosity of 0.219, a permeability of 1.064 Darcys, a saturation of 0.48 at the open face, and a saturation at the front of 0.14.

The analysis described above allows the derivation of a scaling equation for time. The production is given by

$$Q_w = \int_0^t (q_w)_o \, dt \tag{21}$$

where $(q_w)_o$ is the flow at the open face. From Eqs. 5 and 6, this flow rate is given by

$$(q_w)_o = \frac{a_c \, \phi \, A S_{wo}^2}{2\sqrt{t}} \tag{22}$$

It follows that

$$Q_w = a_c \, \phi \, A S_{wo}^2 \sqrt{t} \tag{23}$$

and using Eq. 18

$$Q_w = \phi \, A S_{wo}^2 \sqrt{t} \sqrt{\frac{P_{cp} K k_{rwe}}{\phi \mu_w S_{wo}^2} \int_{S_{wo}}^{S_{wf}} \frac{2 k_{rws} k_{rnws}}{(k_{rnws} + R_m k_{rws})(2S_{wo} - S_w)} dS_w} \tag{24}$$

Scaling the production with the movable pore volume

$$\frac{Q_w}{\phi \, A L_b S_{wo}} = \sqrt{\frac{P_{cp} K k_{rwe} t}{\phi \mu_w L_b^2} \int_{S_{wo}}^{S_{wf}} \frac{2 k_{rws} k_{rnws}}{(k_{rnws} + R_m k_{rws})(2S_{wo} - S_w)} dS_w} \tag{25}$$

where L_b is the sample length. Leverett has shown that the capillary pressure can be scaled using

$$P_{cp} = \sqrt{\frac{\phi}{K}} \sigma P'_{cp} \tag{26}$$

Therefore Eq. 26 may be written as

$$\frac{Q_w}{\phi \, A L_b S_{wo}} = \sqrt{\left(\sqrt{\frac{K}{\phi}} \frac{\sigma P'_{cp} k_{rwe}}{\mu_w L_b^2} \int_{S_{wo}}^{S_{wf}} \frac{2 k_{rws} k_{rnws}}{(k_{rnws} + R_m k_{rws})(2S_{wo} - S_w)} dS_w \right) t} \tag{27}$$

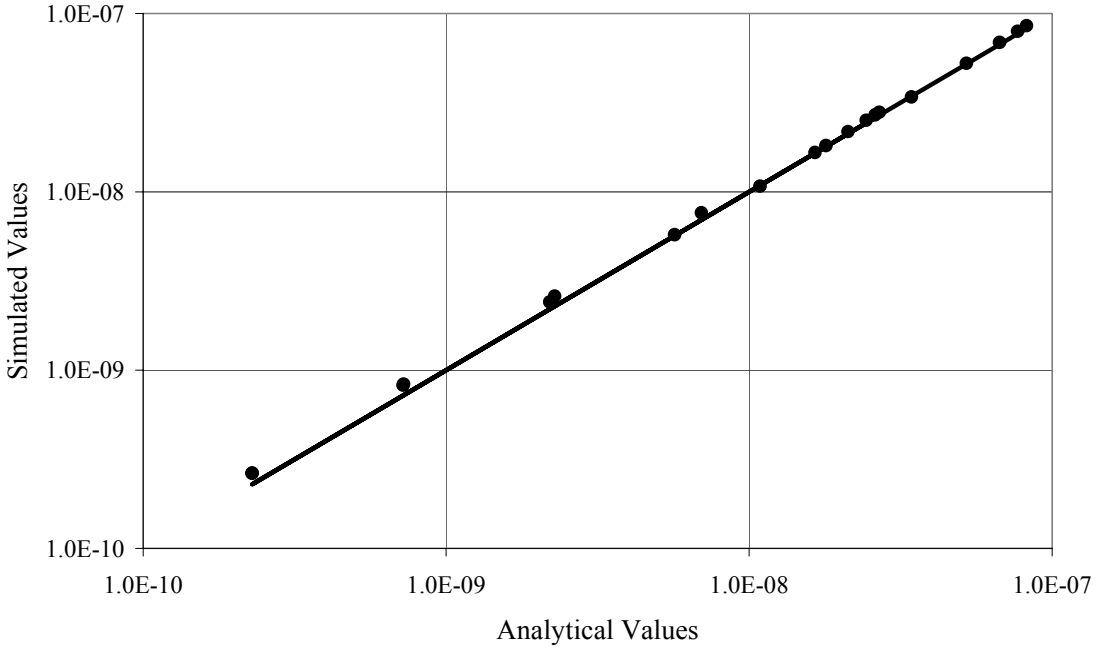

Figure 2 The correlation between the values of the time-scaling factor obtained by simulation and from the analytical equation. The solid line shows the 1:1 correlation; the solid circles are the cross plotted values. Low values correspond to high values of displaced phase viscosity whereas high values correspond to low values of displaced phase viscosity.

It follows that we can define a non-dimensional time as

$$t' = \left(\sqrt{\frac{K}{\phi}} \frac{\sigma P'_{cp} k_{rwe}}{\mu_w L_b^2} \int_{S_{wo}}^{S_{wf}} \frac{2 k_{rws} k_{rnws}}{\left(k_{rnws} + R_m k_{rws}\right)\left(2S_{wo} - S_w\right)} dS_w \right) t \qquad (28)$$

Figure 2 shows the results of Eq. 28 compared with results obtained by numerical simulation (see [7,8] for details of the simulation method). The agreement between the two methods is generally very good. There is some departure at low values of the graph. This is the region of very high displaced phase viscosity. This figure indicates that the analytical solution, although approximate, can be used to derive correlation equations for various parameters.

If the only variable of interest is the viscosity, then the direct dependence of scaling on μ_w is given explicitly in Eq. 28, and all other dependence on viscosities will be contained in the term

$$\lambda\left(\mu_w, \mu_{nw}\right) = \int_{S_{wo}}^{S_{wf}} \frac{2 k_{rws} k_{rnws}}{\left(k_{rnws} + R_m k_{rws}\right)\left(2S_{wo} - S_w\right)} dS_w \qquad (29)$$

Therefore, Eq. 29 rather than Eq. 18 can be used to investigate the viscosity effect.

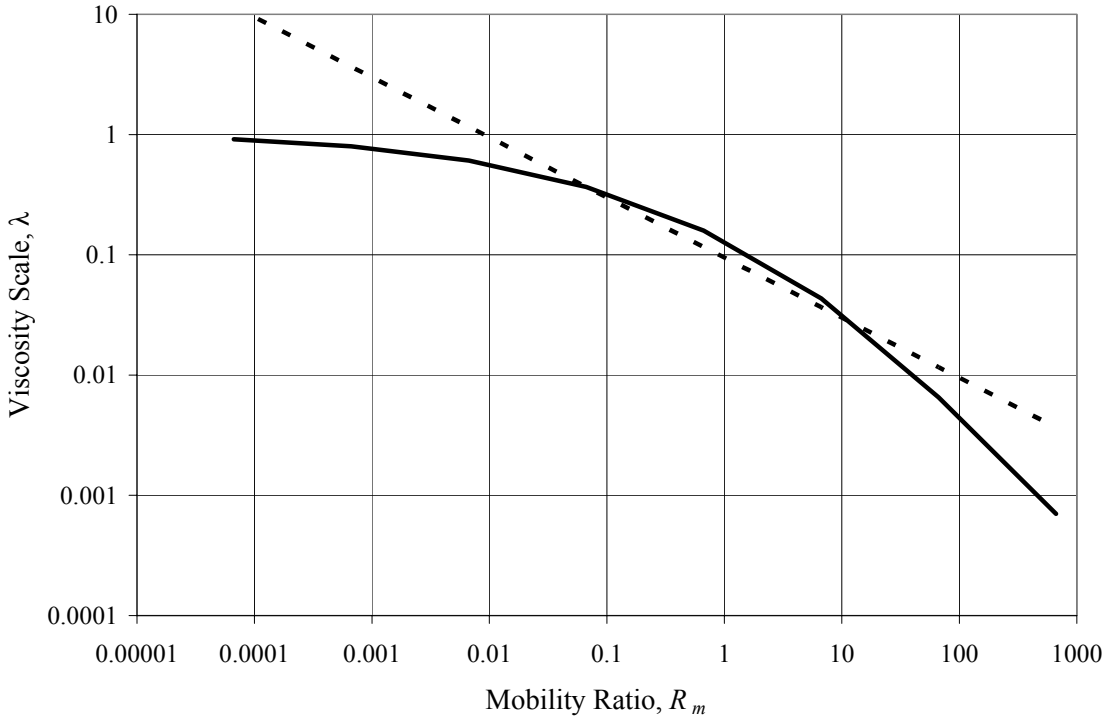

Figure 3 The fit achieved using the square-root of the product of the viscosities. The solid line represents the predictions from Eq. 29 while the dashed line represents the predictions from Eq. 30. The curves have been matched at a viscosity ratio of unity.

Figure 3 shows the results of this analysis. In this plot, the solid line represents the theoretical points predicted by Equation 29. The dashed line, which is straight in this case, represents the predictions from the well known correlation represented by the equation

$$\lambda_N(\mu_w, \mu_{nw}) = \sqrt{\frac{\mu_w}{\mu_{nw}}} \tag{30}$$

These points have been scaled such that the two data sets agree for a viscosity ratio of unity. Figure 3 suggests that Eq. 30 should give reasonable estimates over at least three orders of magnitude of mobility (viscosity) ratio in the region of a mobility ratio of unity.

A more sophisticated model can be obtained by considering a bundle of almost equal radius tubes, hydraulically connected sideways. The tubes are assumed to be almost equal in size because any appreciable size distribution complicates the analysis significantly (see [9]). Some fraction of the tubes, *f,* imbibe the wetting phase and the remaining *1-f* tubes carry non-wetting phase back to the surface (*f* will be the saturation of the wetting phase in the core when the front reaches the end of the sample). There is a capillary

pressure generated at the saturation front of $2\sigma/r$ and a bubble pressure ΔP_c generated at the face of the bundle as the non-wetting phase is produced. At time t let the wetting phase front penetrate a distance L into the tube bundle. This system is shown diagrammatically in Figure 4.

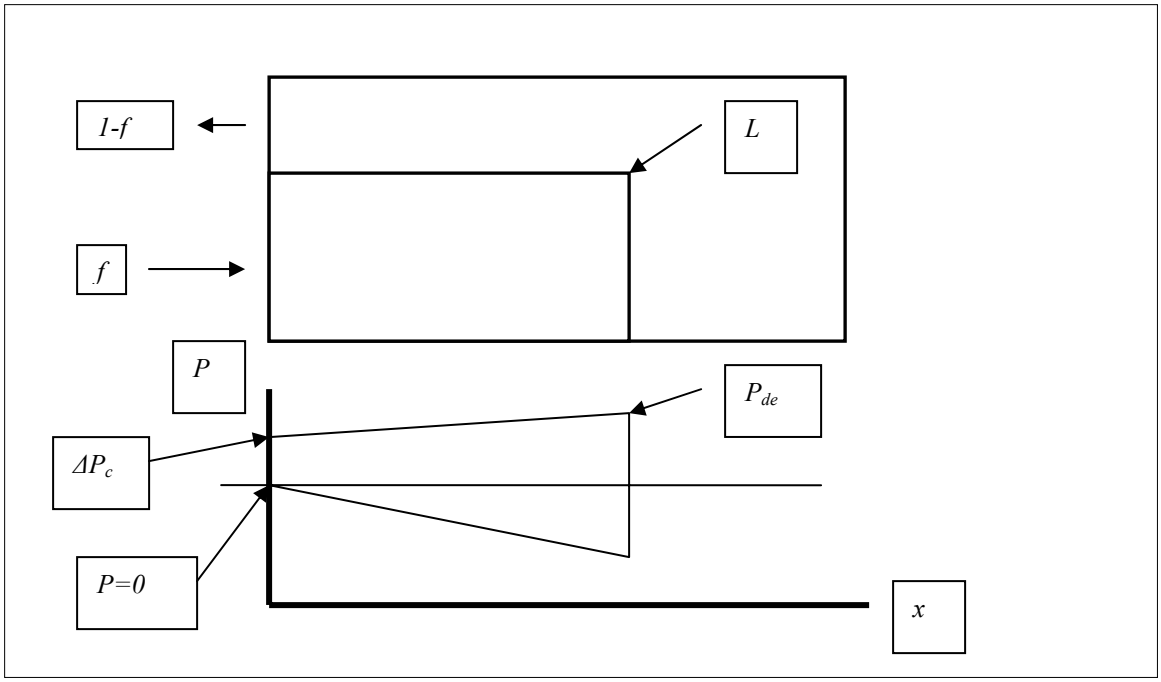

Figure 4 The bundle-of-tubes model for spontaneous countercurrent imbibition.

The pressure profile is also shown in Figure 4. The bubble pressure is assumed to be small, as its only effect is to slow the whole process. We identify P_{de} as the pressure in the dead volume ahead of the front; this is the pressure which drives the wetting phase from the sample. Using the Poiseuille equation, the mean velocity of the wetting phase in one wetting phase filled tube is

$$U_w = \frac{\left(\frac{2\sigma}{r} - P_{de}\right) r^2}{8\mu_w L} \tag{31}$$

while the mean velocity of the non-wetting phase is

$$U_{nw} = \frac{P_{de} r^2}{8\mu_{nw} L} \tag{32}$$

If there are a total of N tubes, then the flow of the wetting phase is $N\pi r^2 f U_w$ and this is equal to the flow of the non-wetting phase $N\pi r^2 (1-f) U_{nw}$. Equating these flows,

$$f\frac{\left(\frac{2\sigma}{r}-P_{de}\right)}{\mu_w}=(1-f)\frac{P_{de}}{\mu_{nw}}=F \tag{33}$$

where F is proportional to the velocity of the front. This is one equation in two unknowns, P_{de} and F. We postulate that P_{de} and F are governed by the condition that F is maximized, that is, we will determine the condition for the fastest moving front. Differentiating the two equations constituting Eq. 33 and setting each to zero, results in the conditions

$$P_{de}=\frac{2\sigma}{r}(1-f) \tag{34}$$

and

$$f=\frac{1}{1+\sqrt{\mu_{nw}/\mu_w}} \tag{35}$$

When μ_{nw} is zero, *f=1*, when μ_w is zero, *f=0*, and when $\mu_{nw}=\mu_w$, *f=0.5*. These values seem reasonable. The velocity of the wetting phase is the rate at which the front advances. Therefore,

$$\frac{dL}{dt}=\frac{2\sigma}{r}\frac{r^2}{8\mu_w L\left(1+\sqrt{\mu_{nw}/\mu_w}\right)} \tag{36}$$

and after performing the integration

$$L^2=\frac{\sigma r}{2\mu_w\left(1+\sqrt{\mu_{nw}/\mu_w}\right)}t \tag{37}$$

The total volume that has flowed at time t is $N\pi r^2 fL$ and

$$Q_w=N\pi r^2\sqrt{\frac{\sigma r}{2\mu_w\left(1+\sqrt{\mu_{nw}/\mu_w}\right)^3}t} \tag{38}$$

The total pore volume is $N\pi r^2 L_b$; therefore the fraction of pore volume recovered is

$$\frac{Q_w}{N\pi r^2 L_b}=\sqrt{\frac{\sigma r}{2\mu_w\left(1+\sqrt{\mu_{nw}/\mu_w}\right)^3 L_b^2}t} \tag{39}$$

This equation implies a viscosity dependence of

$$\lambda_G(\mu_w,\mu_{nw})=\frac{1}{\left(1+\sqrt{\mu_{nw}/\mu_w}\right)^3} \tag{40}$$

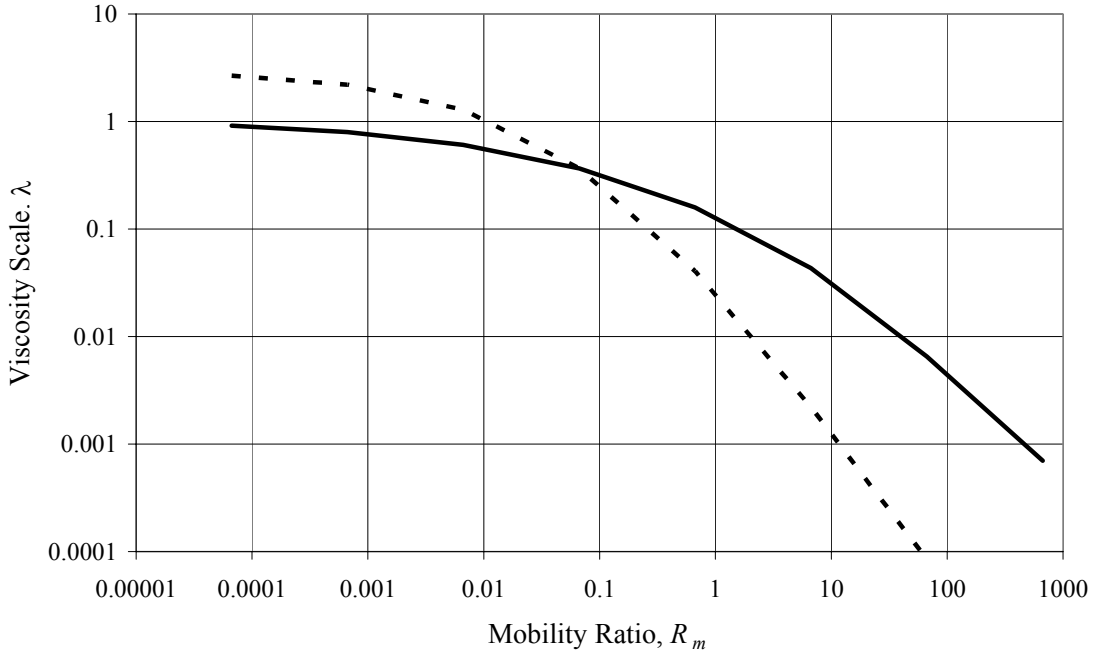

Figure 5 The fit achieved using the tube bundle analysis. The solid line represents the predictions from Eq. 29 while the dashed line represents the predictions from Eq. 39.

Figure 5 shows this dependence with the curve matched at a viscosity ratio of unity. The trends of this curve are roughly correct. However, the limit given by Eq. 20 suggests that it is not the ratio of the viscosity but the mobility ratio that should appear in this equation. Furthermore, when the mobility ratio is large, it should be raised to the power of minus one.

The above discussion suggests a correlation of the form

$$\lambda_D(\mu_w, \mu_{nw}) = \frac{I_{R_m<<1}}{\left(1+\left[\left(I_{R_m<<1}/I_{R_m>>1}\right)R_m\right]^{1/m}\right)^m} \tag{41}$$

where

$$I_{R_m<<1} = \int_{S_{wo}}^{S_{wf}} \frac{2k_{rws}}{(2S_{wo}-S_w)}\,dS_w \tag{42}$$

and

$$I_{R_m>>1} = \int_{S_{wo}}^{S_{wf}} \frac{2k_{rnws}}{(2S_{wo}-S_w)}\,dS_w \tag{43}$$

Figure 6 shows the results for *m=3*. The agreement is very good, particularly in view of the fact that the two lines were not "matched" at any point.

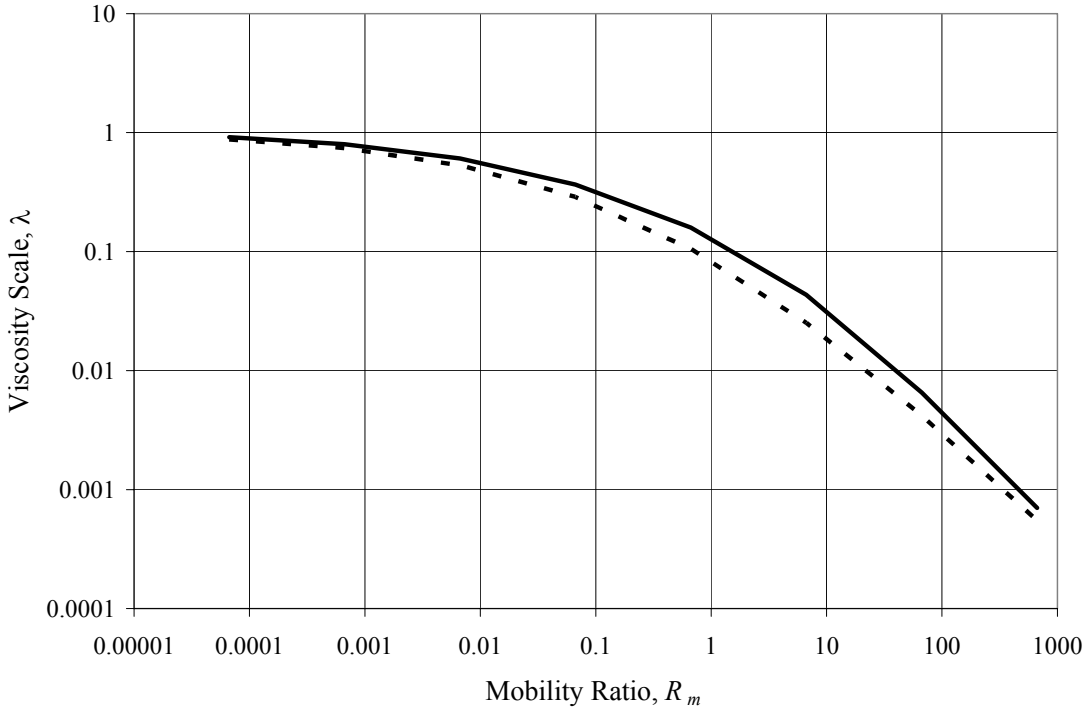

Figure 6. The fit achieved using the tube bundle analysis modified to conform with the limits to Eq. 29. The solid line represents the predictions from Eq. 29 while the dashed line represents the predictions from Eq. 41 with m=3.

The last two integrals depend only on the normalized shapes of the relative permeability curves, and the saturation at the face. If these curves are known, then there is no need for the correlation equation given in Eq. 41 because the scale factor can be calculated directly from Eq. 28 for any mobility ratio. However, Eq. 41 implies that in the absence of information on the relative permeability and capillary pressure curves, the correlation of data for a given rock type should be accommodated by an equation of the form

$$t' = \left(\sqrt{\frac{K}{\phi}} \frac{\sigma C_1}{\mu_w L_b^2} \left(1 + \left[C_2 \frac{\mu_{nw}}{\mu_w} \right]^{1/m} \right)^{-m} \right) t \qquad (42)$$

where C_1, C_2, and m are fitting parameters.

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CONCLUSIONS

The present paper provides a correlation that works for spontaneous imbibition into Berea sandstone for all combinations of fluid viscosities. The analytically derived scaling equation is

$$t' = \left(\sqrt{\frac{K}{\phi}} \frac{\sigma P'_{cp} k_{rwe}}{\mu_w L_b^2} \int_{S_{wo}}^{S_{wf}} \frac{2 k_{rws} k_{rnws}}{\left(k_{rnws} + R_m k_{rws}\right)\left(2S_{wo} - S_w\right)} dS_w \right) t$$

In the absence of information on the relative permeability and capillary pressure curves, this equation can be replaced by the empirical equation

$$t' = \left(\sqrt{\frac{K}{\phi}} \frac{\sigma C_1}{\mu_w L_b^2} \left(1 + \left[C_2 \frac{\mu_{nw}}{\mu_w} \right]^{1/m} \right)^{-m} \right) t$$

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