SCA2003-53: A STRUCTURAL MODEL TO PREDICT TRANSPORT PROPERTIES OF GRANULAR POROUS MEDIA

Guy Chauveteau, IFP, Yuchun Kuang IFP and Marc Fleury, IFP

This paper was prepared for presentation at the International Symposium of the Society of Core Analysts held in Pau, France, 21-24 September 2003

ABSTRACT

The Grain and Pore Throat (GPT) model is a simple but realistic enough pore scale description of statistically homogeneous granular media. The basic assumption is that transport properties are determined only by the characteristics of pore throats and their density. All GPT parameters have a clear physical meaning and are measurable. The theoretical equations derived from this model relate macroscopic transport properties, such as permeability, formation factor and resistivity index, to well-defined geometrical characteristics of flow pathways at pore scale. The analysis shows that all these transport properties vary as $(j - j_1)^a$ where j is porosity (or saturation), j_1 a critical value for which the transport drops to zero and a an exponent which is theoretically derived from transport nature and pore geometrical characteristics. The GPT model predictions cover both diagenetic processes and mechanical stresses effects. This model is also needed to predict 1) the deposition of colloidal particles (minerals suspended in injection water and asphaltenes in heavy oils), 2) the effects of polymer adsorption, 3) the behavior of non-Newtonian fluids in porous media and 4) inertia effects on fluid flow for both grain packs and consolidated media. All GPT model predictions agree with available data.

INTRODUCTION

Only the "Sphere-in-Cell" and the bundle of straight capillaries models are currently used as pore scale geometrical models, because of their simplicity. However the velocity fields derived from these models are too unrealistic to predict reliably most transport properties.
This paper shows that the Grain and Pore Throat model, a concept first proposed some years ago [1], is an efficient tool to predict most transport properties on a clear physical basis. After stating the basic hypotheses at the origin of the model and their experimental confirmation, the validated applications of the GPT model are briefly described.

THE GRAIN AND PORE THROAT MODEL

Basic Considerations at the Origin of the Model

The GPT model concept originates from a very simple observation: the pore-body to pore-throat size ratio is large enough (≥3) in granular media for assuming that the pressure drop is located only in pore throats with a good approximation. In laminar flows, the pressure drop gradient $(\partial p/\partial x)$ along a channel with varying cross-section scales as the power -4 of its diameter as long as the aspect ratio ? of this channel normal to the flow equals1. Obviously, if the cross-section is a slit ? = 8 (film flow), a power -3 must be used.

However, whatever the ρ value is, the $\P p/\P x$ in pore bodies can be neglected. The consequence of this simple analysis is that permeability can be derived only from the characteristics of pore throat and of their density, i.e. their number per cross-section unit area, a parameter directly derived from grain size and porosity (see below Eq.4).

The Permeability Concept According to the GPT Model

Statistically homogeneous media can be modeled as an assemblage of identical GPT cells, such as those described in Fig. 1 and 2, which are hydrodynamically equivalent to the actual geometry. Then the permeability k can be written as :

$$k \propto n\, d_h^4\, l_u / l_h \qquad (1)$$

where d_h and l_h are the hydrodynamic pore throat diameter and length respectively, n the pore throat number per unit cross-section ($n = l_u^{-2}$) and l_u the unit cell dimension, so that:

$$k \mu d_h^4 / l_u l_h \qquad (2)$$

This last scaling law emphasizes that k depends only on three relevant lengths and not on porosity j, except if j is related to one of these lengths for any reason.

For statistically similar porous media (same j and grain shape), l_u/l_h is constant so that:

$$k \propto d_h^4 / l_u^2 \qquad (3)$$

If d_g is the mean grain diameter (here diameter is defined as that of the sphere having the same volume), we have:

$$l_u = [p/6(1-j)]^{1/3} d_g \qquad (4)$$

so that for similar unconsolidated porous media, d_h is proportional to d_g giving:

$$k \mu d_h^4/d_g^2 \mu d_g^2 \qquad (5)$$

For sandstones cemented by mineral overgrowth, either dense as for quartz deposit or "fractal" for pore-lining clay formation, l_u remains constant, so that:

$$k \mu d_h^4 \qquad (6)$$

Relations between Transport Properties and Porosity

An essential characteristic of the pathways through any granular medium is that they are made of restrictions and expansions, currently called "pore throats" and "pore bodies". That means that if all pore sizes are uniformly reduced (for example by mineral deposition or mechanical stresses), pore throats are expected to be closed before pore bodies. In other terms, transport drops to zero while a residual porosity j_1 remains in pore bodies. Then, if A is a characteristic of transport (of fluids or of electric current…),we can write :

$$A \mu (j - j_1)^a \qquad (7)$$

in all cases where a power-law dependence can be justified by the transport physics. A can be permeability, formation factor, resistivity index or diffusion constant and a is determined by both transport nature and the origin of porosity variations. However the equation 7 which is based on a simple analysis of the physics at pore scale is clearly in contradiction with the commonly used Archie and Kozeny-Carman equations: such empirical equations not only must overestimate all transport properties in the low porosity range but the measured power law exponents (m and n in Archie laws) cannot have a physical meaning in contrast with the GPT a values which can be derived from a theoretical analysis at pore scale and thus related to geometrical characteristics.

Determination of the GPT Model Parameters

Typical shapes of grains and pore throats inside granular media are shown in Fig. 1 and 2, together with the three basic GPT characteristic lengths: l_u which is close to d_g as shown by Eq. 2, d_h value which is derived from the permeability reduction R_k due to the deposition of a material (latex or polymer) having a well-known hydrodynamic thickness e_h, by [1]:

$$d_h = 2e_h(1-R_k^{-1/4})^{-1} \quad (8)$$

and l_h which can be calculated from permeability by including the pre-factors in Eq. 2. For a better evaluation of l_h, the pressure drop upstream and downstream of pore throats can be accounted for, leading to a decrease in l_h value of around $0.8d_h$.

However, the l_h/d_h ratio depends only on grain roundness, so that terms l_h is only a minor parameter as compared to the two major parameters d_h and d_g.

Permeability of Unconsolidated Granular Media.

The basic assumptions of the GPT model (Eq.5 and 6) have been validated [1] by carrying out experiments in packs of narrow size distribution spherical and sharp-edged grains. Fig.3 shows that k scales as the power 4 of the two pore throat diameters. For the same d_g at φ = 40 %, sharp-edged grain packs are characterized by d_g/d_h = 4.5 and l_h/d_h = 2/3. However, in spherical grain packs d_h is 1.25 times larger and l_h/d_h 2.4 larger. Another practically important result is that d_{Hg} is always much larger than d_h, even if pore size distribution is narrow, because d_h has an hydrodynamic and thus directional character.

Transport Properties Alterations Induced by Cementation and Fluid Saturation.

The permeability of granular rocks cemented by a non-porous mineral overgrowth with a thickness e_d during diagenesis is decreased as a result of the pore throat size reduction by the same value $2e_d$. When $2e_d$ becomes equal to d_h, the permeability k drops to zero and porosity drops down to j_1. Since the specific surface area remains almost constant during this process, the pore volume decreases linearly with e_d, so we can write:

$$k = k_0[(f-f_1)/(f_0-f_1)]^4 = cd_{g0}^2[(f-f_1)/(f_0-f_1)]^4 \quad (9)$$

where f_0 and d_{g0} are initial porosity and grain diameter respectively before cementation and c a numerical coefficient depending only on grain shape.

Experimental data [1] collected from literature for Fontainebleau sandstones are reported in log-log coordinates in Fig. 4 as suggested by Equation (9) with a residual porosity j_1 = 2% . The best fit gives an exponent equal to 3.98 in very good agreement with Eq. (9).

The Formation Factor GPT exponent (a in Eq. 7) is determined by the pore throat cross-section aspect ratio ρ. The bound values are a = -2 for ρ = 1 (square or circle) and a = -1 for $\rho = \infty$ (slits). For ρ = 2 (elongated rectangle or triangle), a value consistent with the shear rate factor determined by polymer flow (2) and consistent with thin slice observation, we expect a = -1.5 in excellent agreement with the experimental results shown in Fig.6.

The Resistivity Index GPT exponent is also related to the cross-section of the water pathways. When there is continuity of both phases, the aspect ratio of water pathways in pore throats is always large whatever the wettability is. In any cases, the GPT RI exponent a must drop down to values close to -1, as if data are analyzed according to the formalism

suggested by Eq. 7. For experimental validation, see data obtained both on water-wet and oil-wet Fontainebleau sandstones (3) as well as on field sandstones containing pore lining clays (4). For double-porosity media, such as carbonates, f_1 must be defined as the critical porosity separating micro from macro-porosity in high water saturation range.

Permeability Alterations Induced by Mechanical Stresses.

The permeability alterations of rocks subjected to mechanical stresses can also be related to porosity by the formalism suggested by Eq.7. Indeed as long as the strains are isotropic and localized in contact zones, an analysis based on the GPT concept leads to write:

$$k_a = (f - f_1)^{4\lambda/3} \qquad (11)$$

where λ is equal to the pore body-to-pore throat diameter ratio d_p/d_h as long as there is no structure deformation during compaction, as expected for strongly-cemented sandstones. By contrast, if there is a structure deformation, pore bodies are reduced more than pore throats giving a smaller value of ?. This was observed for a weakly consolidated Berea sandstone (Fig.7), while higher values of ?, equal to d_p/d_h, were obtained for more consolidated Berea sandstones.

Inertia Effects in Flows through Granular Media.

In the historical definition of the Reynolds number $Re=vd_g?/\mu$, v *and* d_g are Darcy's velocity and grain diameter respectively. However, this definition is not satisfying because *Re* should be defined where the inertia to viscous forces ratio is maximum, i.e. in pore throats. That becomes easy by using the GPT model. Indeed, for *Re* values higher than1, a dimensionless linear relation between the viscous friction factor lRe and *Re* has been established in scientific literature since a long time:

$$lRe = a\,(1 + b\,Re) \qquad (12)$$

where the parameter b is dimensionless and characteristic of pore throat shape, which does not vary significantly with cementation. But in chemical engineering and oil industry, the Forchheimer equation is still the more commonly used:

$$\P\, p/\P\, x = a\, v + b r v^2 \qquad (13)$$

despite the absence of physical meaning of the dimensional "inertial" Forchheimer. By identifying Equations (11) and (12), we obtain $b = b\, d_g/k$ and then, by using Eq. 5 and 6 :

$$b = b\, k^{-1} \quad \text{and} \quad b = b\, k^{-1/2} \qquad (14)$$

for consolidated and unconsolidated granular media respectively. By this way, the data plotted in Figure 7, collected in Ref.(4) but not yet explained, can be easily interpreted.

Rheology of Non-Newtonian Fluids in Porous Media

The maximum wall shear rate in pore throats was determined from rigid polymer solution flow through sharp-edged granular packs (2). It is 2.5 times larger than that predicted by the bundle of straight capillary model but 2 times smaller by the GPT model if pore throats aspect ratio is 1. The conclusion is that the pore throats aspect ratio is around 2, in agreement with both formation factor GPT exponent (-3/2) and thin section observations.

Permeability Reduction by Deposition of Polymer and of Colloidal Particles .
The simulator PARIS (5) was developed to predict both the kinetics of colloidal particle deposition and the corresponding reduction in permeability. With a well-defined system (a medium modeled by GPT and attractive interactions between pores and particles), PARIS predictions agrees with experiments without any fitting parameter (Fig. 8 and Ref.6).

CONCLUSIONS AND PERSPECTIVES

This short paper shows that the Grain and Pore Throat model, a simple but realistic enough description of the flow cell at pore scale, is an efficient tool to derive numerous transport laws from simple geometrical characteristics of flow pathways. This model points out that transport properties must be related to the difference between porosity (or saturation) and the critical one below which there is no transport, but never directly to porosity (or saturation) as implicitely assumed when Archie and Kozeny-Carman equations are used. In addition the GPT concept makes easy the interpretation of yet non-explained data on a sound physical basis. More detailed analysis and discussion of the GPT model capabilities, including the prediction of multiphase flows as well as of the effects of pore size distribution and pore roughness, will be reported in a forthcoming more extended paper.

REFERENCES

1.Chauveteau G., Nabzar L., El Attar Y. and Jacquin C. “Pore Structure and Hydrodynamics in Sandstones”, SCA 9607, International Symposium of the Society of Core Analysts, Montpellier, France, 8-10 September 1996.
2. G. Chauveteau and K. Sorbie, "Mobility Control by Polymers" in «Basic Concepts in Oil Recovery Processes», Chapter II, Ed. Marc Bavière, in Critical Reports on Applied Chemistry, S.C.I., Elsevier Science Publishers.
3. Fleury M. and Liu F., "Frequency Effect on Resistivity Index Curves using a New Method" 41st Annual SPWLA Symposium, Dallas, June 4-7, 2000.
4. Durand C., Brosse E. and Cerepi A.,“Effects of Pore-Lining Chlorite on Petrophysical Properties of Low -Resistivity Sandstone Reservoirs, June 2001, SPE Reservoir Evaluation and Engineering.
5. Lombard, J.M., Longeron D. and Kalaydjian, F. "Influence of Connate water and Condensate Saturation on Inertial Effects in Condensate Fields," SPE 56485, SPE Annual Technical Conference and Exhibition, Houston, Texas, Oct. 6-9, 1999..
6. Chauveteau G., Nabzar L. and Coste J. P.: “Physics and Modeling of Permeability Damage Induced by Particle Deposition”, SPE 39463, SPE International Symposium on Formation Damage Control, Lafayette, Louisiana, 18-19 February 1998.

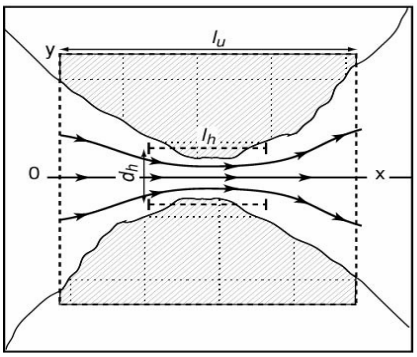


Fig.1 - Modeling actual pore geometry: the 3 GPT characteristic lengths.

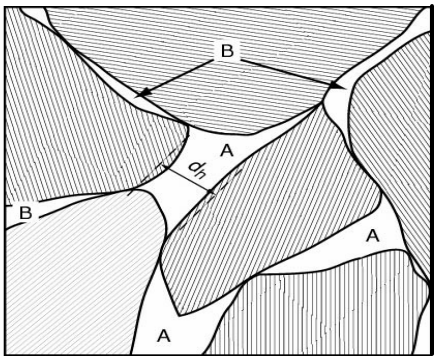


Fig. 2 - Grain and pore throat cross-section showing an aspect ratio around 2.

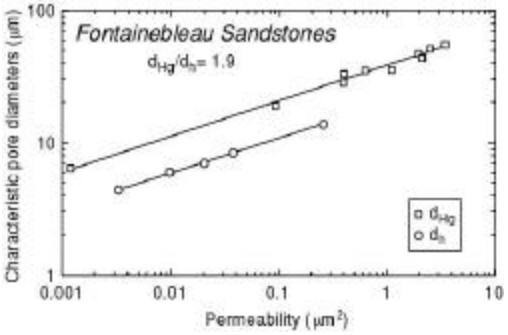


Fig.3 - Permeability varies as the power 4 of the pore throat hydrodynamic diameter (GPT model)

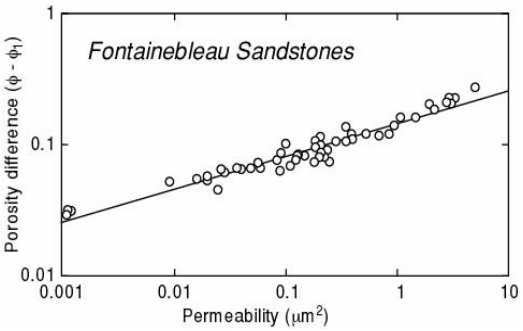


Fig.4 - Permeability varies as the power 4 of porosity difference(GPT prediction)

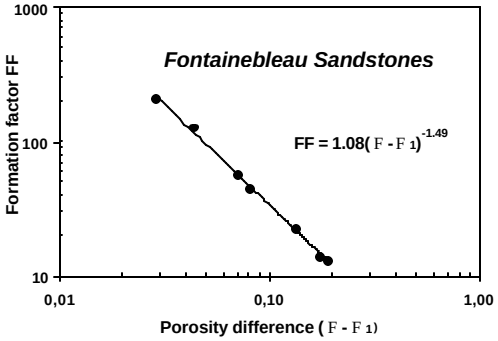


Fig.5 - Formation factor varies as the power -1.5 of porosity difference (GPT model)

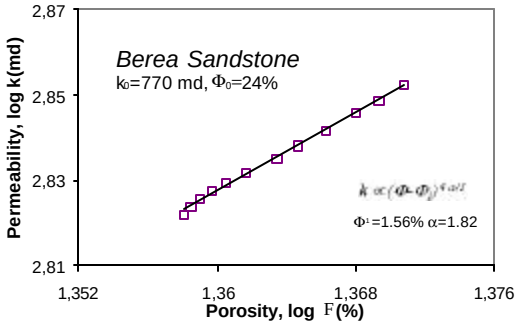


Fig.6 - Permeability variations under isotropic stresses reveals a structure change (small α value)

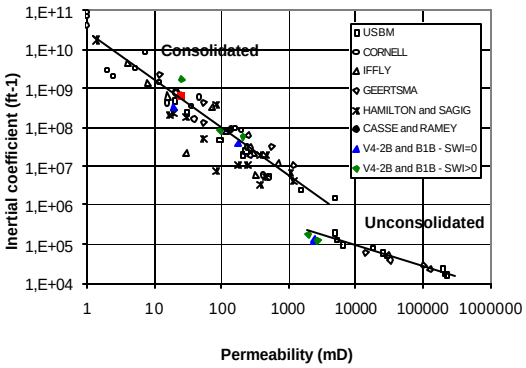


Fig.7 - The Forccheimer inertial coefficient ß varies as predicted by GPT model

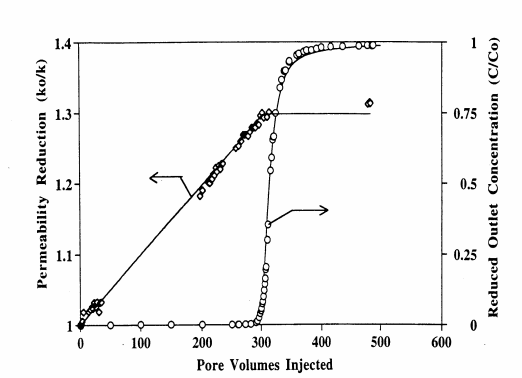


Fig.8-Particle deposition predicted from PARIS simulator without fitting parameter (GPT model)