SPE 84546



Mixed-Wet Drainage and Imbibition Relative Permeabilities From the Diffuse Interface Theory

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This paper was prepared for presentation at the SPE Annual Technical Conference and Exhibition held in Denver, Colorado, 5–8 October 2003.

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Abstract

The Diffuse-Interface model for two-phase flow in porous media is applied to the invesigation of relative permeabilities in unsteady-state flows. It is shown that relative permeabilities have a transient regime where they can be significantly larger than 1. It is also shown that, when normalized by their endpoint values, relative permeabilities can be calculated from formulas involving thermodynamical and capillary pressure functions. The end-point relative permeabilities are linearly related to the fluid velocity, but the coefficients seem to be flow dependent.

Introduction

A new model for two-phase flow in porous media has recently been presented in two papers.^{1,2} We shall here call it "the DImodel." It is described in detail in the next section. Its main characteristic is that it is based on the diffuse interface model of fluid mechanics, where the two phases are manifestations of one and the same fluid, the transition from one phase to the other being taken care of by an equation of state of the van der Waals type. There is just one set of balance equations, in contradistinction to the traditional two-continuum model of twophase flow, where each phase is considered as a separate fluid.

The model is at present restricted to fluids of one chemical component and is thus ideally suited to a steam-water situation. It could also be applied to oil-gas flows in situations where looking at the oil and the gas as the two phases of one and the same flud is considered to be a good approximation. Most of the work done with the DI-model until the present has been directed towards establishing a "dialogue" with the traditional two-continuum model for two-phase flow.

On the one hand, the DI-model does not use relative permeabilities and is thus capable of making statements about these quantities. In fact, expressions for relative permeabilities have been proposed,² giving these as expressions involving the thermodynamical properties of the fluid, the wetting properties of the rock (as embodied in the capillary pressure), and some parameters.

On the other hand, the traditional two-continuum model has provided the DI-model with the means to incorporate experimentally obtained information about wetting, through capillary pressure versus saturation correlations.²

In the present paper we continue the dialogue between the two models by introducing, in the DI-model, information leading to a mixed-wet rock. We then use the DI-model with this information to calculate relative permeabilities in two typically unsteady-state situations. These are a drainage and an imbibition, starting from an initial state where capillary and gravitational forces do not balance and ending in a state where they do.

Description of the model

The model considers a fluid consisting of only one chemical component, but capable of undergoing a phase transition. At the pore level, the two phases (liquid and vapor) are treated as a single fluid with variable density, according to the diffuse-interface model.^{3,4} This is called the *f*-fluid,¹ to distinguish it from the Σ -fluid, which is introduced as a means of accounting for the wetting properties of the rock. The Σ -fluid is a surface fluid with zero velocity, covering the rock surface.

The mass, momentum, and energy balance equations of the f and Σ fluids are upscaled from the pore level to the macroscopic level (also called the Darcy level), by using the Marle⁵ averaging technique. The upscaled equations describing twophase flow in porous media, involve essentially one variable, R, the upscaled f-fluid density. The main characteristics of the model are outlined below.

Thermodynamics The temperature is assumed to be a constant, different from the critical temperature. The fluid consists of one chemical component, with an equation of state of the van der Waals type, i.e., including the possibility of a phase transition. The phases are called liquid and vapor. The equation of state is of the general form

$$P^{b} = R^{2} \frac{d}{dR} \left(\frac{\Psi^{b}}{R} \right), \quad \dots \quad (1)$$

where Ψ^{b} is the intrinsic Helmholtz free energy of the bulk fluid (*bulk* referring to that part of the fluid where the gradient of *R* is negligible). It is of the form

where \overline{M} and \overline{P} are the chemical potential and the pressure of the fluid at equilibrium. It is easy to see that, when assuming uniform temperature as we do here, the value of \overline{M} is irrelevant and that we can put

The W(R)-function has just two minima, occuring at two distinct values, $R = R_v$ and $R = R_l$, such that $W(R_v) = W(R_l) = 0$. R_v and R_l are the densities of the pure vapor and liquid phases.

Depending on the point of view, one can either calculate W from a known equation of state by solving Eq. 1,²

$$W(R) = P^{b}(R_{v})\left(1 - \frac{R}{R_{v}}\right) + R \int_{R_{v}}^{R} P^{b}(R') \frac{dR'}{R'^{2}}, \qquad (4)$$

or postulate a *W*-function having the required essential properties. An often used^{1,2} *W*-function, which we shall also use in the present paper is

$$W(R) = (P_c/R_c^4)(R - R_v)^2(R - R_l)^2, \quad . \quad . \quad . \quad (5)$$

where P_c and $R_c \equiv (R_l + R_v)/2$ are the pressure and density at the critical point.

If, at the pore scale, the wetting is incomplete (i.e., the wetting angle is between 0 and 180°), then to a good approximation, the only remaining trace of the Σ -fluid at the Darcy level is a function I(R) of the upscaled f-fluid density R. This function is a Helmholtz free energy describing the interaction of the f and Σ fluids, which can also be seen as a free energy describing the wetting properties of the rock. It can be shown² that

where P^c is the empirical capillary pressure function. This has been termed the "incomplete wetting approximation",² and is the approximation we are assuming in the present paper.

The determination of the *I*-function is described in a later subsection (see especially Eqs. 21-27).

Flow equations The central equation describing fluid flow at the Darcy level is the classical mass balance equation

$$\frac{\partial R}{\partial t} + \nabla \cdot (R\mathbf{V}) = 0. \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (7)$$

Here V is the Darcy-level velocity:

$$\mathbf{V} = -\frac{KR}{\phi\eta} \nabla \left(\frac{d\Psi}{dR} - \Lambda \nabla^2 R + G \right), \quad \dots \quad \dots \quad (8)$$

where

$$\Psi(R) = \Psi^{b}(R) + I(R) \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (9)$$

is the bulk fluid total Helmholtz free energy, and

is the gravitational potential energy. (**g** is vertical, points upwards, and $|\mathbf{g}| = g$ is equal to the acceleration due to gravity.) *K* is the absolute permeability, Λ a constant, ϕ the porosity, and η the fluid viscosity. The formula for η that has been used in the previous publications^{1,2} is a modified form of a formula proposed by Arrhenius:

where η_l and η_v are the viscosities of the pure liquid and vapor phases, and S_l and S_v are the liquid and vapor saturations. These are interpretated as follows:

$$S_l = \frac{R - R_v}{R_l - R_v}, \qquad S_v = \frac{R_l - R}{R_l - R_v}.$$
 (12)

Note that the flow equation, obtained by combining Eqs. 7 and 8,

$$\frac{\partial R}{\partial t} = \nabla \cdot \left(\frac{KR^2}{\phi \eta} \nabla \left(\frac{d\Psi}{dR} - \Lambda \nabla^2 R + G \right) \right), \quad . \quad (13)$$

is (when G is identically zero) the Cahn-Hilliard equation.

It is, as a rule, convenient for the numerical solution, to transform this equation into two coupled equations of the second order in the space derivatives. This is done by setting

Eq. 13 then becomes

ı

$$\frac{\partial u_1}{\partial t} + \nabla \cdot \left[-\frac{K u_1^2}{\phi \eta} \nabla u_2 \right] = 0, \quad \dots \quad \dots \quad \dots \quad (15a)$$
$$\nabla \cdot [\Lambda \nabla u_1] = \frac{d\Psi}{dR} - u_2 + G. \quad \dots \quad (15b)$$

Boundary Conditions A well-posed problem for Eq. 13 (or, alternatively, for Eqs. 15) is obtained by supplying an initial condition $R(\mathbf{x}, 0) = F(\mathbf{x})$, and a boundary condition. The latter must be one of the following:

(a)
$$H = 0, \dots, \dots, \dots, \dots, \dots$$
 (16a)
(b) $u_1 = \alpha_1$

$$(e) \begin{cases} \mathbf{n} \cdot \mathbf{v} u_1 = G_1 \\ \mathbf{n} \cdot \nabla u_2 = G_2, \end{cases} \quad (16e)$$

where: **n** is the unit normal to the boundary pointing out; *H* is a function of u_1 and u_2 ; α_1 and α_2 are two constants; and G_1 and G_2 are functions of **x**, and of u_1 and u_2 and their derivatives.

A boundary condition on u_1 in the present model is equivalent, because of the equation of state, to a condition on the pressure. A boundary condition on $\mathbf{n} \cdot \nabla u_2$ is a condition on the velocity. The boundary conditions involving $\mathbf{n} \cdot \nabla u_1$ are new to this model and have been discussed in a previous publication.¹ The boundary conditions which are relevant to reservoir studies are thus the ones labelled (c) and (e) above. For one-dimensional studies, as the ones presented in the present paper, it is natural to use $G_1 = 0.^1$

Wetting and the *I*-function In summary, and provided the function I(R) is known, the density and velocity of a biphasic flow in a porous medium where

- the porous medium is characterized by constants K, ϕ , Λ ,
- the fluid has one chemical component and is characterized by constants R_l , R_v , P_c , η_l , η_v , and by a Helmholtz free energy given by Eqs. 2, 3, and 5 (alternatively Eq. 4 and a known equation of state),

can be obtained by solving Eqs. 15 (with definining Eqs. 9–12) with an initial condition on $R(\mathbf{x}, t)$ and boundary conditions of type (c) or (e) above.

Relative permeabilities do not exist in the model. The wetting properties of the rock are entered by means of the *I*-function.

The determination of the *I*-function now follows. The principle behind the determination of this function has been stated and illustrated in a previous publication,² for the case of a vaporwet rock. It is applied here to the case of a rock whose wetting properties are a mixture of vapor and liquid-wetting. The determination of *I* is based on Eq. 6 relating it to capillary pressure. This function is here assumed to be of the form⁶

where

Here S_l , S_{lr} , and S_{vr} are the liquid saturation, the residual liquid saturation, and the residual vapor saturation, respectively. The constants C_l and C_v are usually referred to as entry pressures and $1/a_l$ and $1/a_v$ as pore size distributions. It will be shown that the DI-model imposes some conditions linking the *C*'s, the *a*'s, and the residual saturations.

We now use Eqs. 12 to get the saturations in the above formulas: S_l is given directly by Eq. 12 (left), and

$$S_{lr} = \frac{R_v^* - R_v}{R_l - R_v} \iff R_v^* = R_v + (R_l - R_v)S_{lr}, \quad (19a)$$

$$S_{vr} = \frac{R_l - R_l^r}{R_l - R_v} \iff R_l^* = R_l - (R_l - R_v)S_{vr}.$$
 (19b)

Eqs. 18 become

$$P_l^c = C_l \left(\frac{R_l - R_v^*}{R - R_v^*}\right)^{a_l}, \qquad \dots \qquad \dots \qquad \dots \qquad \dots \qquad (20a)$$
$$\left(\frac{R^* - R_v}{R}\right)^{a_v}$$

The integration of Eq. 6, using Eq. 17, gives

where

$$I_{a}(R) = R \int \frac{P_{a}^{c}}{R^{2}} dR - \alpha_{a} + \beta_{a}R \qquad (a = l, v). \quad . \quad (22)$$

The bounds and constants of integration will be determined, together with constants C_l and C_v , in such a way that W + I as a function of R is a distorted version of W in the following sense: it must have two minima at $R = R_v^*$ and $R = R_l^*$, of value zero.

It should be pointed out that the fact that it is possible to determine the constants in this manner, with empirical functions like those in Eqs. 18, shows that the present model is consistent with experiments.

It should also be pointed out that the determination of I for all relevant fluid densities (say all R > 0) is not possible with the empirical functions just mentioned. The reason is that these functions are defined for $S_{lr} < S_l < 1 - S_{vr}$, so that Eqs. 20 are only valid for $R_v^* < R < R_l^*$. The determination of Iover a broader interval, which is necessary for the integration of Eq. 13, must be done with some physical principle in mind.

We first obtain I_l and I_v for $R_v^* < R < R_l^*$.

 I_l is that part of the *I*-function which is due to the "liquidwet part" of the capillary pressure function. We impose the condition that $W + I_l$ should be "as much as possible equal to W" for *R* in the neighborhood of R_l^* :

$$I_l(R_l^*) = 0, \quad I_l'(R_l^*) = 0$$

(where the prime denotes derivation). Using these conditions to determine the constants in Eq. 22 with a = l one easily gets

$$I_{l}(R) = C_{l} J_{l}(R), \qquad \dots \qquad \dots \qquad \dots \qquad \dots \qquad \dots \qquad (23a)$$
$$J_{l}(R) = \left(\frac{R_{l} - R_{v}^{*}}{R_{l}^{*} - R_{v}^{*}}\right)^{a_{l}} \left(1 - \frac{R}{R_{l}^{*}}\right)$$
$$-R \int_{R}^{R_{l}^{*}} \left(\frac{R_{l} - R_{v}^{*}}{R' - R_{v}^{*}}\right)^{a_{l}} \frac{dR'}{R'^{2}}. \qquad \dots \qquad (23b)$$

 I_v is determined in the same way: $W + I_v$ should be "as much as possible equal to W" for R in the neighborhood of R_v^* :

$$I_v(R_v^*) = 0, \quad I'_v(R_v^*) = 0.$$

One gets

$$I_{v}(R) = C_{v}J_{v}(R), \quad ... \quad \quad ... \quad ... \quad ... \quad ... \quad ... \quad ... \quad$$

$$+R \int_{R_v^*}^R \left(\frac{R_l^* - R_v}{R_l^* - R'}\right)^{a_v} \frac{dR'}{R'^2}.$$
 (24b)

It turns out that J_l and J_v are monotonic functions, respectively decreasing and increasing, for $R_v^* < R < R_l^*$ (see Fig. 1). It is then easy to obtain a function W + I having two minima of value zero at $R = R_v^*$ and $R = R_l^*$. It suffices to put

$$C_{l} = -\frac{W(R_{v}^{*})}{J_{l}(R_{v}^{*})}, \qquad C_{v} = -\frac{W(R_{l}^{*})}{J_{v}(R_{l}^{*})}. \qquad (25)$$

The W+I-function is shown in **Fig. 2**, together with W, $-C_v J_v$, and $-C_l J_l$.



Figure 1: Functions $J_l(R)$ and $J_v(R)$ for $R_v^* < R < R_l^*$.

It will be noted that, to obtain a W + I-function with the required characteristics, we have had to impose some conditions linking the C_l and C_v to some other central parameterts in the problem. We see in fact that

$$C_l = C_l(R_l, R_v, S_{lr}, S_{vr}, a_l),$$

$$C_v = C_v(R_l, R_v, S_{lr}, S_{vr}, a_v).$$



Figure 2: Functions W(R) and W(R) + I(R).

A number of additional problems must be solved before one can tackle the numerical integration of Eqs. 15. The first one is the extension of W + I outside the interval $[R_v^*, R_l^*]$. Fig. **2** illustrates another problem: the extended W + I-function will in general not be differentiable at its minima and a number of standard numerical solvers find this unacceptable.* Eqs. 23b and 24b point out the third problem: if one of the parameters a_l , a_v is chosen larger than or equal to 1, then one of the denominators in Eqs. 25 would diverge, and the corresponding I_l or I_v -function would collapse to zero for all R. If both parameters are chosen larger than or equal to 1, then the entire I-function would be identically zero.

A way to solve the last two problems is to smooth out the *I*-function, or rather its two components I_l and I_v as shown on **Fig. 3**. We define a function J_l^s as follows:

$$J_{l}^{s} = \begin{cases} j_{l}(R), & R_{v}^{*} \leq R < R_{v}^{*} + 2\epsilon \\ J_{l}|_{R_{v}^{*} + \epsilon} & R_{v}^{*} + 2\epsilon \leq R \leq R_{l}^{*}, \end{cases}$$
(26)

where $J_l|_{R_v^*+\epsilon}$ is J_l with R_v^* replaced by $R_v^* + \epsilon$. A typical value for ϵ , or rather for its dimensionless counterpart $\tilde{\epsilon}$ (see the Nomenclature section) is 0.01. (For the sake of readability, the value of ϵ in Fig. **3** is greatly exagerated.) $j_l(R)$ is a second degree polynomial in R which is such that J_l^s and its derivative are continuous at $R = R_v^* + 2\epsilon$: it can easily be seen that j_l has just one remaining degree of freedom. A function J_v^s is defined in a similar manner, with a polynomial $j_v(R)$. We now define a smoothed *I*-function by

and determine the constants C_l^s , C_v^s , and the two degrees of freedom in j_l and j_v , with the four conditions

$$W(R_a^*) + I^s(R_a^*) = 0$$

W'(R_a^*) + I^{s'}(R_a^*) = 0 (a = l, v)

^{*}We have used the FEMLAB (registered trademark of COMSOL AB) solver, which uses the finite element method.



Figure 3: The functions W(R) and the two parts of function I^s (see Eq. 27). The broken lines represent second degree polynomials in R (see Eq. 26. For clarity, the value of ϵ is chosen unrealistically large.

(the prime denotes derivation). The calculations are elementary and are not given here.

We now look at the problem of extending the I^s -function, to the left of R_v^* and to the right of R_l^* . We denote this function by I^{se} . The extension is done as follows. To the left of R_v^* we take $W + I^{se}$ to be equal to W, translated by the amount $R_v^* - R_v$: this is the gas phase region and we expect it to behave approximately as a pure gas. To the right of R_l^* we want the liquid to behave as a nearly incompressible fluid and we impose therefore that

$$W(R) + I^{se}(R) = \frac{P_c}{R_c^2} \frac{(R - R_l^*)^2}{4\tilde{\epsilon}},$$

where $\tilde{\epsilon}$ is the small parameter used previously. The function $W(R) + I^{se}(R)$ is shown in **Fig. 4**. This function is needed for the numerical solution of the differential equations of the model.

Relative permeabilities As mentioned in the Introduction expressions for the relative permeabilities have been found, ² involving the thermodynamical properties of the fluid, the wetting properties of the rock (as embodied in the capillary pressure), and some parameters. To write these expressions we need the derivative of the capillary pressure with respect to R. The smoothing of the *I*-function performed above implies that the capillary pressure function is slighty modified and must be recalculated as (see Eq. 17)

$$\frac{dP^{cs}}{dR} = \frac{dP_l^{cs}}{dR} + \frac{dP_v^{cs}}{dR}, \quad \dots \quad \dots \quad \dots \quad \dots \quad (28)$$



Figure 4: Functions W(R) and $W(R) + I^{se}(R)$. The later results from the smoothing shown in Fig. 3, and from an extension outside the interval $[\tilde{R}_v^*, \tilde{R}_l^*]$.

where, according to Eqs. 27 and 6,

$$\frac{dP_l^{cs}}{dR} = R \frac{d^2}{dR^2} C_l^s J_l^s(R), \qquad \dots \qquad \dots \qquad \dots \qquad (29a)$$

$$\frac{dP_v^{cs}}{dR} = R \frac{d^2}{dR^2} C_v^s J_v^s(R). \qquad (29b)$$

The relative permeabilities are then given by 2

$$k_{rl} = \frac{S_l^*}{1 + \gamma (1 - S_l^*) / (\tilde{g} \, \tilde{R}_l^*)}, \qquad \dots \qquad \dots \qquad (30a)$$
$$1 - S^*$$

$$k_{rv} = \frac{1 - S_l}{1 + \gamma(S_l^*) / (\tilde{g}\tilde{R}_v^*)}, \quad \dots \quad \dots \quad \dots \quad \dots \quad (30b)$$

where

and γ is the generalization to the mixed-wet case of the one given previously:²

$$\gamma(S_l^*) = -\left[\frac{d\tilde{P}_v^{cs}}{d\tilde{R}}\sqrt{\frac{2}{\tilde{\Lambda}}[\tilde{W}(\tilde{R}) + \tilde{I}^s(\tilde{R})]}\right]_{\tilde{R} = \tilde{R}(S_l^*)} - \left[\frac{d\tilde{P}_l^{cs}}{d\tilde{R}}\sqrt{\frac{2}{\tilde{\Lambda}}[\tilde{W}(\tilde{R}) + \tilde{I}^s(\tilde{R})]}\right]_{\tilde{R} = \tilde{R}(1-S_l^*)}, (32)$$

where

and the tilded quantities are dimensionless: see the Nomenclature section.

Relative permeabilities in unsteady-state flows

As pointed out above, the DI-model does not use the relative permeability concept and is therefore well-suited to investigate the subject. Such investigations have already been carried out in two previous publications.^{1,2}

In the first reference,¹ a truly steady state process was set up: a one-dimensional ganglionic flow of two phases. In the framework of that process it was shown that the generalized two-continuum model with viscous coupling is compatible with the present model.

In the second reference,² a nearly steady state situation was studied, with drainage of a vapor wet rock. The above theoretical expressions for relative permeabilities were found in connection with that study and it was shown that they are in good agreement with what one expects.

In the present paper we present results from two unsteadystate flows, a drainage and an imbibition. The calculations have been performed in one space dimension and the results are presented in terms of dimensionless quantities. These are defined in the Nomenclature section. For future reference we give here the main equations, written in these quantities.

The dimensionless velocity, and the dimensionless momentum (to be used in a later section) are

$$\tilde{V} = -\frac{\tilde{u}_1}{\tilde{\eta}(\tilde{u}_1)} \frac{\partial \tilde{u}_2}{\partial \tilde{x}}, \qquad \tilde{\Gamma} = \tilde{u}_1 \tilde{V}. \qquad \dots \qquad \dots \qquad (34)$$

The flow equations (see Eqs. 15) are

$$\frac{\partial \tilde{u}_1}{\partial \tilde{t}} + \frac{\partial}{\partial \tilde{x}} \left[-\frac{\tilde{u}_1^2}{\tilde{\eta}} \frac{\partial \tilde{u}_2}{\partial \tilde{x}} \right] = 0, \quad \dots \quad \dots \quad (35a)$$

$$\frac{\partial}{\partial \tilde{t}} \left[\tilde{t}_1 \frac{\partial \tilde{u}_1}{\partial \tilde{t}_1} \right] = d\tilde{\Psi} \quad \tilde{t}_1 = \tilde{t}_1 \tilde{t}_2 \tilde{t}_1 \tilde{t}_2 \tilde{t}_2 \tilde{t}_1 \tilde{t}_2 \tilde{t}_$$

$$\frac{\partial}{\partial \tilde{x}} \left[\tilde{\Lambda} \frac{\partial u_1}{\partial \tilde{x}} \right] = \frac{d\Psi}{d\tilde{R}} - \tilde{u}_2 + \tilde{g}\tilde{x}. \quad . \quad . \quad (35b)$$

where $\tilde{u}_1 = \tilde{R}$. The unit of length is such that $0 \le \tilde{x} \le 1$ and we have taken the *x*-axis to point downwards so that *g* is negative.

The constants have been chosen as follows:

$$\tilde{R}_l = 1.6, \quad \tilde{R}_v = 0.4, \quad S_{lr} = 0.4, \quad S_{vr} = 0.1, \quad (36)$$

 $\tilde{\Lambda} = 0.01, \quad \tilde{g} = -0.5, \quad \eta_v / \eta_l = 0.1.$

Note that this leads to

(see Fig. 4). These are now the densities of the liquid and vapor phases allowed by the wetting properties of the rock.

The two remaining free parameters, a_l and a_v , have been given the following values:

$$a_l = 0.50, \quad a_v = 1.15, \quad \text{for drainage,} \\ a_l = 1.00, \quad a_v = 1.40, \quad \text{for imbibition.}$$
 (38)

The resulting capillary pressure curves are shown in Fig. 5.



Figure 5: The capillary pressure curves used for drainage (upper curve) and imbibition (lower curve).

Relative permeabilities for drainage We consider drainage flow in a one-dimensional porous medium of mixed wetting properties. We want the flow to be manifestly unsteady-state at all times. This we achieve by arranging the drainage to take place at diminishing velocities, leading to a static equilibrium. Specifically, we consider a medium which is initially filled mostly with liquid underlying a thin later of vapor, with gravity forces larger that capillary forces so that drainage takes place, the amount of vapor increasing at the top and the amount of liquid decreasing at the bottom. We fix the boundary conditions so that this drainage process approaches, as $\tilde{t} \to \infty$, a static final state with mostly vapor overlying a thin layer of liquid.

The mathematical formulation is as follows.

 $R(\tilde{x}, 0)$ (or, equivalently $\tilde{u}_1(\tilde{x}, 0)$) is given as a momotonically increasing function of \tilde{x} , rising rapidly from \tilde{R}_0 at $\tilde{x} = 0$ to a plateau close to \tilde{R}_1 , and reaching \tilde{R}_1 at $\tilde{x} = 1$. Here $\tilde{R}_0 \approx \tilde{R}_v^*$ and $\tilde{R}_1 \approx \tilde{R}_l^*$. The determination of the exact values of \tilde{R}_0 and \tilde{R}_1 is given below.

The pressure is kept constant at both ends, which, in the present model, means imposing boundary conditions

$$\begin{aligned} \tilde{u}_1(0,\tilde{t}) &= \tilde{R}_0, \qquad \frac{\partial \tilde{u}_1}{\partial \tilde{x}} \bigg|_{\substack{\tilde{x}=0,\forall \tilde{t} \\ \tilde{u}_1(1,\tilde{t}) &= \tilde{R}_1, \qquad \frac{\partial \tilde{u}_1}{\partial \tilde{x}} \bigg|_{\substack{\tilde{x}=1,\forall \tilde{t} \\ \tilde{x}=1,\forall \tilde{t}}} = 0. \end{aligned}$$
 (39)

The solution of Eqs. 35 will represent the drainage flow described above if the values of \tilde{R}_0 and \tilde{R}_1 allow the existence of the state of equilibrium specified. To make sure that this is the case we first solve these equations with no-flow boundary conditions at top and bottom and with $\tilde{u}_1(\tilde{x}, 0)$ equal to a constant *C*. By trial and error we find a value of *C* that leads to a state that can reasonably be described as "mostly vapor overlying a thin layer of liquid." With the numerical values given in Eqs.36–38 we found

$$R_0 = 0.7902, \qquad R_1 = 1.4824.$$



Figure 6: Drainage. Left plot: fluid density versus \tilde{x} for $\tilde{t} = 0$ (broken curve) to $\tilde{t} = 0.4$ (rightmost curve), in steps $\Delta \tilde{t} = 0.02$. Right plot: fluid velocity versus \tilde{x} for $\tilde{t} = 0.02$ to $\tilde{t} = 0.4$, in steps $\Delta \tilde{t} = 0.02$.

The plots illustrating the solution of the drainage problem are shown in **Fig. 6**: a plot of the density (left) and a plot of the fluid velocity (right), both versus \tilde{x} , for values of \tilde{t} in the range [0, 0.4]. It is interesting to note that the part of the fluid that is predominantly vapor flows upwards, while the fluid that is predominantly liquid flows downwards. We have, in other words, countercurrent flow.

"Experimental" relative permeabilites are calculated from the results of the drainage calculation and compared to the theoretical formulas given above (Eqs. 30–33b). We start with the definitions in terms of dimensionless quantities:²

$$\tilde{V}_{l}^{*} = \frac{k_{rl(\exp)}}{\tilde{\eta}(\tilde{R}_{l}^{*})} \left(-\frac{\partial \tilde{p}_{l}^{*}}{\partial \tilde{x}} + \tilde{g} \tilde{R}_{l}^{*} \right), \qquad (40a)$$

$$\tilde{\pi}_{*}^{*} = \frac{k_{rv(\exp)}}{\partial \tilde{p}_{v}^{*}} \left(-\frac{\partial \tilde{p}_{v}^{*}}{\partial \tilde{x}} + \tilde{p} \tilde{\pi}_{v}^{*} \right)$$

$$\tilde{V}_{v}^{*} = \frac{\kappa_{Fv}(\exp)}{\tilde{\eta}(\tilde{R}_{v}^{*})} \left(-\frac{\delta p_{v}}{\delta \tilde{x}} + \tilde{g}\tilde{R}_{v}^{*} \right). \qquad (40b)$$
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These formulas mean that we look at the fluid in the transition region (i.e., the region where $\tilde{R}_v^* < \tilde{R} < \tilde{R}_l^*$) as a mixture of two phases, each phase having a density singled out by a minimum of the function $W + I^{se}$ (Fig. 4). \tilde{V}_a^* is the velocity of phase a, and \tilde{p}_a^* is the pressure in phase a (a = l, v). The present model does not provide these quantities, and we know of no rigorous derivation. To obtain them, we proceed by generalizing the method described previoulsy² to the mixed-wet case. With this in mind we introduce the following notation for the velocities and momenta at, respectively, $S_l^* = 0$ and $S_l^* = 1$:

$$\tilde{V}_{0} = \tilde{V}|_{S_{l}^{*}=0}, \quad \tilde{V}_{1} = \tilde{V}|_{S_{l}^{*}=1},
\tilde{\Gamma}_{0} = \tilde{\Gamma}|_{S_{l}^{*}=0}, \quad \tilde{\Gamma}_{1} = \tilde{\Gamma}|_{S_{l}^{*}=1}. \quad \dots \quad \dots \quad (41)$$

We now postulate that²

$$\tilde{V}_l^* = \frac{\tilde{\Gamma}_0 - \tilde{\Gamma}}{\tilde{\Gamma}_0 - \tilde{\Gamma}_1} \tilde{V}_l, \qquad \tilde{V}_v^* = \frac{\tilde{\Gamma} - \tilde{\Gamma}_1}{\tilde{\Gamma}_0 - \tilde{\Gamma}_1} \tilde{V}_0, \quad . \quad . \quad (42)$$



Figure 7: Drainage relative permeabilities versus S_l^* . Left plot: Experimental relative permeabilities calculated for the different time steps shown in Fig. 6: the points approach the horizontal axis as time increases. Right plot: Gray crosses: experimental values, normalized by the endpoint values; full lines: theoretical values, Eqs. 30.



Figure 8: Drainage case. Left: $k_{rl(exp)}$ at $S_l^* = 1$ versus dimensionless fluid velocity at $S_l^* = 1$. Right: $k_{rv(exp)}$ at $S_l^* = 0$ versus dimensionless fluid velocity at $S_l^* = 0$.

where the right-hand sides are obtained by using the solution to the differential equations of the model. It is further postulated that

$$\frac{\partial \tilde{p}_l^*}{\partial \tilde{x}} = -\gamma_{\exp}(1 - S_l^*), \qquad \frac{\partial \tilde{p}_v^*}{\partial \tilde{x}} = -\gamma_{\exp}(S_l^*). \quad . \quad (43)$$

where

$$\gamma_{\exp}(S_l^*) = \left[\frac{\partial \tilde{P}_v^{cs}}{\partial \tilde{R}} \frac{\partial \tilde{R}}{\partial \tilde{x}}\right]_{S_l^*} + \left[\frac{\partial \tilde{P}_l^{cs}}{\partial \tilde{R}} \frac{\partial \tilde{R}}{\partial \tilde{x}}\right]_{1-S_l^*} \dots \quad (44)$$

In this expression, the derivatives of \tilde{P}_l^{cs} and \tilde{P}_v^{cs} are given by Eqs. 29, while the partial derivative of \tilde{R} is obtained by using the solution of the partial differential equations.

Thus, at chosen time steps, the solution of the partial differential equations are used to calculate $k_{rl(exp)}$ and $k_{rv(exp)}$ at values of \tilde{x} where the density \tilde{u}_1 is intermediate between \tilde{R}_v^* and \tilde{R}_l^* . These values are shown in **Fig. 7**. The left-hand plot shows the "raw" values and it is seen that a large number of them are larger than 1. A closer look reveals that this is characteristic for short times and is an indication that Eqs. 40 are too simple to describe the transient behavior of the flow. However, when the experimental values are normalized by the end-point values, i.e., when plotting

$$\frac{k_{rl(\exp)}}{k_{rl(\exp)} \text{ at } S_l^* = 1} \quad \text{and} \quad \frac{k_{rv(\exp)}}{k_{rv(\exp)} \text{ at } S_l^* = 0}$$

versus S_l^* for given \tilde{t} , all points gather remarkably well around the theoretical curves given by Eqs. 30. See Fig. 7 (right).

The left plot also shows that the relative permeabilities become gradually flatter as the fluid decelerates to zero velocities. We have plotted the end-point relative permeabilities versus the fluid velocity: see **Fig. 8**. The straight lines shown are the leastsquares fits. Specifically, it is found that

where the errors on the coefficients are calculated by using the usual formulas of linear regression, assuming the end-point relative permeabilities to be normally distributed. The errors originate from the numerical integration of the differential equations.

Relative permeabilities for imbibition We consider imbibition in a one-dimensional porous medium of mixed wetting properties. Preliminary results have been presented elsewhere.* As for the case of drainage, we want the flow to be manifestly unsteady state at all times and achieve this by arranging imbibition to take place at diminishing velocities, leading to a static equilibrium. Specifically, we consider a medium which is initially filled mostly with vapor overlying a thin layer of liquid, with capillary forces larger that gravity forces so that imbibition takes place, the amount of vapor decreasing at the top and the amount of liquid increasing at the bottom. We fix the boundary conditions so that imbibition approaches, as $\tilde{t} \to \infty$, a static final state with mostly liquid underlying a thin layer of vapor.

The mathematical formulation is quite similar to the drainage case: $\tilde{R}(\tilde{x}, 0)$ (or, equivalently $\tilde{u}_1(\tilde{x}, 0)$) is given as a momotonically increasing function of \tilde{x} , where \tilde{u}_1 stays close to \tilde{R}_0 for most values of $\tilde{x} < 1$, then rises rapidly to \tilde{R}_1 , reaching \tilde{R}_1 at $\tilde{x} = 1$. Here $\tilde{R}_0 \approx \tilde{R}_v^*$ and $\tilde{R}_1 \approx \tilde{R}_l^*$. The determination of the exact values of \tilde{R}_0 and \tilde{R}_1 is similar to the drainage case: Eqs. 35 are first solved with no-flow boundary conditions at top and bottom and with $\tilde{u}_1(\tilde{x}, 0)$ equal to a constant *C*. By trial and error we find a value of *C* that leads to a state that can reasonably be described as "mostly liquid underlying a thin layer of vapor." With the numerical values given in Eqs. 36–38 we arrive at $\tilde{R}_0 = 0.8427$, and $\tilde{R}_1 = 1.4866$. These values then determine the initial condition and the boundary conditions (through Eqs. 39) for the imbibition problem.



Figure 9: Imbibition. Left plot: fluid density versus \tilde{x} for $\tilde{t} = 0$ (broken curve) to $\tilde{t} = 0.4$ (leftmost curve), in steps $\Delta \tilde{t} = 0.02$. Right plot: fluid velocity versus \tilde{x} for $\tilde{t} = 0.02$ to $\tilde{t} = 0.4$, in steps $\Delta \tilde{t} = 0.02$. The dotted curved is referred to in Fig. 10.

The plots illustrating the solution of the imbibition problem are shown in **Fig. 9**: a plot of the density (left) and a plot of the fluid velocity (right), both versus \tilde{x} , for values of \tilde{t} in the range [0, 0.4]. It will be noted that flow is countercurrent here as in the drainage case. The dotted curve shows a curious oscillation in the velocity, taking place at both ends of the transition region. These might be due to the smoothing of the *I*-function, or to some calculational error that is damped out at later times. As will be seen below, the relative permeabilities calculated at the value of \tilde{t} corresponding to the dotted curve deviate somewhat from the relative permeabilities obtained at other \tilde{t} -values.

"Experimental" relative permeabilites are calculated from the results of the imbibition calculation, in a manner which is completely similar to the one used for drainage, and compared to the theoretical formulas given by Eqs. 30 and 32: the "raw" values for $k_{rl(exp)}$ and $k_{rv(exp)}$ are shown in **Fig. 10** (left). The right-hand plot shows

$$\frac{k_{rl(\exp)}}{k_{rl(\exp)} \text{ at } S_l^* = 1} \quad \text{and} \quad \frac{k_{rv(\exp)}}{k_{rv(\exp)} \text{ at } S_l^* = 0}$$

versus S_l^* for given \tilde{t} . The black crosses in the right-hand plot show the relative permeabilities calculated at the time corresponding to the dotted curve in Fig. 9.

As in the case of drainage, the left plot shows that the relative permeabilities become gradually flatter as the fluid decelerates to zero velocities. The end-point relative permeabilities are plotted against the fluid velocity on **Fig. 11**, where the straight

^{*}P. Papatzacos and S. Skjaeveland, *Imbibition Relative Permeabilities from the Diffuse-Interface Theory*, presented at the 1-st International Scientific Conference "Oil Recovery 2003", Moscow, 19–23 May, 2003.



Figure 10: Imbibition relative permeabilities versus S_{1}^{*} . Left plot: Experimental relative permeabilities calculated for the different time steps shown in Fig. 9: the points approach the horizontal axis as time increases. Right plot: Gray crosses: experimental values, normalized by the endpoint values; full lines: theoretical values, Eqs. 30; black crosses: experimental values resulting from the dotted curve in Fig. 9

lines

$$k_{rl(\exp)(S_l^*=1)} = (3.295 \pm 0.006) V_{(S_l^*=1)} + (0.011 \pm 0.002), \quad \dots \quad \dots \quad (46a)$$

$$k_{rv(\exp)(S_l^*=0)} = (1.81 \pm 0.04) \tilde{V}_{(S_l^*=0)} + (-0.01 \pm 0.02), \quad \dots \quad \dots \quad (46b)$$

are the least-squares fits. The comments following Eqs. 45 apply here as well.

Conclusions

We have presented some investigations into the relative permeability concept by using a model where the concept is not used. These investigations were carried out for non-steady-state flows. The conclusions to be drawn are as follows.

- 1. The relative permeability curves depend on the fluid velocities.
- 2. The normalized relative permeability curves are in good agreement with theoretical expressions giving them in terms of the thermodynamic properties of the fluid, and of the wetting properties inferred by the capillary pressure correlations.
- 3. Each end-point relative permeability can be significantly, although transiently, larger than 1. In drainage as in imbibition, it is the end-point relative permeability of the phase that flows fastest that shows this behavior.
- 4. Each end-point relative permeability depends linearly on the velocity of the fluid at the corresponding normalized



Figure 11: Imbibition case. Left: $k_{rl(exp)}$ at $S_l^* = 1$ versus dimensionless fluid velocity at $S_I^* = 1$. Right: $k_{rv(exp)}$ at $S_I^* = 0$ versus dimensionless fluid velocity at $S_I^* = 0$.

saturation (0 or 1). It is improbable that the coefficients are independent of the flow type.

The last statement is based on the comparison of the linear regression Eqs. 45 and 46.

Nomenclature

- $a_l =$ Defined by Eq. 18a
- $a_v =$ Defined by Eq. 18b
- C_l = Defined by Eq. 18a
- C_1^s = Defined by Eq. 27
- C_v = Defined by Eq. 18b
- C_v^s = Defined by Eq. 27 F = Arbitrary function of **x**
- $\mathbf{g} = \mathbf{U}$ pwards pointing vector, $|\mathbf{g}| = g$
- g = Acceleration due to gravity
- $\tilde{g} = (R_c L/P_c)g$
- G = Gravitational potential. See Eq. 10
- G_1 = Arbitrary function of u_1, u_2 , and their derivatives. See Eqs. 16c, 16d, and 16e
- G_2 = Same definition as G_1
- H = Arbitrary function of u_1 and u_2
- I = Free energy of interaction between the fluid proper and the Σ -fluid. See Eq. 6
- $\tilde{I} =$ I/P_c
- I^{s} Smoothed *I*-function. See Eq. 27 =

$$I^s = I^s / P_c$$

- I^{se} = Smoothed and extended *I*-function
- $I_l =$ See Eqs. 21, 22, and 23
- I_v = See Eqs. 21, 22, and 24
- $j_l(R)$ = Second degree polynomial in *R*. See Eq. 26 J_l = See Eq. 23b
 - J_l^s = Smoothed J_l . See Eq. 26
- $j_v(R)$ = Second degree polynomial in R
 - $J_v =$ See Eq. 24b
 - $J_v^s =$ Smoothed J_v .

K = Absolute permeability

 k_{rl} = Relative permeability to liquid

 k_{rv} = Relative permeability to vapor

- L = Unit of length
- M = Fluid chemical potential at equilibrium
- \mathbf{n} = Unit normal to the boundary, pointing out
- \tilde{p}_l^* = Pressure in liquid phase (with residual vapor), divided by P_c . Eq. 40a
- \tilde{p}_v^* = Pressure in vapor phase (with residual liquid), divided by P_c . Eq. 40b
- \bar{P} = Fluid pressure at equilibrium
- $P_{\tilde{e}_{i}}^{b}$ = Pressure of bulk fluid
- $\tilde{P}^b = P^b / P_c$
- P^c = Capillary pressure. See Eqs. 17 and 18
- P_c = Critical pressure
- P_l^c = Defined by Eqs. 17 and 18a
- P_v^c = Defined by Eqs. 17 and 18b
- $P^{cs} = \text{Defined by Eqs. 28 and 29}$
- P_l^{cs} = Defined by Eq. 29a
- $P_v^{lcs} =$ Defined by Eq. 29b
- $\tilde{P}_l^{cs} = P_l^{cs} / P_c$
- $\tilde{P}_v^{cs} = P_v^{cs}/P_c$
- R = Density of fluid
- $\tilde{R} = R/R_c$
- R_c = Critical density of fluid
- R_l = Density of liquid
- R_l^* = Density of liquid with residual vapor. Eq. 19b
- $\tilde{R}_l = R_l/R_c$
- $\tilde{R}_I^* = R_I^* / R_c$
- $\vec{R_v}$ = Density of vapor
- R_{ν}^{*} = Density of vapor with residual liquid. Eq. 19a
- $\tilde{R}_v = R_v/R_c$
- $\tilde{R}_v^* = R_v^*/R_c$
- $\tilde{R}_0 = \text{Density at } \tilde{x} = 0$
- $\tilde{R}_1 =$ Density at $\tilde{x} = 1$
- $S_l =$ Liquid saturation. Eq. 12 (left)
- $S_l = Liquid saturation. Eq. 12 (left)$
- $S_v =$ Vapor saturation. Eq. 12 (right)
- $S_l^* =$ Normalized liquid saturation. Eq. 31
- S_{lr} = Residual liquid saturation. Eq. 19a
- S_{vr} = Residual vapor saturation. Eq. 19b
 - t = Time

 $\tilde{t} = (KP_c/(\phi\eta_c L^2))t$

- $u_1 = R.$ See Eq. 14a
- $u_2 =$ Defined by Eq. 14b
- $\tilde{u}_1 = R$
- $\tilde{u}_2 =$ See Eqs. 35
- $\mathbf{V} =$ Velocity. See Eq. 8
- V = Velocity (one dimensional flow)
- $\tilde{V} = (\phi \eta_c L/(KP_c))V$. Dimensionless version of V
- \tilde{V}_l^* = Dimensionless velocity of liquid (with residual vapor). Eq. 40a
- $\tilde{V}_v^* =$ Dimensionless velocity of vapor (with residual liquid). Eq. 40b
- V_0 = Defined by Eq. 41 (left)
- \tilde{V}_1 = Defined by Eq. 41 (right)
- W = See Eqs. 2, 4, 5

 $\tilde{W} = W/P_c$ $\mathbf{x} =$ Vector of space coordinates x = Space coordinate (one dimensional flow) $\tilde{x} = x/L$ α = (possibly with index) Constant β = (possibly with index) Constant ϵ = Small density, used to smooth the *I*-function. See Eq. 26 $\tilde{\epsilon} = \epsilon/R_c$ γ = Defined by Eq. 32 $\gamma_{exp} =$ ("Experimental" γ) Defined by Eq. 44 $\tilde{\Gamma}$ = Dimensionless fluid momentum. See Eq. 34 (right) $\tilde{\Gamma}_0$ = Defined in Eq. 41 $\tilde{\Gamma}_1$ = Defined in Eq. 41 η = Viscosity of fluid. See Eq. 11 $\tilde{\eta}(\tilde{R}) = \eta/\eta_c = (\eta_v/\eta_l)^r, \quad r = (\tilde{R}_l + \tilde{R}_v - 2\tilde{R})/(2(\tilde{R}_l - 1)^r))$ $R_v))$ $\eta_v =$ Viscosity of vapor η_l = Viscosity of liquid $\eta_c = \sqrt{\eta_v \eta_l}$ Λ = Constant related to microscopic interfacial tension $\tilde{\Lambda} = (R_c^2/(P_c L^2))\Lambda$ ϕ = Porosity Ψ = Helmoltz free energy. See Eq. 9 Ψ^{b} = Bulk free energy. See Eqs. 1, 2, 9 $\tilde{\Psi} = \Psi/P_c$

Subscripts

- c = Critical
- exp = "Experimental"
 - l = Liquid
 - v = Vapor

Superscripts

- ' = Derivative. Also, indicates an integration variable
- b = Bulk
 - c = Capillary

Overstrikes

~= Dimensionless variable

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