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Relative Permeability From Capillary Pressure

Paul Papatzacos, Stavanger University College, P.O. Box 8002, N-4068 Stavanger Svein M. Skjæveland, SPE, Stavanger University College, P.O. Box 8002, N-4068 Stavanger

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Abstract

A new theory is reviewed for single-component, two-phase flow in porous media. It includes wettability and capillary pressure as integral parts of the thermodynamic description and does not make use of the relative permeability concept. However, by providing a capillary pressure correlation, we are able to extract relative permeabilities and to show good consistency between rock property correlations.

Introduction

A new theory for two-phase flow in porous media has recently been developed and presented in a paper¹ referred to below as P^1 . A short review is given in the next section.

At the pore level, the two phases, liquid and its vapor, are treated as a single fluid with variable density, according to the diffuse-interface model.^{2,3} Wettability^{4,5} and capillary pressure are included in the thermodynamic description while phase pressures and relative permeabilities are irrelevant. Although the fluid description is restricted to a single-component system, practical examples may be found in the steam-water literature.^{6,7}

The upscaling from pore level equations to the macroscopic, Darcy-level equations is done by the Marle⁸ averaging technique. The equation of state is of the van der Waals type and describes two-phase flow by a single partial differential equation of the Cahn-Hilliard type coupled to a set of functions for the thermodynamic properties of the fluid, including capillary pressure. The differential equation is solved by a numerical procedure. To our knowledge, this theory is the only one published that includes wettability and capillary pressure as integral parts of the thermodynamical description of two-phase flow in porous media.

Capillary pressure and relative permeability both depend on the same fluid-fluid and rock-fluid interaction energies. If the capillary pressure vanishes, the residual saturations approach zero and the relative permeability of a phase becomes equal to its saturation. Relative permeability models in the literature are therefore often inferred from a capillary pressure correlation coupled with pore network models, e.g., the Corey-Burdine relative permeability correlations from the Brooks-Corey power-law capillary pressure correlation and the bundleof-tubes model.⁹

Introducing a capillary pressure correlation of the Brook-Corey type, we generate relative permeability curves from the theory without any further specifications of the pore network. The curves compare favorably with the Corey-Burdine type, thus validating their simple power-law form. The solution of the Cahn-Hilliard equation is made for a downwards primary drainage process where non-wetting water diplaces wetting vapor, for a range of capillary pressure levels and pore-size distribution indices.

The theory and methods can be used to check consistency between models for capillary pressure and relative permeabilities, e.g., hysteretic fluid flow processes with wettability incorporated in the thermodynamic description.

Theoretical Background

At pore level, P^1 assumes a fluid consisting of a single chemical component, capable of existing in either a one-phase state which is a liquid or a vapor, say, or in a two-phase state which would then be a coexistence of a liquid and a vapor. The equation of state is of the van der Waals type. The diffuse-interface theory of two-phase flow^{2,3} is then adopted. In short terms, this theory, called DI below, is as follows. It postulates that, in the case of a coexistence of two phases, a transition region exists between the phases, where the fluid density varies continuously, thus allowing a single-fluid description. (Note that this is in strong contrast to the traditional view of a two-phase system as two distinct fluids separated by a surface.) The transition region in DI is thin and the gradient of density is large in that region. Accordingly, DI incorporates, in the thermodynamical description of the fluid, an additional energy proportional to the squared gradient of the density, $|\nabla \rho|^2$, the effect of which is twofold: (i) the fluid dynamics of DI consist of the Navier-Stokes equations modified by an additional term proportional to $\nabla \nabla^2 \rho$;^{2,3} (ii) the thermodynamics of DI needs to emphasize that pressure and chemical potential have the traditional meanings *in the bulk fluid* (meaning far from the transition region) and this is done by letting pressure and chemical potential of the bulk fluid carry the superscript *b*.

Seppecher⁵ has shown that the wetting properties in DI are described by the Cahn theory of wetting.⁴ It is important for the appreciation of what follows to point out that Cahn describes wetting through a very short-range interaction between the fluid and the solid. This interaction gives rise in DI to an extra boundary condition, of the Neumann type, which answers to the extra term in the Navier-Stokes equations. This boundary condition on the density ρ which states that $\nabla \rho$ along the normal to the solid surface is proportional to the cosine of the wetting angle.^{1,5} Thus wetting implies a modification of the fluid density near the surface, the nearness being characterized by a length $\ell \approx 10^{-10}$ m. (ℓ is also the thickness of the transition region between phases, at pore level.¹) This extreme localization makes it possible, in the process of averaging to macroscopic level, to look at wetting as having the same effect as a surface fluid, called the Σ -fluid, with density ρ_{Σ} , obeying its own flow equations.1

The averaging of the pore level equations over many pores inside a representative volume is done in P¹ by assuming that the following parameters are uniform throughout the porous medium: porosity ϕ , pore surface per unit volume A_{Σ} , and temperature.

This averaging produces the equations for two-phase flow in porous media arising, as stated, from DI. The dependent variables in these equations are the macroscopic averages of ρ and ρ_{Σ} , namely R an R_{Σ} . In P¹, ρ and R carry a subscript f and are said to be quantities of the *f*-fluid, as a necessary distinction from the Σ -fluid introduced above; the subscript f is here suppressed for simplicity, and ρ and R are said to be quantities of the *fluid proper* when the necessity to distinguish from the Σ -fluid arises. The central equation is a mass balance equation for R:

where V is the macroscopic (Darcy-like) velocity:

$$\mathbf{V} = -\frac{KR}{\phi\eta} \nabla (M^b + M^c - \Lambda \nabla^2 R + G). \quad . \quad . \quad . \quad (2)$$

In this expression, *K* is the absolute permeability, Λ is a constant, and *G* is the gravitational potential giving rise an acceleration equal to $-\nabla G$; η is the fluid viscosity, assumed to be a known function of *R* (see at the end of this section); M^b is the bulk chemical potential and is a function of *R*; M^c is the chemical potential due to wetting and is a function of *R* and R_{Σ} .

Chemical potentials replace pressures as the naturally occuring quantities in the Darcy-like velocity, but we shall see that pressures can be reinstated in certain circumstances. The information which is necessary to determine these chemical potentials is contained in the Helmholtz free energy of the bulk fluids (fluid proper and Σ -fluid), which has the form

$$\phi\Psi(R,R_{\Sigma}) = \phi\Psi^{b}(R) + A_{\Sigma}F_{\Sigma}(R_{\Sigma}) + \phi I(R,R_{\Sigma}).$$
(3)

Here Ψ^b is the free energy of the averaged fluid proper, F_{Σ} is the free energy of the averaged surface fluid, and *I* is the energy of interaction between the two fluids. The M^b and M^c are defined below, together with the corresponding chemical potentials of the Σ -fluid:

$$M_{\Sigma}^{c} = \frac{\phi}{A_{\Sigma}} \frac{\partial I}{\partial R_{\Sigma}}.$$
 (4d)

For neutral wetting the interaction energy I is identically zero so that M^c drops out of Eq. 2. The Σ -fluid becomes irrelevant and the problem is completely defined by Eqs. 1, 2, and 4a. For non-neutral wetting $I(R, R_{\Sigma})$ determines M^c as a function of R and R_{Σ} . There is a need for an additional equation which, according to \mathbb{P}^1 is

where the left-hand side is defined by Eq. 4c while the righthand side is

P¹ concludes that the equations for two-phase flow in porous media resulting from DI are Eqs. 1, 2, and 5. As additional input, all thermodynamic functions appearing on the right-hand side of Eq. 3 must be known, as weel as the fluid viscosity η in Eq. 2.

It is also pointed out in P^1 that the chemichal potentials can be replaced, in Eq. 2, by the more familiar pressures, through the following formulas:

$$R\nabla M^c = \nabla P^c. \qquad (7b)$$

Here P^b is the bulk pressure, related to the density *R* through an equation of state, while P^c is the capillary pressure. Eq. 7a is a thermodynamical relation valid at constant temperature. Eq. 7b on the other hand is just a formal definition which is not compatible with P^c being a function of *R* and R_{Σ} . Indeed, the assumption $P^c = P^c(R, R_{\Sigma})$ leads directly to

$$\frac{\partial}{\partial R_{\Sigma}}\frac{\partial P^{c}}{\partial R} - \frac{\partial}{\partial R}\frac{\partial P^{c}}{\partial R_{\Sigma}} = \frac{\partial M^{c}}{\partial R_{\Sigma}},$$

so that, strictly speaking, P^c is not a function of state. We shall return to this presently.

It is of some interest to point out that the combination of Eqs. 1 and 2, and of Eqs. 3, 4a, and 4b, gives

$$\frac{\partial R}{\partial t} = \nabla \cdot \left(\frac{KR^2}{\phi \eta} \nabla \left(\frac{\partial \Psi}{\partial R} - \Lambda \nabla^2 R + G \right) \right). \quad . \quad . \quad (8)$$

When *G* is identically zero this reduces to the Cahn-Hilliard equation.

This completes the review of P^1 . What follows are further developments to the model.

The Incomplete-Wetting Approximation. It is shown in P^1 (Eqs. 35) that, at equilibrium,

$$M^{b} + M^{c} + G - \Lambda \nabla^{2} R = \overline{M}, \qquad (9a)$$
$$M_{\Sigma} + M_{\Sigma}^{c} + G = \overline{M}, \qquad (9b)$$

where \overline{M} is a constant (see below, Eq. 13). Assuming equilibrium with zero macroscopic velocity everywhere, we get from Eq. 6, that $M = M^b - \Lambda \nabla^2 R$ so that Eqs. 9 above can be written

$$M + M^c + G = \overline{M},$$

 $M_\Sigma + M_\Sigma^c + G = \overline{M}.$

These equations, together with Eq. 5, now give $M^c = M^c_{\Sigma}$ which, with Eqs. 4b and 4d yields

$$\frac{\partial I}{\partial (\phi R)} = \frac{\partial I}{\partial (A_{\Sigma} R_{\Sigma})}.$$
 (10)

It is easy to show that this is a necessary and sufficient condition for *I* to be a function of $\phi R + A_{\Sigma}R_{\Sigma}$.

It is shown in P¹ that $\phi R/(A_{\Sigma}R_{\Sigma})$ (the ratio of averaged fluid proper to averaged Σ -fluid per unit volume) is of the same order of magnitude as $q = \phi/(\ell A_{\Sigma})$. Using the Karman-Kozeny equation¹⁰ ($K = \phi^3/(5A_{\Sigma}^2)$) one gets

$$q = \sqrt{5K/\phi}/\ell.$$

We shall, in this paper, say that we have *complete wetting* when the wetting angle is either 0 or 190 degrees; otherwise, we shall say that we have *incomplete wetting*.

It is known¹ that for incomplete wetting, $\ell \approx 10^{-10}$ m so that q is quite large and the dependence of I on $A_{\Sigma}R_{\Sigma}$ can be neglected. For complete wetting on the other hand, experiments¹¹ indicate that ℓ may increase by a factor of many hundreds: the dependence of I on $A_{\Sigma}R_{\Sigma}$ can then become important for permeabilities of the order of the millidarcy.

Accordingly, we call I = I(R) the incomplete-wetting approximation. We assume, for the rest of this paper, that this approximation is valid.

A direct consequence of the incomplete-wetting approximation is that

$$M^c = \frac{dI}{dR}.$$
 (11)

The equations for two-phase flow in porous media resulting from DI are now Eqs. 1 and 2 where the chemical potentials are defined by Eqs. 4a and 11. In addition, the F_{Σ} contribution to the free energy Ψ , Eq. 3, can be neglected so that one can write

The two functions on the right-hand side, Ψ^b and *I*, must be known since M^b and M^c are their derivatives. They are determined below. The fluid viscosity η appearing in Eq. 2 must also be known.

The Helmholtz Free Energy of the Bulk Fluid. The incomplete-wetting approximation has no bearing on the calculations of this section.

Usually, the relevant thermodynamic properties of the fluid are known through an equation of state $P^b = P^b(R)$. It would then be easy to get the needed M^b -function by using Eq. 7a. It will, however, be made clear in a later section that knowledge of the free energy $\Psi^b(R)$ is important, so that this section concentrates on a method to calculate it. It has been shown¹² that, for van der Waals type $P^b(R)$ curves, Ψ^b has the form

$$\Psi^{b}(R) = W(R) + \bar{M}R - \bar{P}, \quad \dots \quad \dots \quad \dots \quad \dots \quad (13)$$

where W(R) has two minima where it is zero; \overline{M} and \overline{P} (the chemical potential and the pressure of the fluid at equilibrium) are functions of the temperature alone so that they can be considered constant for calculations at uniform temperature. **Fig. 1** shows a typical equation of state of the van der Waals type and its resulting *W*-function. It is stated here for later reference that a continuous P^b versus *R* curve implies a smooth *W* function. However, solutions to the Cahn-Hilliard equation, Eq. 8, are known¹³ where, for physical reasons, *W* that is not smooth at the minima. The only important feature of *W* is then said to support two phases, ¹³ the densities of which are the values of the density at which the minima of *W* occur.

The densities of the phases, say vapor and liquid, are denoted by R_v and R_l , and are such that the Maxwell construction (also called *the Maxwell equal area rule*¹⁴ when referring to the pressure versus volume diagram) applies:

Note that the \overline{P} of Eq. 13 is either $P^b(R_v)$ or $P^b(R_l)$.

By definition,

$$P^{b} = R^{2} \frac{d}{dR} \left(\frac{\Psi^{b}}{R} \right)$$

Using Eq. 13 on the right-hand side one easily gets

$$W(R) = P^{b}(R_{v}) \left(1 - \frac{R}{R_{v}}\right) + R \int_{R_{v}}^{R} P^{b}(R') \frac{dR'}{R'^{2}}.$$
 (15)

Note that $W(R_v) = 0$ by choice of integration constants, and that $W(R_l) = 0$ because of the Maxwell construction. It is also eaily checked that the derivative W'(R) vanishes at R_v and R_l .



Figure 1: A van der Waals type equation of state (top) and the *W*-function (bottom)

The expression for M^b now follows from its definition, as given by Eq. 4a:

Since only the gradient of M^b appears in the flow equation the value of \overline{M} is irrelevant and one can set

Note that we must have $M^b(R_v) = M^b(R_l) = \overline{M} = 0$. This is easily checked by using partial integration in Eq. 16, and referring to Eq. 14. The determination of M^b through the equation of state is thus completed. The determination of M^c follows.

The Interaction Energy due to Wetting. The incomplete-wetting approximation is now assumed, i.e., we assume that the interaction energy depends exclusively on R. An unambiguous definition of the capillary pressure P^c as a function of R is then possible by Eqs. 7b and 11. One gets

$$\frac{dP^c}{dR} = R \frac{dM^c}{dR}.$$
 (18)

Apparently, this determines M^c when the capillary pressure is known, but we shall see that the determination of I is an essen-

tial first step. To get I we use the defining Eq. 4b, now rewritten as

$$M^c = \frac{dI}{dR},$$

in Eq. 18:

$$\frac{d^2I}{dR^2} = \frac{1}{R}\frac{dP^c}{dR}.$$

The formal integration of this equation gives

The bounds of integration and the constants α and β depend on the wetting properies, as will be shown below. It is convenient to start by defining saturation.

The averaged fluid is at equilibrium in a two-phase state if its density R is between R_v and R_l (see **Fig. 1**). The volume fractions of the two phases can be taken as the definitions of the vapor and liquid saturations, S_v and S_l . This implies that

$$S_l = \frac{R - R_v}{R_l - R_v}.$$
 (20b)

If the rock is vapor-wet, there will be a residual vapor saturation, S_{vr} , due to the fact that the density of what is now the liquid phase is $R_l^* < R_l$. According to Eq. 20a,

$$S_{vr} = \frac{R_l - R_l^*}{R_l - R_v}.$$
 (21)

If the rock is liquid-wet there will, correspondingly, be a residual liquid saturation

$$S_{lr} = \frac{R_v^* - R_v}{R_l - R_v}, \quad \dots \quad (22)$$

where $R_v^* > R_v$ is now the density of the vapor phase. See **Fig. 2**. From a different point of view, one can say that the



Figure 2: Shifts in the densities of the liquid and vapor phases due to the wetting properties. The *W*-function is shown in broken line.

shifted densities are inferred from measured residual saturations by

The free energy Ψ , Eq. 12, reduces to Ψ^b , Eq. 13, for neutral wetting (I = 0). The phase transition mechanism is then regulated by W. For non-neutral wetting this mechanism is regulated by W + I. One then expects W + I to be a modified W-function, the modification consisting in a shift of the minima along the *R*-axis.

We now turn to the determination of *I*. We shall in this paper limit ourselves to the case of a vapor-wet medium.



Figure 3: Expected form of the W + I function for vaporwet rock. Note that it is not necessarily smooth at R_l^* . The *W*-function is shown in broken line.

The Interaction Energy for Vapor-Wet Rock. The rock being vapor-wet one expects a residual vapor saturation in a process where, say, the porous medium is filled initially with vapor which is then displaced by liquid. The equilibrium value of the averaged vapor saturation remains unchanged, equal to R_v , but the averaged liquid density will be $R_l^* < R_l$. We then want I(R) to be such that W + I has the form shown in **Fig. 3**. Obviously, there is a minimum of three necessary conditions on I:

$$I(R_v) = 0, \dots, \dots, \dots, \dots, \dots, \dots, \dots, \dots$$
 (24a)
 $I'(R_v) = 0, \dots, \dots, \dots, \dots, \dots, \dots, \dots, \dots, \dots$ (24b)

where I' is the derivative of I. The two first conditions imply that W + I resembles W near $R = R_v$, the third implies that W + I vanishes at $R = R_l^*$. Since, for vapor-wet medium, $P^c(R_v)$ is a finite constant (usually called the entry pressure) we can write Eq. 19 as

$$I(R) = R \int_{R_v}^{R} \frac{P^c(R')}{R'^2} dR' + \beta R - \alpha.$$

 β and α are determined by Eqs. 24a and 24b. One finds

$$I(R) = P^{c}(R_{v})\left(1 - \frac{R}{R_{v}}\right) + R \int_{R_{v}}^{R} \frac{P^{c}(R')}{R'^{2}} dR'. \quad (25)$$

We now use a capillary pressure versus saturation correlation of the form

$$P^{c} = C \left(\frac{1 - S_{vr}}{S_{v} - S_{vr}}\right)^{a}, \qquad 0 < a < 1, \dots .$$
 (26)

where C is the entry pressure while 1/a is the pore size distribution index (a = 0 corresponding to uniform pore size). Using

Eqs. 20a and 21 one finds that

$$I(R) = CJ(r), \quad \dots \quad (27a)$$

$$J(R) = 1 - \frac{R}{R_v} + R \int_{R_v}^{R} \frac{1}{R'^2} \left(\frac{R_l^* - R_v}{R_l^* - R'}\right)^a dR'.$$
 (27b)

Plots of J(R) are shown on **Fig. 4**.



Figure 4: Function J(R) for vapor-wet rock, Eq. 27b, plotted versus R for the numerical values indicated.

Eq. 24c now turns out to be a condition on C:

$$C = -\frac{W(R_l^*)}{J(R_l^*)}.$$
 (28)

We see that C < 0. Eq. 28 is an equation between R_v , R_l^* , C, and a, involving the thermodynamic function W of the fluid. Referring to Eq. 23b we see that we can look upon Eq. 28 as an equation containing the pure liquid and vapor densities R_v and R_l as parameters, and linking the two constants C and a to the residual vapor saturation S_{vr} . Referring to **Fig. 5** we see that Cgoes to zero when S_{vr} goes to zero, since R_l^* goes then to R_l .

The *I* function is now determined for the vapor-wet case and W + I has the form shown in **Fig. 5**. Note that the determination of the constant *C* in the capillary pressure correlation, Eq. 26, is essential to make sure that W + I function has two minima thus supporting the existence of two phases.

The function $M^{c}(R)$ for the vapor-wet case is now known through its definition, Eq. 11.

The capillary pressure function being defined for $S_v > S_{vr}$ only, *I* and consequently M^c are only known for $R_v < R < R_l^*$ (see Eqs. 27). For most numerical applications one must supply a function M^c which is valid outside this interval. Obviously, it seems natural to use Eq. 27b for $R < R_v$ also (this was done in the plots of **Figs. 4 and 5**). For $R > R_l^*$, however, a continuation must be found that is best adapted to the physics of the problem. An example is given in elsewhere in this paper.



Figure 5: Vapor-wet case: function I(R) resulting from P^c versus saturation given by Eq. 26, plotted versus R. Functions W and W + I are also shown. Concerning the broken lines, see the text following Eqs. 37.

The Fluid Viscosity. We shall use a formula for viscosity proposed by Arrhenius and used in P^1 , namely

where S_l and S_v are the liquid and vapor saturations defined by Eqs. 20.

Application: Relative Permeabilities

The numerical experiments described here are restricted to onedimensional situations. The coordinate axis (or *x*-axis) points in the direction of the gravitational force. The solution R(x, t)to the following equation is sought (see Eqs. 1 and 2):

$$\frac{\partial R}{\partial t} = \frac{\partial}{\partial x} \left\{ \frac{KR^2}{\phi \eta} \frac{\partial}{\partial x} \left[\frac{d\Psi}{dR} - \Lambda \frac{\partial^2 R}{\partial x^2} + G \right] \right\}. \quad . \quad (30)$$

In this equation, G = -gx (g > 0 is the axceleration due to gravity); η is the fluid viscosity given by Eq. 29; Ψ is given by Eqs. 12, 13, and 17. Thus

Following P¹ we now introduce dimensionless variables. The dimensionless counterpart of any quantity q is denoted \tilde{q} . The definitions of the dimensionless variables are given in the Nomenclature section. The dimensionless version of Eq. 30 is

$$\frac{\partial \tilde{R}}{\partial \tilde{t}} = \frac{\partial}{\partial \tilde{x}} \left\{ \frac{\tilde{R}^2}{\tilde{\eta}} \frac{\partial}{\partial \tilde{x}} \left[\tilde{M}^b + \tilde{M}^c - \tilde{\Lambda} \frac{\partial^2 \tilde{R}}{\partial \tilde{x}^2} - \tilde{g} \tilde{x} \right] \right\}.$$
 (32)

We shall also need the dimensionless version of the velocity, Eq. 2:

$$\tilde{V} = -\frac{\tilde{R}}{\tilde{\eta}(\tilde{R})} \frac{\partial}{\partial \tilde{x}} \left[\tilde{M}^b + \tilde{M}^c - \tilde{\Lambda} \frac{\partial^2 \tilde{R}}{\partial \tilde{x}^2} - \tilde{g} \tilde{x} \right]. \quad . \quad . \quad (33)$$

The *I* appearing in Eq. 31 will be an extended version of Eqs. 27 and 28. As for *W*, we shall use the following (called the pseudo van der Waals form in P^1):

$$\tilde{W}(\tilde{R}) = (\tilde{R} - \tilde{R}_v)^2 (\tilde{R} - \tilde{R}_l)^2, \quad \dots \quad \dots \quad \dots \quad (34)$$

so that

$$\tilde{M}^b = 2(\tilde{R} - \tilde{R}_v)(\tilde{R} - \tilde{R}_l)(2\tilde{R} - \tilde{R}_v - \tilde{R}_l). \quad . \quad . \quad (35)$$

Since we are not trying to replicate the behavior of any specific fluid we have chosen a \tilde{W} that is very easy to use: it allows us to choose \tilde{R}_v and \tilde{R}_l at will; otherwise, using van der Waals or more realistic equations of state (Peng-Robinson and others), these constants must be obtained through calculations involving the Maxwell rule.

Vapor-Wet Rock. Gravity Drainage by Liquid Phase. We consider a one dimensional porous medium where flow occurs vertically. It is initially filled with vapor and it is assumed that liquid is provided at the top, at a constant pressure, and that the vapor can flow out at the bottom, also at constant pressure.

The mathematical formulation, inside the P^1 model, is as follows.

- The \tilde{x} -axis points downwards and $0 \le \tilde{x} \le 1$.
- Initial condition:

where $\tilde{R}_0 \approx \tilde{R}_l^*$ for small values of \tilde{x} , and $\tilde{R}_0 \approx \tilde{R}_v$ otherwise.

• Boundary conditions:

$$\tilde{R}(0,\tilde{t}) = \tilde{R}_l^*, \quad \dots \quad \dots \quad \dots \quad \dots \quad (37a)$$
$$\tilde{R}(1,\tilde{t}) = \tilde{R}_v, \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (37b)$$

$$\frac{\partial \tilde{R}}{\partial \tilde{x}}(0,\tilde{t}) = 0, \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (37c)$$

The *I*-function which is needed to define M^c , is defined here by Eqs. 27 and 28, where we have chosen

$$\tilde{R}_v = 0.4, \quad \tilde{R}_l = 1.6, \quad a = 0.5, \quad S_{vr} = 0.4, \quad . \quad (38)$$

implying that $\tilde{R}_l^* = 1.12$. In addition, we have chosen

$$\eta_v/\eta_l = 0.1, \qquad \tilde{g} = 0.5, \qquad \tilde{\Lambda} = 0.01. \dots \dots (39)$$

The solution is expected to show a gradual sinking of the liquid displacing the gas so that there should be, after some time has elapsed, a layer of liquid with density \tilde{R}_l^* at the top, a twophase transition region, and a layer of gas with density R_v at the bottom. The above value of $\tilde{\Lambda}$ has been chosen so that the three regions should have about the same thickness. (It is shown in P¹ that the thickness of the transition region is about $2\sqrt{\Lambda}/(\tilde{R}_l - \tilde{R}_v)$, as referred to the unit of length.) We have used a commercially available program using finite elements (FEM-LAB, operating under MATLAB)¹. The program requires continuous and smooth functions as input, implying that: (i) $\tilde{R}_0(\tilde{x})$ must be smooth (infinitely differentiable is best) and; (ii) *I* must be continued smoothly, to the right of \tilde{R}_l^* , as indicated on **Fig. 5** by the broken lines. The most important consequence, related to the second smoothing above, implies removing the infinity from the P^c curve.



Figure 6: Vapor wet rock. Gravity drainage by liquid phase. Plot of $\tilde{R}(\tilde{x}, \tilde{t})$ vs. \tilde{x} for (left to right) $\tilde{t} = 0.05$ to 0.5 in steps of 0.05. Equations solved: 32, 36, and 37; numerical values: 38 and 39. Inset: long-time solution of the same equations with the same numerical values as above, except that $\tilde{g} = 0$.

The result of the calculation of $\tilde{R}(\tilde{x}, \tilde{t})$ is shown in **Fig. 6**. The following facts emerge from this figure:

- The transition from the liquid (*R̃* ≈ *R̃*^{*}_l) plateau to the steep-slope region is fast when compared to the transition from steep-slope region to the vapor (*R̃* ≈ *R̃*_v) plateau. This is obviously due to the medium being vapor wet.
- $\tilde{R}(\tilde{x}, \tilde{t})$ behaves like a wave, travelling at constant velocity and without distortion, so that $\tilde{R}(\tilde{x}, \tilde{t}_1)$ is (up to calculational errors) a translated version of $\tilde{R}(\tilde{x}, \tilde{t}_2)$.
- Each R(x, t) is a translated version of the curve shown in the inset. The latter is the long-time solution of Eqs. 32, 36, and 37 with the numerical values given in 38 and 39, except that g = 0. In other words, the curve in the inset is the static equilibrium solution without gravity. As such, it must (and does) satisfy Eq. 9a with G = 0. Keeping in mind Eq. 17 and using dimensionless quantities, we see that the curve in the inset obeys

$$\tilde{M}^b + \tilde{M}^c - \tilde{\Lambda} \frac{d^2 \tilde{R}}{d \tilde{x}^2} = 0$$

 $^1{\rm FEMLAB}$ is a registered trademark of COMSOL AB; MATLAB is a registered trademark of TheMathWorks Inc.

We mention here for future reference that, when one takes into account Eqs. 4a and 11, the following first integral can be found:

obeying conditions 37.

We shall return to these properties of the solution later.

We now present a method for using the above solution to calculate relative permeabilities to vapor and liquid, k_{rv} and k_{rl} . These do not exist in the P¹ model, so that their calculation presupposes that they can be defined with quantities that can be calculated in the model. The definitions, in terms of dimensionless quantities, are

$$\tilde{V}_{v} = \frac{k_{rv}}{\tilde{\eta}(\tilde{R}_{v})} \left(-\frac{\partial \tilde{p}_{v}}{\partial \tilde{x}} + \tilde{g}\tilde{R}_{v} \right), \quad \dots \quad \dots \quad \dots \quad (41a)$$

One here assumes that, at a point in the two-phase region, the fluid consists of a vapor phase with density \tilde{R}_v , viscosity $\tilde{\eta}(\tilde{R}_v)$, velocity \tilde{V}_v and pressure \tilde{p}_v , and a liquid phase where the corresponding quantities are primed and have subscript l (the prime as used here reminds that the medium is vapor wet so that the liquid phase is a mixture of liquid and residual vapor).

We now consider a time \tilde{t}_0 at which the solution has, as closely as possible, the characteristics of an infinite one-dimensional system consisting of liquid (and residual vapor) to the left, a transitional two-phase region in the middle, and vapor to the right, the whole flowing from left to right as a standing wave. Referring to **Fig. 6** we see that a number of solutions with values of \tilde{t}_0 around 0.5 have the required characteristics. The considerations which now follow are valid for solutions at such \tilde{t}_0 's.

The phase densities and viscosities to use in Eqs. 41 are already well defined and the main problem is to derive phase velocities and phase pressures. We have not found a rigorous derivation, so that the results presented are based on guesses based on the physics of the situation.

Basing the definition of phase velocities on the concept of momentum one can say that the local momenta carried by each phase should add to the the fluid momentum $\tilde{R}\tilde{V}$:

$$\tilde{R}_v \tilde{V}_v + \tilde{R}_l^* \tilde{V}_l^* = \tilde{R} \tilde{V}. \qquad (42)$$

This equation does not allow the calculation of the two phase velocities but we can use it as a guide: any formulas we derive for \tilde{V}_v and \tilde{V}_l^* should verify it.

We use the following shorthand notation in what follows:

$$\Gamma_{v}^{*}(\tilde{x}, \tilde{t}_{0}) = \frac{\tilde{R}(0, \tilde{t}_{0})\tilde{V}(0, \tilde{t}_{0}) - \tilde{R}(\tilde{x}, \tilde{t}_{0})\tilde{V}(\tilde{x}, \tilde{t}_{0})}{\tilde{R}(0, \tilde{t}_{0})\tilde{V}(0, \tilde{t}_{0}) - \tilde{R}(1, \tilde{t}_{0})\tilde{V}(1, \tilde{t}_{0})}.$$
 (43)

Note that Γ_v^* is a normalized vapor momentum, constructed on the same lines as the normalized vapor saturation

$$S_{v}^{*}(\tilde{x}, \tilde{t}_{0}) = \frac{\tilde{R}(0, \tilde{t}_{0}) - \tilde{R}(\tilde{x}, \tilde{t}_{0})}{\tilde{R}(0, \tilde{t}_{0}) - \tilde{R}(1, \tilde{t}_{0})} = \frac{\tilde{R}_{l}^{*} - \tilde{R}}{\tilde{R}_{l}^{*} - \tilde{R}_{v}}, \quad . \quad . \quad (44)$$

where the second equality follows from Eqs. 37a and 37b.

The simplest expressions for the local phase velocities, satisfying Eq. 42, are

$$\tilde{V}_{v}(\tilde{x}, \tilde{t}_{0}) = \tilde{V}(1, \tilde{t}_{0})\Gamma_{v}^{*}(\tilde{x}, \tilde{t}_{0}), \qquad (45a)$$
$$\tilde{V}_{v}^{*}(\tilde{x}, \tilde{t}_{0}) = \tilde{V}(0, \tilde{t}_{0})[1 - \Gamma_{v}^{*}(\tilde{x}, \tilde{t}_{0})], \qquad (45b)$$

$$V_l^*(\tilde{x}, \tilde{t}_0) = V(0, \tilde{t}_0)[1 - \Gamma_v^*(\tilde{x}, \tilde{t}_0)].$$
 (45b)

A confirmation of the fact that these expressions are the correct choices comes from a comparison of the plot of Γ_{u}^{*} versus \tilde{x} with that of S_v^* versus \tilde{x} : see Fig. 7. The two curves are



Figure 7: Plot of Γ_v^* vs. \tilde{x} (solid line) and of S_v^* vs. \tilde{x} (dotted line) at $\tilde{t} = 0.5$. See the caption of Fig. 6.

practically undistinguishable, meaning that V_v and V_l^* are proportional to, respectively, S_v^* and $1 - S_v^*$. We shall see later that this leads to relative permeabilty curves with the desired straight-line behavior when the capillary pressure vanishes.

The numerical solution leads then to

We can go one step further and express the boundary velocities
in terms of densities by using Eq. 33. Indeed, since the density
at
$$\tilde{t}_0$$
 has flat plateaus near the left and right boundaries (**Fig. 6**),
the first, second, and third derivatives of \tilde{R} with respect to \tilde{x}
vanish at $\tilde{x} = 0$ and $\tilde{x} = 1$ (this is verified to be the case, up to
numerical errors) we find that

$$\tilde{V}(0,\tilde{t}) = \frac{\tilde{R}_l^*}{\tilde{\eta}(\tilde{R}_l^*)}\tilde{g}, \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (47a)$$

$$\tilde{V}(1,\tilde{t}) = \frac{\tilde{R}_v}{\tilde{\eta}(\tilde{R}_v)}\tilde{g}, \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (47b)$$

so that

$$\tilde{V}_{v}(\tilde{x}, \tilde{t}_{0}) = \frac{\tilde{R}_{v}}{\tilde{\eta}(\tilde{R}_{v})} \tilde{g} S_{v}^{*}, \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (48a)$$

We now turn to the definitions of the phase pressures, or rather of the pressure gradients, needed in Eqs. 41.



Figure 8: $P^b + P^c$ versus density, corresponding to the W+Icurve of Fig. 5 (broken line). Compare with Fig. 1.

The required definitions are based first on the observation that, since the phase separating mechanism is now based on the W + I-function shown in Fig. 5 (broken curve) then the gradient of $P^b + P^c$ at equilibrium is zero, in analogy with the fact that the gradient of P^b at equilibrium is zero when only W is present: see Fig. 1 where the horizontal line between A and B is the equilibrium P^b , and compare with Fig. 8. This means that

$$\left[\frac{\partial \tilde{P}^b}{\partial \tilde{x}}\right]_{eq} = -\frac{\partial \tilde{P}^c}{\partial \tilde{x}}.$$
 (49)

The right-hand side of this equation is easily obtained in terms of $\partial \tilde{R} / \partial \tilde{x}$, keeping in mind that \tilde{P}^c is a function of S_v^* only, and that S_v^* is defined Eq. 44. We know, however, that due to the properties of the solution, $\partial \tilde{R} / \partial \tilde{x}$ is given by Eq. 40. Thus $\partial \tilde{P}^b / \partial \tilde{x}$ at equilibrium can be written in terms of the derivative of the capillary pressure with respect to the normalized vapor saturation, and of the thermodynamic functions W and I. We introduce the following notation

$$\gamma(S_v^*) = \frac{[gR_v(R_l - R_v)]^{-1}}{(1 - S_{vr})} \frac{dP^c}{dS_v^*} \sqrt{\frac{2[W(\tilde{R}) + I(\tilde{R})]}{\Lambda}}, (50)$$

where the argument of the W and I functions is

and obtain

$$\left[\frac{\partial \tilde{P}^b}{\partial \tilde{x}}\right]_{eq} = -\tilde{g}\tilde{R}_v\gamma(S_v^*).$$

Note, incidentally, that γ is dimensionless and can be written with tilded as well as with non-tilded quantities. We now postulate that

Using these equations, together with Eqs. 48 in Eqs. 41, we obtain

$$k_{rl} = \frac{1 - S_v^*}{1 + (R_v/R_l^*)\gamma(1 - S_v^*)}.$$
 (53b)

These equations express relative permeabilities in terms of, essentially, the equation of state and the capillary pressure and, in addition, of the parameter Λ which is related to the thickness of the two-phase region. Note that the relative permeability curves reduce to two straight lines when the capillary pressure is zero, since γ then vanishes.



Figure 9: Vapor wet rock. Gravity drainage by liquid phase. Relative permeabilities, k_{rv} and k_{rl} , versus S_v^* for a = 0.1 (full line), a = 0.5 (broken line), and a = 0.9 (dotted line).

Fig. 9 shows a plot of k_{rv} and k_{rl} versus S_v^* , for three values of *a*; the other parameters needed are those of Eqs. 38 and 39. Concerning thermodynamic functions, P^c is given by Eq. 26, *W* by Eq. 34, and *I* by Eqs. 27 and 28. Note in particular that the *I*-function used in **Fig. 9** is not the smoothed one (the broken line in **Fig. 5**): the latter was only introduced for the numerical solution of Eq. 32.

Discussion

The Incomplete-Wetting Approximation. We presented above the incomplete-wetting approximation and assumed that it holds for the calculations presented in the paper. We have, however, used a capillary pressure versus saturation correlation, Eq. 26, which holds for a completely vapor-wet medium. This was done in the interest of simplicity and has lead to results that make sense both mathematically and physically, leading to the conclusion that the model presented is robust. The conclusions are, however, restricted to cases where the incomplete wetting approximation holds, i.e., essentially cases where the permeability is well above the millidarcy.

Work is in progress on the strictly incomplete-wetting case, with a capillary pressure correlation of the form ¹⁵

$$P^{c} = C_{v} \left(\frac{1 - S_{vr}}{S_{v} - S_{vr}}\right)^{a_{v}} + C_{l} \left(\frac{1 - S_{lr}}{S_{l} - S_{lr}}\right)^{a_{l}}$$

Comparison with Corey-Burdine. A comparison of the relative permeability curves presented in this paper with the well known Corey-Burdine expressions is shown in **Fig. 10**. We remind that the Corey-Burdine formulas are, with our notation for the normalized vapor saturation,

$$k_{rv} = (S_v^*)^{2a+1+m}, \ldots \ldots \ldots \ldots \ldots \ldots \ldots (54b)$$

where m is the tortuosity exponent. The Corey-Burdine curves



Figure 10: Relative permeability curves given by Eqs. 53 (solid line) and by Eqs. 54 (broken lines), plotted versus normalized vapor saturation S_v^* . For the numerical values of the various parameters see the text.

shown are drawn with a = 0.5 and m = 2. Our curves were obtained with the parameters given in Eqs. 38 and 39. Considering the difference in the approaches and the difference in the dependence on numerical parameters, the agreement between the two sets of curves is somewhat surprising. However, the agreement is probably more than a coincidence since the parameters appearing in the γ -function, Eq. 50, have been tuned to each other as described above: we have produced a solution which has the characteristics of an infinite one-dimensional system consisting of three regions (liquid, two-phase, vapor) of about the same thickness.

The Entry Pressure in Terms of Residual Saturation. The entry pressure *C* is given above by Eq. 28. It depends on the equation of state of the fluid through *W* (see Eq. 15) and on the capillary pressure correlation through *J* (see Eqs. 26 and 27b). It can easily be seen that *C* depends on the fluid parameters R_l and R_v . More importantly, *C* is a function of two variables, *a* and S_{vr} .



Figure 11: The dimensionless entry pressure \tilde{C} versus S_{vr} , for different values of *a*. See text for details.

For the sake of a simple illustration, we have chosen the \tilde{W} -function given by Eq. 34 and have plotted the resulting dimensionless \tilde{C} -function in **Fig. 11** for $\tilde{R}_v = 0.4$ and $\tilde{R}_l = 1.6$. This plot can be used for a quick determination of the residual vapor saturation in the case of a vapor-wet porous medium, whenever the constant *a* and the entry pressure *C* are either known or can easily be determined experimentally.

Conclusions

We have presented a new theory for two-phase flow in porous media for a fluid consisting of one chemical component.

We have also presented some conclusions of the theory, in the case of a vapor-wet medium. These are as follows:

- The relative permeabilities for a wave-like flow type can be expressed in terms of the equation of state of the fluid and of the capillary pressure correlation.
- There is a relation between the pore size distribution parameter, the entry pressure, and the residual vapor saturation. This can be used for a quick determination of the residual vapor saturation.

Nomenclature

- a = See Eq. 26
- A_{Σ} = Pore surface per unit volume
- C = Entry pressure. See Eq. 26
- $\tilde{C} = C/P_c$
- F_{Σ} = Free enbergy of the Σ -fluid. Eq. 3
- g = Axceleration due to gravity
- G = Gravitational potential giving rise to an acceleration - ∇G
- $\tilde{g} = (R_c L/P_c)g$

$$\tilde{G} = (R_c/P_c)G = \tilde{g}\tilde{x}$$

- I = Free energy of interaction between the fluid proper and the Σ -fluid. Eqs. 3, 12
- $\tilde{I} = I/P_c$
- J = See Eqs. 27
- K = Absolute permeability
- k_{rl} = Relative permeability to liquid
- k_{rv} = Relative permeability to vapor

- ℓ = Thickness of transition region between phases (pore level)
- L = Unit of length
- m = Tortuosity exponent
- M = Eq. 6
- \overline{M} = Fluid chemical potential at equilibrium
- M^b = Chemical potential of bulk fluid (macroscopic)
- $\tilde{M}^b = (R_c/P_c)M^b$
- M^c = Chemical potential of bulk fluid, due to capillary effects or wetting (macroscopic)

$$\tilde{M}^c = (R_c/P_c)M^c$$

- $M_{\Sigma} = \text{Eq. 4c}$
- $M_{\Sigma}^c = \text{Eq. 4d}$
- \tilde{p}_l^* = Dimensionless pressure in averaged liquid phase (with residual vapor)
- \tilde{p}_v = Dimensionless pressure in averaged vapor phase
- \bar{P} = Fluid pressure at equilibrium
- P^b = Pressure of bulk fluid (macroscopic)

 $\tilde{P}^b = P^b/P_c$

- P^c = Capillary pressure (macroscopic)
- $\tilde{P}^c = P^c / P_c$
- P_c = Critical pressure of averaged fluid
- $q = \phi/(\ell A_{\Sigma})$
- R = Density of averaged fluid (macroscopic level)
- $R = R/R_c$
- R_c = Critical density of averaged fluid
- R_l = Density of averaged liquid
- $\tilde{R}_l = R_l/R_c$
- $\tilde{R}_v = R_v/R_c$
- R_v = Density of averaged vapor
- R_l^* = Density of averaged liquid with residual vapor
- $\tilde{R}_l^* = R_l^* / R_c$
- \tilde{R}_0 = Density of averaged fluid at initial time
- R_{ν}^{*} = Density of averaged vapor with residual liquid
- R_{Σ} = Density of averaged Σ -fluid (macroscopic level)
- S_l = Liquid saturation. Eq. 20b. Also $(\tilde{R} \tilde{R}_v)/(\tilde{R}_l \tilde{R}_v)$
- S_v = Vapor saturation. Eq. 20a. Also $(\tilde{R}_l \tilde{R})/(\tilde{R}_l \tilde{R}_v)$
- $S_v^* = (\ddot{R}_l^* \ddot{R}) / (\ddot{R}_l^* \ddot{R}_v)$. Normalized vapor saturation (vapor wet medium)
- S_{vr} = Residual vapor saturation
- S_{lr} = Residual liquid saturation
- t = Time
- $\tilde{t} = (KP_c/(\phi\eta_c L^2))t$
- \tilde{t}_i = Particular values of \tilde{t} (i = 1, 2, 3)
- **V** = Velocity (macroscopic level)
- V = Velocity (macroscopic level) in one dimension
- $\tilde{V} = (\phi \eta_c L / (K P_c)) V$. Dimensionless version of V
- \tilde{V}_l^* = Dimensionless velocity of averaged liquid (with residual vapor)
- \tilde{V}_v = Dimensionless velocity of averaged vapor
- W = Eq. 13, 15
- $W = W/P_c$. Eq. 34
- *x* = Space coordinate (one dimensional system)
- $\tilde{x} = x/L$
- α = Integration constant

- β = Integration constant
- $\gamma = \text{Eq. 50}$
- Γ_{v}^{*} = Normalized vapor momentum. Eq. 43

$$\eta = \text{Viscosity of averaged fluid. Eq. 29} \\ \tilde{\eta}(\tilde{R}) = \eta/\eta_c = (\eta_v/\eta_l)^r, \quad r = (\tilde{R}_l + \tilde{R}_v - 2\tilde{R})/((2(\tilde{R}_l + \tilde{R}_v))))$$

- η_v = Viscosity of averaged vapor
- Viscosity of averaged liquid $\eta_l =$
- $\eta_c =$
- $\sqrt{\eta_v \eta_l}$ Constant (macroscopic) related to interfacial ten- $\Lambda =$ sion (microscopic)

$$\Lambda = (R_c^2/(P_c L^2))\Lambda$$

- ρ = Fluid density (pore level)
- ρ_{Σ} = Density of Σ -fluid (pore level)
- ϕ = Porosity
- Ψ = Helmoltz free energy. Eqs. 3, 12
- Ψ^{b} = Bulk free energy. Eqs. 3, 12, 13

Subscripts

- c = Critical
- l = Liquid
- v = Vapor
- $\Sigma = \Sigma$ -fluid

Superscripts

- ' = Derivative. Also, indicates an integration variable
- b = Bulk
- c = Capillary
- * = Normalized variable

Overstrikes

~= Dimensionless variable

Abbreviations

- $P^1 = Reference 1$
- DI = Diffuse-interface

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Paul Papatzacos (paul.papatzacos@tn.his.no) is professor of applied mathematics at Stavanger University College, Stavanger, Norway. He holds a PhD degree in theoretical physics from Trondheim University. His interests are in analytical modeling of multiphase flow in porous media.

Svein M. Skjaeveland (s-skj@ux.his.no) is a professor of reservoir engineering at Stavanger University College. He holds PhD's in physics from the Norwegian Institute of Technology and in petroleum engineering from Texas A & M University.