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A Model to Quantify Residual Saturation Distribution in Heterogeneous Reservoirs

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Abstract

To study capillary trapping phenomena in 2 and 3D a mathematical model is developed facilitating rapid fine grid computation of the saturation fields in heterogeneous reservoirs. The reduction in computation time is achieved due to the fact that the model is much simpler than a conventional black oil simulator. Influence of wettability type, magnitude of capillary pressure, rate, and gravity forces are studied in 2D and 3D analytically (qualitatively) and by way of computations (quantitatively). It is shown that at infinite time in the strongly water wet reservoirs all the mobile oil is recovered, while in the oil-wet and mixed reservoirs a certain amount of capillary trapped oil exists. The capillary trapped oil saturation has a tendency to increase in the direction from injection to production wells, though it reveals a non-monotonous behavior locally.

The developed model is useful in calculation of recoverable reserves, and in optimization of waterflooding strategy to minimize remaining oil in heterogeneous reservoirs including the fractured ones.

Introduction

The oil remaining in the domains of reservoir swept by waterflooding consists of (1) the oil trapped on the microscopic level, i.e., dispersed inside single pores, and (2) the oil trapped due to small to medium scale heterogeneities which may have a size of 1 cm to 10 m. The first part of the remaining oil corresponds to the zero value of the oil relative permeability. With realistic pressure drops it can only be mobilized by utilization of surfactants. The second part of the unrecovered oil which is further referred to as capillary trapped oil, is mobile, i.e., its relative permeability is non-zero.

Generally, the distribution of residual oil depends on the balance of capillary, viscous and gravity forces and on reservoir heterogeneity. For a given reservoir heterogeneity field the fluid velocity in different points of space strongly differs. The velocity ratio between high permeable zones and low permeable ones, close to the wells and away from them may easily reach several orders of magnitude. This makes it unreasonable to expect capillary dominance in the whole flow domain, which is often assumed. A theoretical and numerical study of capillary trapped oil distribution in a 3D heterogeneous reservoir with account for capillary, gravity and viscous forces is undertaken. It extends to 3-D flow geometry previous studies of capillary trapping phenomena in 2-phase flow[2,7] which where limited to 1-D.

Calculation of residual oil distribution

The capillary trapped oil distribution is calculated in a heterogeneous reservoir based on the information about heterogeneity field, and relative permeabilities and capillary pressure assigned to every lithological type. It is not assumed that capillary pressure is correlated to absolute permeability in any particular way. Any type of correlation can be included in the model.

The equation (7) describing capillary trapped oil distribution in 3D is derived and analyzed in the Appendix. In the general case of water phase mobility and capillary pressure, when the Equation (7) is non-linear it is solved numerically, by finite difference method [1]. Since the case described by the proposed model is a very particular one, i.e., only one phase is moving, steady state flow, the computations can be arranged more efficiently and accurately then in a standard black oil simulator, like, e.g., *ECLIPSE*. In order to reproduce the results a conventional simulator would have to be run in a transient mode until steady-state is obtained, while the developed model describes directly the final steady-state condition.

A *FORTRAN* program, *REST2D*, was developed to efficiently calculate residual saturation distribution by way of numerical solution of Equation (7).

To test the numerical method and the developed code a number of tests have been performed for a 2D cross-sectional model. In all considered cases the relative permeability to water, water viscosity, and phase densities were the same. Capillary pressure was correlated to the absolute permeability through the Leverett *J*-function. It is worth noting that this type of capillary pressure correlation is chosen neither for physical reasons nor for numerical convenience, but just as one of reasonable models. The question of capillary pressure correlation, especially for mixed wet rocks, is not being addressed in the present paper, it deserves special study and is discussed elsewhere[3,5].

Test case 1: Field scale

The problem is formulated in vertical cross section. The sizes of the domain are 98m × 28 m. The heterogeneity field consists of a high permeable (*1D*) background with low permeable (*10mD*) inclusions, the pattern of low permeable zones is regular, see Figure 3. Capillary pressure function and the relative permeability to water are displayed in Figure 1. The size of the grid is 100 × 15. The injection is distributed along the left boundary (4 lower grid blocks), and the production is through the 12 lower blocks on the right boundary. The solution is displayed in Figure 4. The CPU time for this problem was about 3 minutes.

Comparison with a standard numerical model.

The purpose of this part of the study is to compare residual oil saturation computed by a standard finite difference model with results obtained by the *REST2D* program . The *ECLIPSE* black oil simulator is used as an industry-standard black oil simulator. The reservoir is initially filled with oil and water is injected at a constant rate until oil production becomes approximately zero.

Grid definition. The reservoir is divided into 100 blocks in the horizontal direction and 15 blocks in the vertical direction. Grid block lengths in the horizontal direction is 1 meter, aside from the first and last columns of blocks which have length 0.5 meter. One column of small blocks of length 0.0001 meter is added to the grid in order to be able to specify zero capillary pressure for blocks containing production wells. Hence, the total number of columns in the grid is 101. All blocks in the vertical direction have length 2 meters, except the first and last rows of blocks which have length 1 meter.

Reservoir properties. Two values of absolute permeability, 0.01 Darcy and 1 Darcy, are distributed in the grid in a regular pattern described above. Porosity is 0.2 throughout the reservoir.

Fluid properties. The fluids are incompressible with water viscosity equal to 1 cp. and oil viscosity equal to 2 cp.

Relative permeabilities and capillary pressure. Relative permeability of water as a function of water saturation S_w is given by S_w^2 and relative permeability of oil is given by $(1-S_w)^2$. Oil/water capillary pressure is a function of absolute permeability in addition to water saturation using the Leverett *J*-function correlation. Zero capillary pressure is specified for the column of cells containing production wells.

Specification of wells. Four injectors are located in blocks in the first column and the four layers at the bottom. Water injection rate is constant and equal to 1 m^3/(m day) for the well at the bottom and 3 m^3/(m day) elsewhere. Twelve producers are located in the twelve layers at the bottom in the last row of blocks. All wells produce at a constant bottom hole pressure.

Development period. The development period is 2083 days. Oil recovery after 2083 days is 90.26 %. Although the oil production rate is not exactly zero at the end of the simulation, it seems to have stabilized. Oil recovery increases to 90.87 % if the simulation period is extended to 8333 days.

Results. Figure 5 depicts the distribution of simulated water saturation in the grid after 2083 days. The agreement with the saturation distribution obtained by the *REST2D* is good except for a few blocks in the vicinity of production wells. Oil recovery (90.26 %) obtained by simulation is close to the recovery computed using the *REST2D* (92.0%).

CPU time for the simulation is 28 minutes for the 2083 days case.

Test case 2: Smaller scale

The problem is formulated in a cross section. The size of the domain is 80cm × 60 cm. The size of the grid is 41 × 31. The absolute permeability field consists of a low permeable (10 *mD*) inclusion into a high permeable (1 *D*) background, see Figure 6. The upper and lower boundaries are impermeable. On the left and right boundaries constant pressures are fixed. The injection is through the left boundary of the domain, and the production is through the right boundary where capillary pressure is set to zero. Inside the domain capillary pressure is correlated with absolute permeability through the Leverett *J*-function. Capillary pressure curve corresponding the absolute permeability of 1 *Darcy* together with the water relative permeability is shown in Figure 2.

6 different pressure drops are considered, see Table 1. The resulting water saturation fields are presented in Figure 7 - Figure 9. The remaining oil saturation depends on the pressure drop applied to the rock. It decreases from 61% at low rate, which is close to capillary limit value of 72%, down to 33% at high rate. The dependence of the remaining oil on the average total velocity is shown in Figure 10. The effective permeability to water, Figure 11, increases with the average water saturation increasing.

Discussion

S_{or} measurement in the lab and of cut off problem.

Normally, residual oil saturation is measured in the lab by a centrifuge test. Relative permeability to oil remains greater then zero unless the flow stops. This point corresponds to residual oil saturation. In practice it is never reached, but approached by the capillary pressure curve which asymptotically goes to minus infinity. The possibilities to measure of both capillary pressure and relative permeability lowest values are limited depending on the method used and the accuracy of the equipment. The lowest measurable relative permeability may be about 10^{-4} in flooding experiments while in centrifuge experiments it may be as low as 10^{-7} [4]. Though low k_{ro} values are not measurable in the lab, in the simulation of flow on the field scale, the slow processes controlled by low oil relative permeability values may become significant on later stages of the waterflooding process.

In the proposed model the oil relative permeability does not enter. The oil relative permeability is supposed to be reaching zero value simultaneously with the capillary pressure becoming minus infinity. The resulting solution therefore corresponds to an infinite time.

A reasonable way to relate the resulting saturation field to a finite time is to introduce a cut-off value for the oil relative permeability and/or capillary pressure. This work is planned to perform in the future.

Conclusions

1. A two-phase, model algorithm, and prototype *FORTRAN* code is developed to compute capillary trapped oil in a heterogeneous reservoir in 2D.
2. The correctness and efficiency of the model is tested against an industry standard simulator.

Appendix: Model description

The model to describe capillary trapped oil in two-phase flow is derived based on the Darcy's law for each of the phases and a capillary pressure relationship:

$$\mathbf{u}_i = -\kappa\lambda_i(\nabla p_i - \mathbf{g}\rho_i), \quad i = 1,2$$
$$p_2 - p_1 = P_c(S) \qquad \text{.................. (1)}$$

By summing up the 2 expressions for individual phase velocities and elimination of phase pressures in a standard manner an expression for the total velocity is obtained.

$$\mathbf{u}_1 = f_1\mathbf{u}_t + \kappa f_2\lambda_1\nabla P_c - \kappa\mathbf{g}\lambda_1 f_2(\rho_2 - \rho_1) \text{........(2)}$$

Assume only the first phase (water) is mobile, i.e.

$$\mathbf{u}_1 = \mathbf{u}_t \text{..(3)}$$

Then with account for Equation(3), Equation(2) reads

$$f_2(-\mathbf{u}_t + \kappa\lambda_1\nabla P_c - \kappa\mathbf{g}\lambda_1(\rho_2 - \rho_1)) = 0 \text{ ....(4)}$$

Equation (4) implies an alternative:

either

1. $f_2 = 0$,

or

2. $-\mathbf{u}_t + \kappa\lambda_1\nabla P_c + \kappa\mathbf{g}\lambda_1(\rho_1 - \rho_2) = 0$.

The first case is trivial (relative permeability to the second phase is zero). If the relative permeability to the second phase is greater then zero the following equation holds:

$$\kappa\lambda_1\nabla P_c + \kappa\mathbf{g}\lambda_1(\rho_1 - \rho_2) = \mathbf{u}_t \text{......................(5)}$$

If the fluids are incompressible the conservation law reads

$$div\mathbf{u}_t = Q \text{..(6)}$$

where Q is the density of sources and sinks, positive Q corresponds to injection. Substitution of Equation (5) into the conservation law (6) gives the following equation describing the distribution of residual saturation in a 3D heterogeneous reservoir:

$$div[\kappa\lambda_1\nabla P_c + \kappa\mathbf{g}\lambda_1(\rho_1 - \rho_2)] = Q \text{......................(7)}$$

Leverett *J*-function approach

For simplicity we assume porosity constant. Besides simplicity as already mentioned, an additional argument to use this assumption is the fact that porosity usually changes significantly less than permeability (30% compared to 100 times).

$$P_c = \alpha J(S)/\sqrt{\kappa} \text{..(8)}$$

Substitution of the expression for capillary pressure (8) into Equation (7) gives

$$div\lambda_1(\tau\nabla J - J\nabla\tau + \tau^2\mathbf{g}\Delta\rho) = \frac{Q}{\alpha} \text{..................(9)}$$

$$\Delta\rho = \rho_1 - \rho_2, \quad \tau = \sqrt{\kappa}$$

Equation (9) is a non-linear elliptic equation with respect to J If the relative permeability to water is constant the equation becomes linear. Another case when the equation may be reduced to a linear one is considered below.

Linear case

If

$$\lambda_1 = cJ^k, \quad k \neq -1 \text{...............................(10)}$$

the Equation (9) becomes linear:

$$\frac{1}{k+1}div\sqrt{\kappa}\nabla H - divH\nabla\sqrt{\kappa} = \frac{Q}{c\alpha} \text{..............(11)}$$

with $J^{k+1} = H$.
If, in particular,

$$k = -2, \text{ i.e. } \lambda_1 = \frac{c}{J^2}, \quad c > 0$$

e.g.,

$$\lambda_1 = S^2 / \mu_1, \quad J \propto S^{-1}$$

which corresponds to a **strongly water-wet system,** then:

$$\nabla^2(\sqrt{\kappa}H) = -\frac{Q}{c\alpha} \quad (12)$$

Implementing the principle of the maximal value for the Laplace equation[6] the following conclusion can be drawn.
In this case injection wells (positive Q) will correspond to maximal H-value) and (since $H=1/J=S$) therefore maximal water saturation (in homogeneous reservoir).
If, in contrast to the previous, $\kappa > 0$, $c < 0$ which corresponds to a **strongly oil-wet system,** e.g., $\lambda_1 = S^2 / \mu_1$, $J = -1000S^4$, then in homogeneous reservoir the equation (11) becomes

$$\frac{1}{k+1} div\sqrt{\kappa}\nabla H = \frac{Q}{c\alpha} \quad (13)$$

Since c is negative, in homogeneous reservoir H is maximal at Q>0, i.e. at injection wells, and therefore **water saturation is minimal.**

f = fractional flow function
g = acceleration of gravity vector
J = Leverett J-function
k_r = relative permeability
p = pressure
P_c = capillary pressure
Q = density of sources and sinks
S = water saturation
t = time
u =fluid velocity vector
x = coordinate
α = parameter in Eq. (8)
κ = absolute permeability
λ = mobility
μ = viscosity
ρ = density

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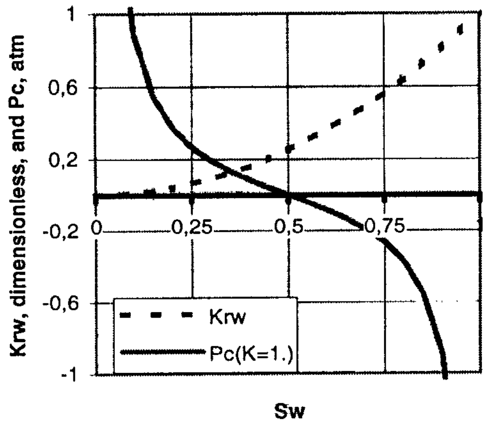

Figure 1. Relative permeability to water and capillary pressure for the test case 1.

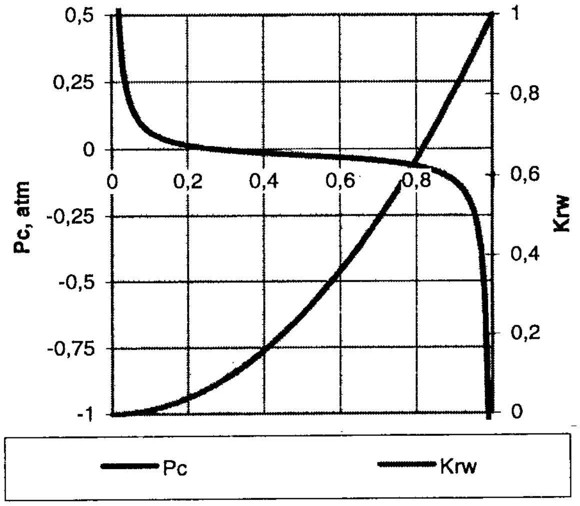

Figure 2. Relative permeability to water and capillary pressure used in the test case 2.

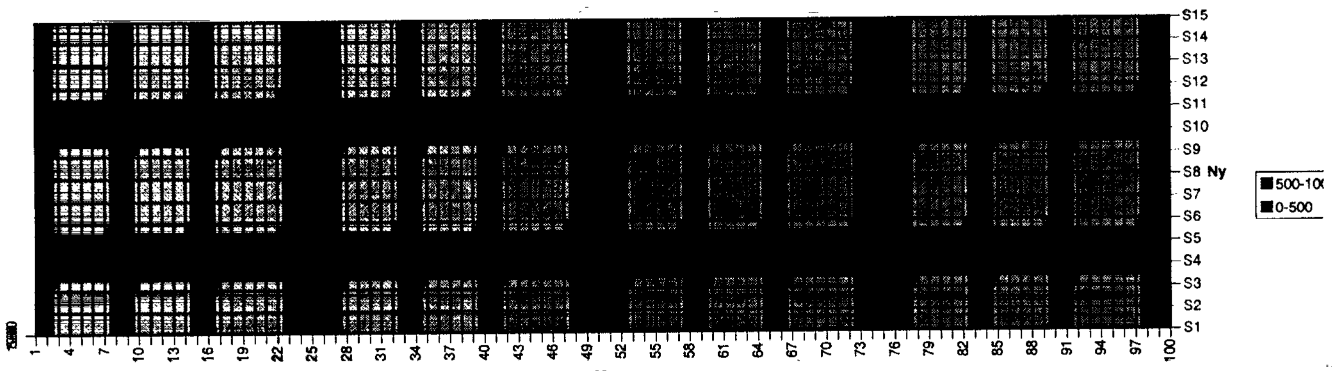

Figure 3. Absolute permeability field for the test case 1. Darker color corresponds to 1D.

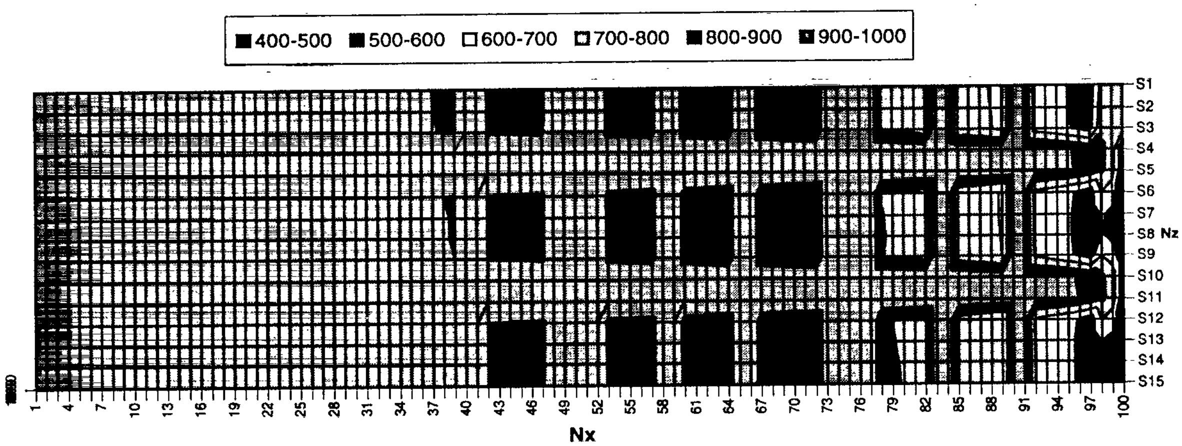

Figure 4. Water saturation distribution, test case 1

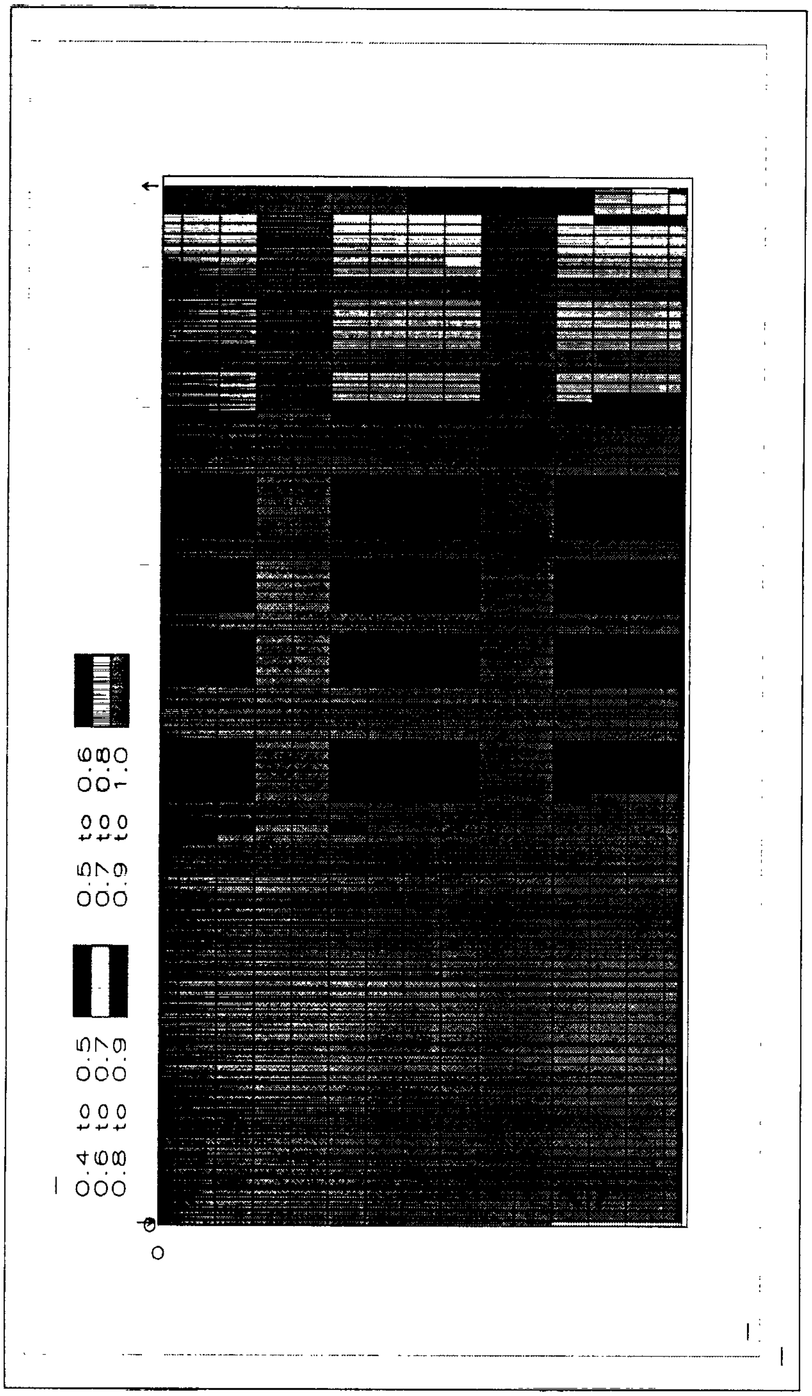

Figure 5. Water saturation field from *ECLIPSE* solution.

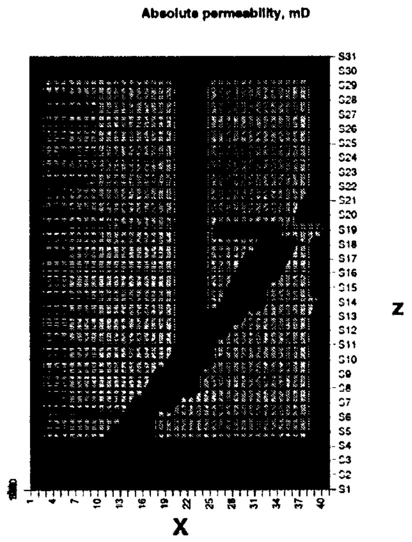

Figure 6. Absolute permeability field for the test case 2. Darker color corresponds to 1D.

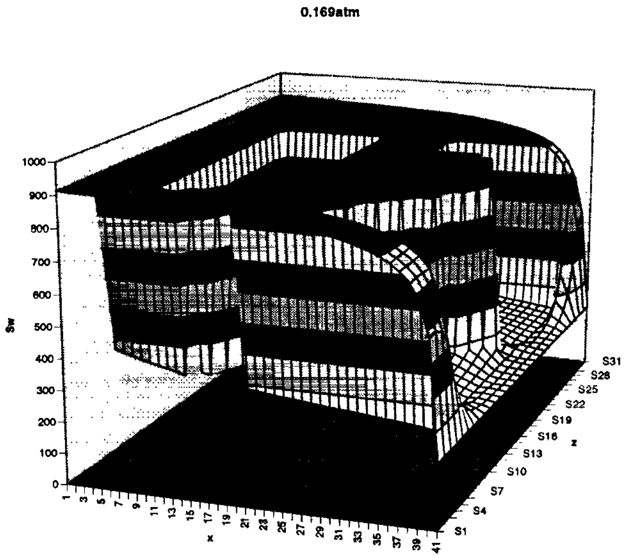

Figure 8. Water saturation distribution at pressure drop 0.169 atm

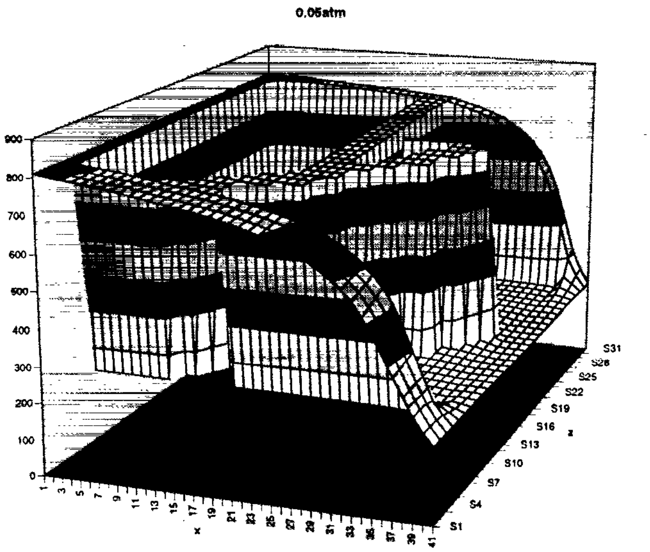

Figure 7. Water saturation distribution at pressure drop 0.05 atm

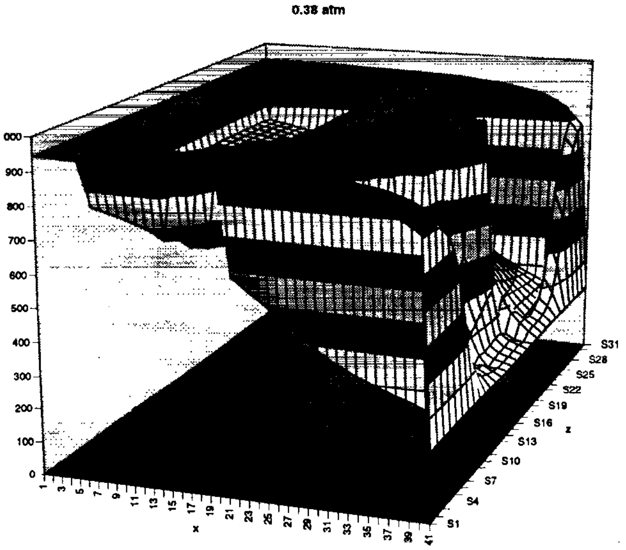

Figure 9. Water saturation distribution at pressure drop 0.38 atm.

TABLE 1 - SIMULATION RESULTS, TEST CASE 2

N	Pressure drop, atm	Rate, sq.cm/sec	Velocity, cm/sec	Velocity, m/day	Average water saturation	Effective permeability to water, Darcy	Average oil saturation
1	0,05	2,40E-03	0,00004	0,03456	0,39	0,036	0,61
2	0,075	4,79E-03	7,98E-05	0,06898	0,42	0,0479	0,58
3	0,113	8,68E-03	0,000145	0,12499	0,45	0,0576	0,55
4	0,169	1,49E-02	0,000248	0,21456	0,484	0,0661	0,516
5	0,253	2,45E-02	0,000408	0,3528	0,562	0,0726	0,438
6	0,38	3,96E-02	0,00066	0,57024	0,671	0,0781	0,329

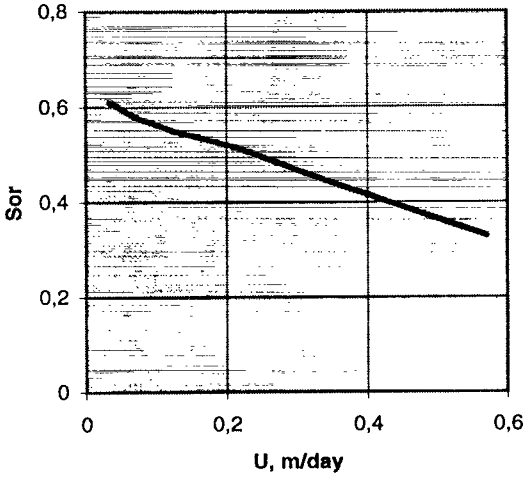

Figure 10. Capillary trapped oil as a function of total rate

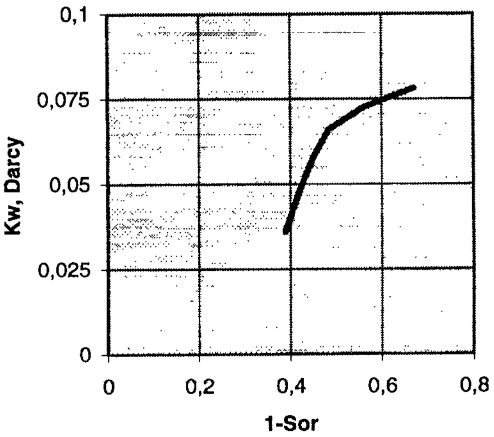

Figure 11. Water phase permeability corresponding to different capillary trapped oil saturations