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Analyzing Pressure Buildup Data by the Rectangular Hyperbola Approach

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SUMMARY

This paper examines applicability and limitations on the use of rectangular hyperbolas to analyze pressure buildup data, with emphasis on the determination of average pressure and flow capacity. It is shown that the method can be used with confidence only if it is applied to data that can also be analyzed by conventional semilog methods, and that it for such data is essentially equivalent to the conventional methods in terms of information needed and information obtained. If we use semilog data, then we can determine the flow capacity from the slope of the hyperbola, and we can determine the average pressure indirectly from the asymptote, provided we know the drainage area and the MBH function of the reservoir. Following stabilized flow we only need the shape factor in addition to the area. If we use the direct approach, and assume that the asymptote is equal to the average pressure, then we need the same type of information to make a proper choice of interval where the hyperbola should match the buildup curve. For this direct approach we will normally get an estimate of average pressure that is less than m/1.151 psi (kPa) above the last wellbore pressure being used in the analysis, where m is the conventional semilog slope. Moreover, if we use only semilog buildup data following pseudosteady-state flow, then we can only get an accurate estimate of average pressure by this approach if the shape factor is close to 21, or higher.

If nothing is known about the reservoir, then the hyperbola method can be used to get a rough estimate of the average pressure, but with a high degree of uncertainty if we only have data from a short buildup period. This claim follows from the many examples included in this paper of asymptotes determined from hyperbolas matched to dimensionless synthetic buildup data plotted vs. interval midpoints.

INTRODUCTION

The Miller-Dyes-Hutchinson[1] (MDH), Matthews-Brons-Hazebroek[2] (MBH), and Dietz[3] methods can be used to determine the average, or static, reservoir pressure for closed reservoirs, and the method of Kumar and Ramey[4] can be used for constant-pressure squares. These methods are based on an indirect use of exact pressure solutions, and hence require knowledge of the size and shape of the drainage area, and of the outer boundary condition. For a given test, all or part of this information might be missing, in which case approximations must be used to carry out the analysis. This leads to uncertainties in estimates of average pressure and other parameters that depend on this information.

A different approach to pressure buildup analysis was suggested by Mead.[5] He observed that pressure buildup curves closely resemble rectangular hyperbolas, and therefore asserted that the average reservoir pressure should be equal to the horizontal asymptote of a hyperbola matched to a buildup curve. Mead supported his assertion by examples.

Hasan and Kabir[6] explored Mead's empirical results further, and presented a theoretical justification for the hyperbola approach to buildup analysis when both the drawdown and buildup transients are in the infinite-acting period. Their work was based on a truncated series expansion of the logarithmic solution. Hasan and Kabir successfully applied the method to examples with different boundary and flow conditions, and concluded that the rectangular hyperbola approach can generally be used to determine the average, or static, reservoir pressure directly from field data, and also that good estimates can be obtained for flow capacity and skin. This without prior knowledge of the size, shape, and type of the reservoir being tested. An analysis of the inherent limitations on the method was not included in Ref. 6.

The general conclusions in Ref. 6 attracted criticism from Humphreys[7] and Bowles and White.[8] In their replies, Hasan and Kabir[9,10] acknowledged the superiority of Horner[11] analysis of infinite-acting reservoirs, but reaffirmed the validity of the method for other cases, again supported by examples.

The main objective of this paper is to present a general analysis of the validity of the rectangular

References and illustrations at end of paper.

hyperbola approach to pressure buildup analysis. For semilog data this is accomplished by deriving analytical expressions for the relationship between the semilog slope, m, and the slope of the hyperbola, and between the average reservoir pressure and the asymptote of the hyperbola. These expressions concern point values, but are also applicable for short intervals. For buildup data beyond the infinite-acting period we only illustrate the limitations on the method through the use of examples. Our main conclusion is that the rectangular hyperbola method is essentially equivalent to the conventional methods in terms of information needed and information obtained, but that it is more difficult to use. All our examples are based on the use of synthetic dimensionless buildup data generated by algorithms presented by Larsen.[12]

THEORETICAL BACKGROUND

The basic assumption in Refs. 5 and 6 is that buildup curves can be approximated by rectangular hyperbolas. It follows that parameters a, b, and c must exist such that the buildup pressure, p_{ws}, can be expressed in the form

$$p_{ws}(\Delta t) = a + \frac{c}{b + \Delta t}, \qquad (1)$$

where Δt is the buildup time.

If Eq. 1 is satisfied for large values of Δt, then a must be equal to the average, or static, reservoir pressure, $\bar{p}$. If it is satisfied in the infinite-acting period, where Horner or MDH analysis can be used, then we can determine the average pressure indirectly from a, and the conventional semilog slope, m, directly from b and c. From m we can then determine kh.

Considering first the semilog slope, note that if both Horner analysis and the hyperbola can be applied in a neighborhood of $\Delta t = \alpha'$, then we must have

$$m \log\frac{\Delta t}{t + \Delta t} + p^* = a + \frac{c}{b + \Delta t}, \qquad (2)$$

where p^* is the Horner false pressure. Setting the derivatives, and hence slopes, equal at $\Delta t = \alpha'$, we get

$$\frac{m}{\ln 10}\left(\frac{1}{\alpha'} - \frac{1}{t + \alpha'}\right) = -\frac{c}{(b + \alpha')^2}, \qquad (3)$$

and hence

$$\frac{m}{\ln 10} = -\frac{(t + \alpha')c\alpha'}{t(b + \alpha')^2} \qquad (4)$$

Similarly, if MDH analysis applies in a neighborhood of $\Delta t = \alpha'$, then we must have

$$m \log \Delta t + p_{1hr} = a + \frac{c}{b + \Delta t}, \qquad (5)$$

where p_{1hr} is the pressure at one hour on the MDH semilog straight line. Setting the derivatives equal at $\Delta t = \alpha'$, we then get

$$\frac{m}{\ln 10} = -\frac{c\alpha'}{(b + \alpha')^2} \qquad (6)$$

Note that Eqs. 4 and 6 are equivalent if $\alpha' \ll t$. It follows from these equations that we can determine the semilog slope, m, and hence the flow capacity, kh, from any hyperbola matched to buildup data from the infinite-acting period, either by Eq. 4, for short producing times, or by Eq. 6 for longer producing times. But this assumes that we <u>only</u> use data from the infinite-acting period, and such data are easier to identify and analyze by the Horner and MDH methods.

As for the relationship between the average pressure, $\bar{p}$, and the asymptote a in Eq. 1, we show below that this depends on α', i.e., on the position of the interval. Hence, to get $\bar{p}$ from a we need to know the difference $\bar{p} - a$ as a function of α'. This is considered in detail below through the use of dimensionless variables.

Theoretical justifications for Eq. 1 were presented by Hasan and Kabir[6] for buildup data from infinite-acting reservoirs, under the assumption that the asymptote equals the average pressure, but their work was otherwise based on verification through the use of examples. A general analysis of the validity of Eq. 1 was not included. Such an analysis is obviously needed, and this is the main objective of the present paper, both along analytical and along numerical lines.

To determine the inherent limitations on the use of Eqs. 1, 4, and 6 in buildup analysis, it is natural to work with dimensionless variables. Therefore, let

$$p_{wD}(t_{DA}) = \frac{kh}{141.2qB\mu}\left[p_i - p_{wf}(t)\right], \qquad (7)$$

denote dimensionless drawdown pressure, where q is the surface rate, p_i the initial pressure, p_{wf} the flowing wellbore pressure, and

$$t_{DA} = \frac{0.000264kt}{\phi\mu c_t A} \qquad (8)$$

the dimensionless time based on the drainage area, A. Moreover, let

$$p_{Ds}(\Delta t_{DA}) = \frac{kh}{141.2qB\mu}\left[p_i - p_{ws}(\Delta t)\right] \qquad (9)$$

denote dimensionless buildup pressure, where the dimensionless buildup time, Δt_{DA}, is based on Eq. 8.

Our theoretical considerations are limited to the analysis of bounded single-well homogeneous reservoirs of regular shape and uniform thickness. The reservoirs are assumed filled with a single fluid of small and constant compressibility and produced by a fully penetrating well with constant rate prior to shut-in. Wellbore storage and skin effects are not considered. In short, we assume that the standard pressure solutions can be used. Since we get

$$p_{Ds}(\Delta t_{DA}) = p_{wD}(t_{DA} + \Delta t_{DA}) - p_{wD}(\Delta t_{DA}), \qquad (10)$$

by superposition, it follows that we can set up simple expressions for the buildup pressure within special flow regimes. This applies to both closed reservoirs and reservoirs with constant pressure, or mixed no-flow/constant pressure outer boundary.

Let us now rewrite Eq. 1 in the dimensionles form

$$p_{Ds}(\Delta t_{DA}) = a_D + \frac{c_D}{b_D + \Delta t_{DA}}, \qquad (11)$$

where

$$a_D = \frac{kh}{141.2qB\mu}(p_i - a), \qquad (12)$$

$$b_D = \frac{0.000264kb}{\varphi\mu c_t A}, \qquad (13)$$

and

$$c_D = \frac{kh}{141.2qB\mu}\,\frac{0.000264k(-c)}{\varphi\mu c_t A} \qquad (14)$$

Assuming Eq. 11 to be valid on a chosen interval, our objective is to determine restrictions on the positioning of the interval, and also on the type of reservoir being analyzed, in order for the parameters a_D, b_D, and c_D to give sufficiently accurate estimates of the parameters being sought.

Considering the position of the interval, we have chosen to use α to denote <u>the logarithmic midpoint of the interval being used to determine the hyperbola parameters</u> in terms of dimensionless data, and α' in the same meaning for real data. This only affects the hyperbola parameters obtained by least-squares methods applied to synthetic buildup data, since the analytical derivations only refer to the interval midpoint, and not to the end points. The α used in Ref. 6 has a similar meaning, but only in terms of Horner time ratios.

The main objective of the rectangular hyperbola approach to pressure buildup analysis is to determine the average pressure of the reservoir. Let us therefore introduce the notation

$$\bar{p}_{Ds} = \frac{kh}{141.2qB\mu}(p_i - \bar{p}) \qquad (15)$$

for the dimensionless average, or static, reservoir pressure. From material balance we know that

$$\bar{p}_{Ds} = 2\pi t_{DA} \qquad (16)$$

for closed reservoirs, while we for reservoirs with all or part of the outer boundary kept at constant pressure must have

$$\bar{p}_{Ds} = 0, \qquad (17)$$

since the static pressure is then equal to the initial pressure. For both cases we get

$$a_D - \bar{p}_{Ds} = \frac{kh}{141.2qB\mu}(\bar{p} - a) \qquad (18)$$

We can therefore determine the average pressure from the asymptote when kh and the left-hand side of Eq. 18 are known.

It should be obvious that Eq. 18 is a function of the midpoint of the interval being used to determine a_D, and that the main assumption in Refs. 5 and 6, namely that the right-hand side equals 0, can only be satisfied at very late buildup times, or at isolated earlier buildup times. It will also be shown below that if we have sufficient information to use Eq. 18 to determine the average pressure from infinite-acting buildup data, then we can more easily use conventional methods. Moreover, if we use infinite-acting data, then we <u>must</u> use Eq. 18, either directly or indirectly. If we use data beyond the infinite-acting period, then the analytical expressions of this paper cannot be used, but Eq. 18 still applies. For such data we only consider the relationship between buildup pressures used in the analysis and the average reservoir pressure when the right-hand side of Eq. 18 equals 0.

To get the flow capacity, kh, from buildup data following a short producing period, it follows from Eq. 4 that we must have

$$\varrho = \frac{t_{DA}}{t_{DA} + \alpha}, \qquad (19)$$

where ϱ is defined by the equation

$$\varrho = \frac{2\alpha c_D}{(b_D + \alpha)^2} \qquad (20)$$

If α is small compared to t_{DA}, then we just get

$$\varrho = 1, \qquad (21)$$

and this is equivalent to Eq. 6. The symbol ϱ has been introduced to simplify labels on plots.

To derive expressions for the parameters of the hyperbola, Hasan and Kabir[6] used a truncated series expansion of the Horner solution for infinite-acting reservoirs. A more general approach is to require the two sides of Eq. 11 to have the same value, slope, and curvature at $\Delta t_{DA} = \alpha$, i.e., to require the derivatives of order 0, 1, and 2 to be equal. We then get a hyperbola that must match the buildup curve in a neighborhood of α. In other words, we just need to set

$$p_{wD}(t_{DA}+\alpha) - p_{wD}(\alpha) = a_D + \frac{c_D}{b_D+\alpha}, \quad \ldots \ldots (22)$$

$$p'_{wD}(t_{DA}+\alpha) - p'_{wD}(\alpha) = \frac{-c_D}{(b_D+\alpha)^2}, \quad \ldots \ldots (23)$$

and

$$p''_{wD}(t_{DA}+\alpha) - p''_{wD}(\alpha) = \frac{2c_D}{(b_D+\alpha)^3} \quad \ldots \ldots (24)$$

The solutions to these equations are

$$a_D = p_{wD}(t_{DA}+\alpha) - p_{wD}(\alpha) - \frac{2[p'_{wD}(t_{DA}+\alpha) - p'_{wD}(\alpha)]^2}{p''_{wD}(t_{DA}+\alpha) - p''_{wD}(\alpha)}, \quad \ldots \ldots (25)$$

$$b_D = -\alpha - \frac{2[p'_{wD}(t_{DA}+\alpha) - p'_{wD}(\alpha)]}{p''_{wD}(t_{DA}+\alpha) - p''_{wD}(\alpha)}, \quad \ldots \ldots (26)$$

and

$$c_D = -\frac{4[p'_{wD}(t_{DA}+\alpha) - p'_{wD}(\alpha)]^3}{[p''_{wD}(t_{DA}+\alpha) - p''_{wD}(\alpha)]^2} \quad \ldots \ldots (27)$$

By substituting Eqs. 26 and 27 into Eq. 20 we also get

$$\rho = -2\alpha[p'_{wD}(t_{DA}+\alpha) - p'_{wD}(\alpha)] \quad \ldots \ldots (28)$$

Effects of α on a_D and ρ are not immediately evident from these expressions, but such effects do become evident when we substitute solutions for special flow regimes.

BUILDUP DATA FROM INFINITE-ACTING RESERVOIRS

As was pointed out by Hasan and Kabir,[10] the Horner method should be used to analyze buildup data from infinite-acting reservoirs. Still, since the rectangular hyperbola approach was applied to this case in Ref. 8, we have also chosen to include an analysis of this case in the present paper.

A reservoir is infinite-acting if $t_{DA} + \Delta t_{DA} < t_{DAeia}$, in which case we get

$$p_{wD}(\Delta t_{DA}) = \frac{1}{2}\ln\frac{4A\Delta t_{DA}}{e^{\gamma}r_w^2} \quad \ldots \ldots (29)$$

and

$$p_{wD}(t_{DA} + \Delta t_{DA}) = \frac{1}{2}\ln\frac{4A(t+\Delta t)_{DA}}{e^{\gamma}r_w^2} \quad \ldots \ldots (30)$$

when wellbore effects are not included, where γ denotes Euler's constant (0.57721...) and r_w the wellbore radius. This form of the logarithmic solution is used here for direct comparison with the solutions for stabilized flow.

For this particular case we have chosen an approach similar to that used in Ref. 6 in order to get a comparison of results. Now, by combining Eqs. 10, 29, and 30, we get

$$p_{Ds}(\Delta t_{DA}) = \frac{1}{2}\ln\frac{t_{DA}+\Delta t_{DA}}{\Delta t_{DA}} \quad \ldots \ldots (31)$$

for the buildup period. From the results in Appendix A, it follows that we can set

$$p_{Ds}(\Delta t_{DA}) = \frac{1}{2}\ln\alpha_H + 1 - \frac{2\alpha_H}{\alpha_H + \frac{t_{DA}+\Delta t_{DA}}{\Delta t_{DA}}}, \quad \ldots \ldots (32)$$

with negligible error for Horner time ratios $(t_{DA} + \Delta t_{DA})/\Delta t_{DA}$ near α_H, or equivalently, for buildup times Δt_{DA} near α, where α and α_H are related by the equation

$$\alpha_H = \frac{t_{DA}+\alpha}{\alpha} \quad \ldots \ldots (33)$$

Eq. 32 can be used directly if the analysis is based on a Horner plot in <u>linear</u> coordinates, preferably rewritten in terms of an inverted Horner time ratio. However, to base the analysis on a linear plot in terms of Δt_{DA}, note that Eqs. 11 and 32 are equivalent if

$$a_D = \frac{1}{2}\ln\frac{t_{DA}+\alpha}{\alpha} - \frac{t_{DA}}{t_{DA}+2\alpha}, \quad \ldots \ldots (34)$$

$$b_D = \frac{\alpha t_{DA}}{t_{DA}+2\alpha}, \quad \ldots \ldots (35)$$

and

$$c_D = \frac{2\alpha(t_{DA}+\alpha)t_{DA}}{(t_{DA}+2\alpha)^2} \quad \ldots \ldots (36)$$

If Eqs. 35 and 36 are substituted into Eq. 20, then we also find that Eq. 19 is satisfied. The same results are obtained if we substitute Eqs. 29 and 30 into Eqs. 25 - 28.

Since Eq. 19 is satisfied, it follows that Eq. 4 can be used to determine kh if the reservoir is infinite-acting. But note that since a_D can be computed directly from Eq. 34, we cannot use Eq. 18 to determine $\bar{p}$ from a. To get the average pressure from Eq. 18 we need a more exact expression for a_D. Such an expression is presented in the section on buildup analysis following non-stabilized flow. However, note that it is still possible that a value of α can be found where $a_D = \bar{p}_{Ds}$, or in terms of real data, that an α' can be found where $a = \bar{p}$. This possibility, and the validity of Eqs. 19 and 34, are illustrated below.

Hasan and Kabir[6] expressed their results in terms of α_H, and in addition assumed that $\Delta t \ll t$ at some step in their derivation. The results derived above are therefore somewhat different from those in Ref. 6, but note that Eq. 32 is equivalent to Eq. 6 in Ref. 6. It was also assumed in Ref. 6 that $a_D = \bar{p}_{Ds}$, which for infinite-acting reservoirs normally means that we should have $a_D \approx 0$. Actually, if we set $a_D = 0$ and $t_{DA} + \alpha = t_{DA}$ in Eq. 34, then we get $\alpha = e^{-2} t_{DA} = 0.135 t_{DA}$. Hence, if the reservoir is truly infinite acting, then we should not expect to get $a = \bar{p}$ if we use buildup data with $\Delta t \ll t$ to determine a.

The validity of Eqs. 19 and 34 is illustrated in Figs. 1 - 3. Buildup data for these examples, and for the other examples used in this paper, have been generated by the algorithms in Ref. 12, and each example represents a reservoir with area given by $\sqrt{A} = 2000 r_w$. For Figs. 1 - 3 we have used buildup data from a closed square with the well at the center and a producing time $t_{DA} = 0.01$ prior to shut-in. For such reservoirs we normally assume that $t_{DAeia} = 0.08$, but this definition depends on what is considered to be a significant deviation.

Fig. 1 illustrates the effect of varying the length of the interval used to determine the parameters of the hyperbola. Two hyperbolas centered at α = 0.0043 are included, one based on 0.04 log cycles of buildup data and one based 1 log cycle of data. The buildup curve is represented by the solid line, the hyperbola based on the short interval by the dashed line, and the hyperbola based on the longer interval by the dotted line. If we use 2 log cycles of data, then we get a hyperbola that almost coincides with the buildup curve. Since the analytical results of this paper concern point values, we get the most accurate results for short intervals. Note, for instance, that $a_D = 0.0628 = 2\pi t_{DA}$ for α = 0.0043, according to Eq. 34, and that we get $a_D = 0.0625$, 0.0487, and 0.0278 from the hyperbolas centered at α = 0.0043 and based on 0.04, 1, and 2 log cycles of buildup data, respectively. The parameters of these hyperbolas were all computed from the least-squares method numbered 2 in Appendix B, and this is used as our standard least-squares method to match hyperbolas to buildup data.

In Fig. 2 we have illustrated the validity of Eq. 34 by plotting values of a_D determined from hyperbolas matched to the buildup data and computed by Eq. 34. Both are plotted as functions of the interval midpoint, α. The solid lines represent values determined from asymptotes of hyperbolas based on intervals of length 0.04 and 1 log cycles, and the dashed line represents values computed by Eq. 34. We have also included the buildup curve, represented by the dotted line, for comparison. Eq. 34 was derived for short intervals, and for such intervals it is seen to be essentially exact in the infinite-acting period, which for the analysis here is seen to end around $\Delta t_{DA} = 0.02$. If intervals of length 1 log cycle are used, then the asympotes of the hyperbolas deviate somewhat from the values given by Eq. 34. Considering the curves in Fig. 2, note that for intervals of a chosen length, there is only one position in the infinite-acting period that will, strictly speaking, give an asymptote equal to the average pressure. But note that we also get $a_D = \bar{p}_{Ds}$ from data influenced by the boundary. Note also that the absolute error introduced by setting $a_D = \bar{p}_{Ds}$ is small for $\alpha > 0.003$, but that the relative error can be significant in this range of data. Moreover, both types of error increase sharply for smaller buildup times.

The validity of Eq. 19 is illustrated in Fig. 3 for the same data used in the Figs. 1 and 2. In this figure we have plotted values of ρ determined from hyperbolas matched to the buildup data, and values of ρ computed from Eq. 19, with both plotted as functions of α. The solid lines represent values determined from hyperbolas based on 0.04 and 1 log cycles of data, and the dashed line values given by Eq. 19. Note that Eq. 19 can be considered to be exact for $\alpha \leq 0.02$ for the shorter intervals, and also to be fairly accurate for $\alpha \leq 0.03$, both for the shorter and longer intervals. It follows that Eq. 4 can be applied to data from infinite-acting reservoirs, but such reservoirs can also be analyzed by the Horner method.

If we use a constant-pressure square instead of a closed square in Figs. 2 and 3, then we get exactly the same results for data not influenced by the outer boundary. For data influenced by the boundary, we get values of a_D <u>below</u> those given by Eq. 34, and values of ρ <u>above</u> those given by Eq. 19.

It should be noted that if Eqs. 11 and 19 are both satisfied at $\Delta t_{DA} = \alpha$, then we get

$$a_D = p_{Ds}(\alpha) - \frac{t_{DA}}{t_{DA} + 2\alpha}, \quad \ldots \ldots \ldots \ldots (37)$$

and hence

$$a = p_{ws}(\alpha') + \frac{m}{1.151}\frac{t}{t + 2\alpha'} \quad \ldots \ldots \ldots (38)$$

in terms of real data. It follows that if we get an accurate estimate of the flow capacity from a chosen hyperbola, then the asymptote cannot be equal to the average pressure unless the wellbore pressures used are less than m/1.151 psi (kPa) below $\bar{p}$.

To complete this section, note that b is known in the infinite-acting period if the producing time is known. Therefore, to analyze buildup data from infinite-acting reservoirs by the rectangular hyperbola method, we can set

$$b = \frac{\alpha' t}{t + 2\alpha'}, \quad \ldots \ldots \ldots \ldots (39)$$

where α' is still the midpoint of the interval in real data, and then make a linear plot of

$$p_{ws}(\Delta t) \quad \text{vs.} \quad \frac{1}{b + \Delta t}$$

Data points near $\Delta t = \alpha'$ should then form a straight line with slope = c and intercept = a (for $\Delta t = \infty$). This is similar to one of the methods suggested in Ref. 6. For data that are not influenced by any boundaries, the approach just outlined is equivalent to a direct use of least squares.

BUILDUP DATA FOLLOWING PSEUDOSTEADY-STATE FLOW

Consider a closed reservoir produced to pseudosteady state prior to shut-in, i.e., with $t_{DA} > t_{DApss}$. We then get

$$p_{wD}(t_{DA} + \Delta t_{DA}) = 2\pi(t + \Delta t)_{DA} + \frac{1}{2}\ln\frac{4A}{e^{\gamma}C_A r_w^2} \quad \ldots \quad (40)$$

for the drawdown transient when we have zero skin. If the buildup transient is in the infinite-acting period, i.e., if $\Delta t_{DA} \leq t_{DAeia}$, then this part is given by Eq. 29. It follows from Eq. 10 that

$$p_{Ds}(\Delta t_{DA}) = 2\pi t_{DA} + 2\pi\Delta t_{DA} - \frac{1}{2}\ln\Delta t_{DA} - \frac{1}{2}\ln C_A \quad \ldots \quad (41)$$

for this range of buildup data.

To replace the right-hand side of Eq. 41 by a hyperbola, we can neglect the term $2\pi\Delta t_{DA}$, as is done when MDH analysis is applied to such buildup data, and apply the results in Appendix A to $\ln\Delta t_{DA}$. We then get $b_D = \alpha$ and $c_D = 2\alpha$. It is, however, more correct to substitute Eqs. 29 and 40 into Eqs. 25 - 28. We then get

$$a_D = 2\pi t_{DA} + 2\pi\alpha - \frac{1}{2}\ln\alpha C_A - (1 - 4\pi\alpha)^2 , \quad \ldots \quad (42)$$

$$b_D = \alpha(1 - 8\pi\alpha), \quad \ldots \quad (43)$$

$$c_D = 2\alpha(1 - 4\pi\alpha)^3, \quad \ldots \quad (44)$$

and

$$\varrho = 1 - 4\pi\alpha. \quad \ldots \quad (45)$$

From Eqs. 16 and 42 it follows that the left-hand side of Eq. 18 takes the form

$$a_D - \bar{p}_{Ds} = 2\pi\alpha - \frac{1}{2}\ln\alpha C_A - (1 - 4\pi\alpha)^2 \quad \ldots \quad (46)$$

From Eq. 45 it follows that Eq. 21 is valid for small values of α, and hence that we can use Eq. 6 to determine the flow capacity from a hyperbola based on early buildup data. If we know the area and shape factor of the reservoir, then we can also determine the average pressure from the asymptote of the same hyperbola by using Eq. 46 to compute the left-hand side of Eq. 18. With this information we can, of course, also use Dietz' method to determine the average pressure. If we do not know the area or the shape factor of the reservoir, and just assume that $a_D = \bar{p}_{Ds}$ for some chosen interval, i.e., for some chosen α, then we can use Dietz' method with the same uncertainty, as is explained below. Considering Eq. 45, note that if $\alpha < 0.004$ for the midpoint of the interval where the hyperbola has been matched to the buildup data, then we should get less than 5% error if we use Eq. 6 to estimate the flow capacity.

Considering the assertions in Refs. 5 and 6, note that if the asymptote of a hyperbola based on infinite-acting data is equal to the average pressure, and the reservoir has been produced to pseudosteady state prior to shut-in, then the dimensionless interval midpoint, α, must satisfy the equation

$$2\pi\alpha - \frac{1}{2}\ln\alpha C_A - (1 - 4\pi\alpha)^2 = 0. \quad \ldots \quad (47)$$

For a closed square with the well at the center we get the shape factor $C_A = 30.8828$ from Table C.1 in Ref. 13. It follows that Eq. 47 has three solutions: α = 0.00651, 0.0581, and 0.0997. Only the first of these is clearly in the infinite-acting period. Actually, if $C_A < 20.95$, then Eq. 47 has only one solution, and only beyond the infinite-acting period. For such reservoirs it follows that a hyperbola based on infinite-acting data cannot have an asymptote equal to the average pressure.

Note that we cannot use Eq. 47 to determine the position where the asymptote should be equal to the average pressure unless k, A, and C_A are known, in which case we can also use Dietz' method to determine the average pressure. In this case we can also compute the right-hand side of Eq. 46, and hence use Eq. 18 to determine $\bar{p}$ from any hyperbola matched to an interval of infinite-acting data. Note also that Eq. 47 can be used to determine, as we have done, restrictions on the type of reservoir and data that can give $\bar{p}$ directly from the asymptote, provided the hyperbola is based on infinite-acting buildup data. For such data following pseudosteady-state flow, we also get the following: If Eq. 47 is satisfied for some $\alpha \ll 1/4\pi$, i.e., if $a_D = \bar{p}_{Ds}$ for such an α, then we must have $e^2\alpha C_A = 1$. But this implies that we can get $\bar{p}_2$ by extrapolating the MDH straight line to $\Delta t = e^2\alpha'$, since this Δt is seen to be the buildup time needed for Dietz' method.

The validity of the results of this section is illustrated in Figs. 4 - 11, with the first three cases representing a closed square produced to $t_{DA} = 1$ prior to shut-in, and with the well located at the center. This reservoir has the shape factor C_A = 30.8828, and $t_{DApss} = 0.1$. It follows from Eq. 46 that $a_D = \bar{p}_{Ds}$ for $\alpha = 0.0065$. In Fig. 4 we have matched a hyperbola to the buildup curve at this α by using Method 2 on an interval of length 0.067 log cycles. From the hyperbola we got $a_D = 6.27$, which is almost equal to $\bar{p}_{Ds} = 2\pi t_{DA} = 6.28$.

Fig. 5 illustrates the validity of Eq. 42, and hence of Eq. 46, for the same closed square considered in Fig. 4. In this figure the solid lines represent values of a_D computed by Method 2 for intervals of length 0.067 and 1 log cycles, the dashed line represents a_D computed by Eq. 42, and the dotted line represents the buildup curve. Note that we only get $a_D = 6.28$ for $\alpha = 0.0065$ in the infinite-acting period, as was pointed out above. However, any value of a_D determined by Method 2 on a short interval with $\alpha \leq 0.03$ can be used to determine $\bar{p}_{Ds}$ from Eq. 46, since we know that $C_A = 30.8828$. From the curves in Fig. 5 we see that the asymptotes are affected if we increase the length of the intervals from 0.067 to 1 log cycle, but that the relative change is not large for this example. Note also that the absolute change will be the same for any producing time in the pseudosteady-state period. Finally, if we assume that $a_D = \bar{p}_{Ds}$, then we can get large errors for short buildup times. Moreover, we can then also use Dietz' method.

In Fig. 6 we have illustrated the validity of Eq. 45 for the closed square considered in Figs. 4 and 5. In this figure the solid lines represent values of ϱ computed by Method 2 for intervals of length 0.067 and 1 log cycles, and the dashed line ϱ computed by Eq. 45. We again find an excellent agreement between values determined from Method 2 applied to short intervals of synthetic data, and values computed by Eq. 45 for $\alpha \leq 0.03$. It follows that we can use Eq. 6 to determine the flow capacity from early buildup data, but such data can of course also be analyzed by conventional methods.

Fig. 7 illustrates the validity of Eq. 42, and hence of Eq. 46, for a closed 2:1 rectangle with the well located 1/4 of the length and 1/2 of the height of the drainage area, and produced to $t_{DA} = 2$ prior to shut-in. The solid line represents values of a_D determined from asymptotes of hyperbolas based on intervals of length 0.05 log cycles, the dashed line represents a_D computed by Eq. 42, and the dotted line represents the buildup curve. We see that Eq. 42 applies in the infinite-acting period, therefore, if we know k and A for this reservoir, then we can use the shape factor $C_A = 4.5141$ (see Ref. 13) to determine the average pressure from the asymptote of any hyperbola matched to a short interval of buildup data in the infinite-acting period. If we set $a_D = \bar{p}_{Ds}$ for this example, then we must use data around $\alpha \simeq 0.064$ to get the correct average pressure, i.e., we must use data well beyond the end of the infinite-acting period. Fig. 8 shows how a hyperbola with asymptote equal to $2\pi t_{DA} = 12.57$ matches the buildup curve when an interval of 0.05 log cycles centered at $\alpha = 0.064$ is used. If we use a longer interval, then we must use a larger value of α to get the same asymptote. Fig. 9 shows that Eq. 45 also applies in the infinite-acting period, hence, if we use early buildup data, then we can use Eq. 6 to determine the flow capacity, and hence k, from a rectangular hyperbola.

In Figs. 10 and 11 we have illustrated the validity of Eqs. 42 and 45 for a closed 4:1 rectangle with the well located 1/4 of the length and 3/4 of the height of the drainage area, and produced to $t_{DA} = 5$ prior to shut-in. For this example it turns out that we must center the hyperbola near $\alpha = 0.12$ to get an asymptote equal to $2\pi t_{DA} = 31.42$ when intervals 0.06 log cycles long are used. However, note that Eqs. 42 and 45 do apply for early buildup data, hence, by using such data we can determine both the flow capacity and the average pressure from a hyperbola, provided we know the area and the shape factor $C_A = 0.1155$. Of course, we can then also use conventional methods. Note the sharp deviations between values determined from hyperbolas and from Eqs. 42 and 45 when boundary effects start to influence the data.

Finally, note that if Eqs. 11, 43, and 44 are satisfied at α, then we get

$$a_D = p_{Ds}(\alpha) - (1 - 4\pi\alpha)^2, \quad \ldots\ldots (48)$$

and hence

$$a = p_{ws}(\alpha') + \frac{m}{1.151}(1 - 4\pi\alpha)^2. \quad \ldots\ldots (49)$$

in terms of real data. It follows that if we get an accurate estimate of the flow capacity from a chosen hyperbola, then the asymptote cannot be equal to the average pressure unless the wellbore pressures used are less than m/1.151 psi (kPa) below $\bar{p}$.

BUILDUP DATA FOLLOWING STEADY-STATE FLOW

Consider a reservoir with constant pressure or mixed no-flow/constant pressure outer boundary, and assume that the reservoir has been produced to steady state prior to shut-in, i.e., that $t_{DA} \geq t_{DAss}$. It follows that

$$p_{wD}(t_{DA} + \Delta t_{DA}) = \frac{1}{2}\ln \frac{16A}{e^{\gamma}C_A r_w^2}, \quad \ldots\ldots (50)$$

where the shape factor C_A is defined relative to the initial pressure, which is the same as the static reservoir pressure for these reservoirs. This is the convention used by Kumar and Ramey[3] for a well in the center of a constant-pressure square and by Larsen[14] for other reservoir configurations.

If $\Delta t_{DA} \leq t_{DAeia}$, then it follows from Eqs. 10, 29, and 50 that

$$p_{Ds}(\Delta t_{DA}) = \frac{1}{2}\ln 4 - \frac{1}{2}\ln C_A - \frac{1}{2}\ln\Delta t_{DA}. \quad \ldots\ldots (51)$$

By direct application of results from Appendix A, or by use of Eqs. 25 - 28, it follows that Eq. 11 is satisfied if

$$a_D = \ln 2 - \frac{1}{2}\ln\alpha C_A - 1, \quad \ldots\ldots (52)$$

$$b_D = \alpha, \quad \ldots\ldots (53)$$

and

$$c_D = 2\alpha. \quad \ldots\ldots (54)$$

Moreover, for these values we also get Eq. 21 satisfied.

Since the dimensionless static pressure is equal to zero, it follows that the left-hand side of Eq. 18 is equal to Eq. 52. Hence, if we can determine kh from Eq. 21, and in addition know the area and shape factor of the reservoir, then we can use Eq. 18 to determine the static pressure from the asymptote of any hyperbola matched to data from the infinite-acting period. With this information we can also determine the static pressure by Dietz' method.

If we use the assumption in Refs. 5 and 6 that $a_D = \bar{p}_{Ds}$, then we must also assume that

$$\ln 4 - \ln \alpha C_A - 2 = 0, \quad (55)$$

or

$$\alpha = \frac{4e^{-2}}{C_A}, \quad (56)$$

provided the hyperbola is matched to infinite-acting data. But we are then indirectly assuming, through Eq. 56, that the static pressure can be obtained by extrapolating the MDH semilog straight line to $\Delta t = e^2\alpha$, i.e., that Dietz' method can be used. Moreover, note that this approach will not work unless, C_A is sufficiently large to give a value of α in the infinite-acting period.

From Refs. 4 and 14 it follows that $C_A = 30.8828$ and $t_{DAss} = 0.25$ for a square with constant-pressure outer boundary and the well at the center. A hyperbola based on a short interval centered at $\alpha = 0.0175$ should therefore yield the asumptote $a_D = 0$, according to Eq. 52. Fig. 12 illustrates this claim for such a constant-pressure square produced to $t_{DA} = 1$ prior to shut-in. With 0.067 log cycles of buildup data centered at $\alpha = 0.0175$ used to determine the parameters of the hyperbola by Method 2, we get $a_D = -0.0013$.

Figs. 13 and 14 illustrate the validity of Eqs. 21 and 52 for the same square considered in Fig. 12. In Fig. 13 we have plotted values of a_D determined from hyperbolas based on intervals of length 0.067 and 1 log cycles, and values of a_D computed from Eq. 52. The solid lines represent values from the hyperbolas, the dashed line values from Eq. 52, and the dotted line the buildup curve. Note that we only get $a_D = 0$ for short intervals near $\alpha = 0.0175$, in the infinite-acting period, as was asserted above. We can, however, use any value of a_D determined from a short interval with $\alpha < 0.03$ to determine $\bar{p}_{Ds}$ from Eqs. 18 and 52, since we know that $C_A = 30.8828$. Note also that the asymptotes are affected if we increase the length of the intervals from 0.067 to 1 log cycle, but that the relative change is not large. Moreover, the absolute change will be the same for any producing time in the steady-state period. Finally, if we assume that $a_D = \bar{p}_{Ds}$, then we can get large errors for short buildup times. Of course the type of data considered here can also be analyzed by the MDH and Dietz methods.

In Fig. 14 we have illustrated the validity of Eq. 21 by plotting values of ϱ determined from hyperbolas based on intervals of length 0.067 and 1 log cycles vs. interval midpoints. Note that we do get $\varrho = 1$ from short intervals of infinite-acting data, as asserted in Eq. 21, and that the error for the intervals of length 1 log cycle is less than 7% in the infinite-acting period. It follows that we can use Eq. 6 to determine the flow capacity from early buildup data, but such data can of course also be analyzed by conventional methods.

Fig. 15 illustrates the validity of Eq. 52 for a square with one side at constant pressure and the well located 3/4 of the length of the reservoir from this side, and at 1/2 of the height. The buildup data were generated with a flow period of $t_{DA} = 5$ prior to shut-in. In Ref. 14 it is shown that $C_A = 0.02602$ and $t_{DAss} = 2.46$ for such reservoirs. In Fig. 15 we have plotted values of a_D determined from asymptotes of hyperbolas based on intervals 0.06 log cycles long, and values of a_D computed from Eq. 52, both as functions of α. The solid line represents the asymptotes, the dashed line Eq. 52, and the dotted line the buildup curve. We see that Eq. 52 applies in the infinite-acting period, and therefore, if we know k and A for this reservoir, then we can use the shape factor $C_A = 0.02602$ to determine the static pressure from the asymptote of any hyperbola matched to a short interval of buildup data in the infinite-acting period. With this information we can also use conventional methods. If we set $a_D = \bar{p}_{Ds}$ for this reservoir, then we must use data around $\alpha = 0.085$ to get the correct static pressure if we use short intervals. If we use longer intervals, then we must use a larger value of α to get the same asymptote. Note also that large errors are introduced if we set $a_D = \bar{p}_{Ds}$ for hyperbolas matched to early buildup data.

Fig. 16 shows that Eq. 21 is satisfied for early buildup data from the reservoir considered in Fig. 15, and hence that we can determine the flow capacity from such reservoirs by using Eq. 6. But this analysis is easier on an MDH plot.

Finally, note that if Eqs. 11 and 21 are both satisfied at α, then we get

$$a_D = p_{Ds}(\alpha) - 1, \quad (57)$$

and hence

$$a = p_{ws}(\alpha') + \frac{m}{1.151} \quad (58)$$

in terms of real data. It follows that if we get an accurate estimate of the flow capacity from a chosen hyperbola, then the asymptote cannot be equal to the average pressure unless the wellbore pressures used are less than m/1.151 psi (kPa) below $\bar{p}$.

BUILDUP DATA FOLLOWING NON-STABILIZED FLOW

This section is restricted to closed reservoirs, due to the lack of general results for reservoirs with constant pressure on all or part of the outer boundary.

If we do not have stabilized flow in the reservoir prior to shut-in, then we cannot use Eqs. 25 - 27 to get the parameters of the hyperbola in Eq. 11 unless we have expressions for the derivatives of the first and second order of p_{wD}. Note, however, that if Eq. 11 is satisfied at α, then we must

$$a_D = p_{wD}(t_{DA} + \alpha) - p_{wD}(\alpha) - \frac{c_D}{b_D + \alpha} \quad (59)$$

Now, if α is in the semilog period, then this equation can also be expressed in the form

$$a_D = 2\pi t_{DA} + 2\pi\alpha - \frac{1}{2}p_{DMBH}(t_{DA} + \alpha) + \frac{1}{2}\ln\frac{t_{DA} + \alpha}{\alpha} - \frac{c_D}{b_D + \alpha} \quad \ldots\ldots (60)$$

This follows from the identity

$$p_{wD}(t_{DA}) - \frac{1}{2}\ln\frac{4At_{DA}}{e^{\gamma}r_w^2} = 2\pi t_{DA} - \frac{1}{2}p_{DMBH}(t_{DA}), \quad \ldots\ldots (61)$$

where p_{DMBH} denotes the MBH function of the reservoir. The left-hand side of Eq. 18 can therefore be expressed in the form

$$a_D - \bar{p}_{Ds} = 2\pi\alpha - \frac{1}{2}p_{DMBH}(t_{DA} + \alpha) + \frac{1}{2}\ln\frac{t_{DA} + \alpha}{\alpha} - \frac{c_D}{b_D + \alpha} \quad \ldots\ldots (62)$$

for closed reservoirs.

Now, to use Eq. 62 in a given analysis, note that if we determine the flow capacity from ρ, then we can convert b, c, and α', determined from a hyperbola based on real data, to the last term in Eq. 62 by setting

$$\frac{c_D}{b_D + \alpha} = \frac{kh(-c)}{141.2qB\mu(b + \alpha')} \quad \ldots\ldots (63)$$

Hence, once we have matched a hyperbola to infinite-acting buildup data, we actually assume everything known on the right-hand side of Eq. 62, except $2\pi\alpha$ and the value of the MBH function. From Eq. 18 it follows that we can determine $\bar{p}$ from a if we know A and the MBH function. We assume then that the flow capacity can be obtained from the slope of the hyperbola. If we just set $\bar{p}$ = a, then we are faced with the problem of choosing the proper interval. This is the same for buildups following stabilized and non-stabilized flow. The possible errors are also similar. Note also that if we set $\bar{p}$ = a, then we are assuming the right-hand of Eq. 62 to be 0. With the flow capacity known, we can then determine the quantity $2\pi\alpha$ minus the MBH function, and this information is sufficient to get the average pressure directly from an MDH plot.

Fig. 17 ilustrates the validity of Eq. 60 for a closed 2:1 rectangle with the well located 1/4 of the length and 1/2 of the height of the drainage area, i.e., for the same reservoir considered in Figs. 7 - 9. For the present example the reservoir was produced to t_{DA} = 0.1 prior to shut-in. From Ref. 13 we know that t_{DApss} = 1.5, and hence that stabilized flow was not reached prior to shut-in. In Fig. 17 we have plotted values of a_D determined from asymptotes by the solid line, values of a_D computed from Eq. 60 by the dashed line, and the buildup curve by the dotted line. The hyperbolas were matched over intervals of 0.06 log cycles, and for such intervals we see that the asymptotes are given by Eq. 60 for α < 0.03, i.e., in the infinite-acting period. It follows that we can determine the average pressure from the asymptote of any hyperbola matched in the infinite-acting period by using Eqs. 18, 62, and 63, provided we know k, A, and the MBH function for the reservoir. But with this information we can also use conventional methods. If we just set $\bar{p}$ = a, and hence indirectly assume the information needed for conventional methods, based on infinite-acting data, to be known, then we need to use late data to get a correct estimate of $\bar{p}$. Fig. 18 shows that Eqs. 34, 42, and 60 can give quite different results if used to estimate the possible response of the hyperbola approach to pressure buildup analysis.

Finally, Fig. 19 illustrates the validity of Eqs. 19 and 45 for the same 2:1 rectangle considered in Figs. 17 and 18. In this figure we have plotted ρ determined from hyperbolas by the solid line, ρ computed by Eq. 19 by the dashed line, and ρ computed by Eq. 45 by the dotted line. Note that Eq. 45 best approximates the values obtained from the hyperbolas for this example, but this depends on the flow period prior to shut-in. Moreover, if we use early buildup data, then we can get a sufficiently accurate estimate of the flow capacity from Eq. 6. For shorter producing times, Eq. 4 should be chosen. But again, such data are easier to identify and analyze by the Horner method.

CONCLUSIONS

1. Rectangular hyperbolas can be used to determine the flow capacity and average pressure from infinite-acting buildup data, but only if the data can also be analyzed by conventional methods. The point is that the methods are essentially equivalent in terms of information needed and information gained.

2. The flow capacity can be determined from the slope of a hyperbola matched to a short interval of infinite-acting buildup data, but it is easier to identify and analyze such data by the Horner and MDH methods.

3. The average reservoir pressure can be determined from the asymptote of a hyperbola matched to a short interval of infinite-acting buildup data if we know the drainage area and shape factor following stabilized flow, or area and MBH function otherwise. But we can then also use conventional methods.

4. If the hyperbola method is based on the assumption that the average pressure is equal to the asymptote of the hyperbola, then, for a chosen length of interval, there is at most one position in the infinite-acting period where the asymptote can be equal to the average pressure. Moreover, except for highly symmetric reservoirs, there will normally be none, in which case we must include data beyond the infinite-acting period to get an asymptote equal to the average pressure. If this approach does apply in the infinite-acting period, then the information we need to choose the correct position can also be used to carry out a conventional analysis.

5. If the average pressure can be obtained directly from the asymptote of a hyperbola, then the average pressure cannot be more m/1.151 psi (kPa) above the last wellbore pressure used in the analysis, where m is the conventional semilog slope.

NOMENCLATURE

A = drainage area, sq ft (m^2)

a = hyperbola asymptote, psi (kPa)

B = formation volume factor, RB/STB (res m^3/std m^3)

b = hyperbola parameter, hours

C_A = shape factor, dimensionless

c = hyperbola parameter, psi hours (kPa hours)

c_t = total system compressibility, psi^{-1}, (kPa^{-1})

e = 2.71828..., base natural logarithm

h = thickness, ft (m)

k = permeability, md

ln = natural logarithm

m = semilog slope, psi/log cycle (kPa/log cycle)

p = pressure, psi (kPa)

q = flow rate, STB/D (stock-tank m^3/d)

r_w = wellbore radius, ft (m)

t = time, hours

α = logarithmic interval midpoint, dimensionless

γ = Euler's constant (0.5772...)

ϕ = porosity

μ = viscosity

ϱ = slope parameter, dimensionless (Eq. 20)

Subscripts

A = area

D = dimensionless

eia = end infinite acting

f = flowing

H = Horner

i = initial

k, n = indices

pss = pseudosteady state

ss = steady state

w = wellbore

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REFERENCES

1. Miller, C. C., Dyes, A. B., and Hutchinson, C. A., Jr.: "The Estimation of Permeability and Reservoir Pressure From Bottom Hole Pressure Build-Up Characteristics," Trans., AIME (1950) 189, 91-104.

2. Matthews, C. S., Brons, F., and Hazebroek, P.: "A Method for Determination of Average Pressure in a Bounded Reservoir," Trans., AIME (1954) 201, 182-191.

3. Dietz, D. N.: "Determination of Average Reservoir Pressure From Build-Up Surveys," J. Pet. Tech. (Aug. 1965) 955-959; Trans., AIME, 234.

4. Kumar, Anil and Ramey, Henry J., Jr.: "Well-Test Analysis for a Well in a Constant-Pressure Square," Soc. Pet. Eng. J. (April 1974) 107-116.

5. Mead, H.N.: "A Practical Approach to Transient Pressure Behavior," paper SPE 9901 presented at the 1981 SPE California Regional Meeting, Bakersfield, March 25-26.

6. Hasan, A. Rashid and Kabir, C. Shahjahan: "Pressure Buildup Analysis: A Simplified Approach," J. Pet. Tech. (Jan. 1983) 178-188.

7. Humphreys, Neil V.: "Discussion of Pressure Buildup Analysis: A Simplified Approach," J. Pet. Tech. (May 1983) 909-911.

8. Bowles, Dennis L. and White, Cristopher: "Discussion of Pressure Buildup Analysis: A Simplified Approach," J. Pet. Tech. (May 1983) 912-915.

9. Hasan, A. R. and Kabir, C. S.: "Authors' Reply to Discussion of Pressure Buildup Analysis: A Simplified Approach," J. Pet. Tech. (May 1983) 911.

10. Hasan, A. R. and Kabir, C. S.: "Authors' Reply to Discussion of Pressure Buildup Analysis: A Simplified Approach," J. Pet. Tech. (May 1983) 916-917.

11. Horner, D. R.: "Pressure Build-Up in Wells," Proc., Third World Pet. Cong., The Hague (1951) Sec. II, 503-523.

12. Larsen, Leif: "A Simple Approach to Pressure Distributions in Geometric Shapes By Images," paper SPE 10088 presented at the 1981 SPE Annual Fall Technical Conference and Exhibition, San Antonio, Oct. 5-7.

13. Earlougher, Robert C., Jr.: Advances in Well Test Analysis, Monograph Series, Society of Petroleum Engineers of AIME, Dallas (1977) 5.

14. Larsen, Leif: "Wellbore Pressures in Reservoirs With Constant-Pressure or Mixed No-Flow/Constant-Pressure Outer Boundary," J. Pet. Tech. (to appear Sep. 1984).

15. Meyer, J. P., Hasan, A. R., and Kabir, C. S.: "Solving the Rectangular Hyperbola Equation For Pressure Buildup Analysis," paper SPE 12353 submitted to SPE (unsolicited, 1984)

SI Metric Conversion Factors

bbl × 1.589 873	E - 01	= m^3
cp × 1.0*	E - 03	= Pa.s
ft × 3.048*	E - 01	= m
psi × 6.894 757	E + 00	= kPa

* Conversion factor is exact.

APPENDIX A

TRUNCATED SERIES APPROACH

For any positive constants x and α, it can be shown that

$$\ln x = \ln\alpha - 2\sum_{n=1}^{\infty} \frac{1}{2n-1}\left(\frac{\alpha - x}{\alpha + x}\right)^{2n-1} \quad \text{(A-1)}$$

The point is that

$$x = \alpha \frac{1 - \frac{\alpha - x}{\alpha + x}}{1 + \frac{\alpha - x}{\alpha + x}}, \quad \text{(A-2)}$$

and hence that

$$\ln x = \ln\alpha + \ln\left(1 - \frac{\alpha - x}{\alpha + x}\right) - \ln\left(1 + \frac{\alpha - x}{\alpha + x}\right) \quad \text{(A-3)}$$

Eq. A-1 can now be obtained from Eq. A-3 by expanding the last two logarithmic terms as power series, and then adding these.

Considering Eq. A-1, note that we must have

$$\ln x \approx \ln\alpha - 2\frac{\alpha - x}{\alpha + x} = \ln\alpha + 2 - \frac{4\alpha}{\alpha + x} \quad \text{(A-4)}$$

if x is sufficiently close to α.

APPENDIX B

DETERMINING HYPERBOLA PARAMETERS

Several methods can be used to determine parameters a, b, and c such that the rectangular hyperbola in Eq. 1 closely matches the real buildup curve on a chosen interval. These methods can be introduced independent of any theoretical justifications of the rectangular hyperbola approach to pressure buildup analysis, and they can be applied to both real and dimensionless data. For the objectives of the present paper, it suffices to consider two such methods.

To describe the methods we have used to determine a, b, and c in Eq. 1, let $x_1, x_2, \ldots, x_n$ denote the n buildup times being used in the analysis, and let y_k be the corresponding buildup pressures. The most direct method to determine a, b, and c is to minimize the following sum of quadratic deviations,

$$F_1(a,b,c) = \sum_{k=1}^{n}\left(y_k - a - \frac{c}{b + x_k}\right)^2 \quad \text{(B-1)}$$

This amounts to solving the equations

$$\frac{\partial F_1}{\partial a} = \frac{\partial F_1}{\partial b} = \frac{\partial F_1}{\partial c} = 0 \quad \text{(B-2)}$$

Unfortunately, this approach does not lead to simple expressions for a, b, and c. Instead, an equation for b that must be solved by some numerical method is obtained. The remaining two parameters can then be determined by direct substitution. In the present paper, this method will be called Method 1.

An alternative method, which is also based on minimizing a sum of quadratic deviations, can be obtained by only considering the numerators in Eq. B-1. We then get

$$F_2(a,b,c) = \sum_{k=1}^{n}(x_k y_k - a x_k + b y_k - ab - c)^2 \quad \text{(B-3)}$$

The corresponding version of Eq. B-2 then leads to the equations

$$\sum_{k=1}^{n}(x_k y_k - a x_k + b y_k - ab - c)(x_k + b) = 0, \quad \text{(B-4)}$$

$$\sum_{k=1}^{n}(x_k y_k - a x_k + b y_k - ab - c)(y_k - a) = 0, \quad \text{(B-5)}$$

and

$$\sum_{k=1}^{n}(x_k y_k - a x_k + b y_k - ab - c) = 0. \quad \text{(B-6)}$$

To express the solutions in closed form we have found it convenient to use the following special notation:

$$\langle x\rangle = \frac{1}{n}\sum_{k=1}^{n} x_k, \quad \langle xy\rangle = \frac{1}{n}\sum_{k=1}^{n} x_k y_k, \text{ etc.,} \quad \text{(B-7)}$$

$$\{\langle x\rangle^2\} = \langle x\rangle^2 - \langle x^2\rangle, \qquad \text{(B-8)}$$

$$\{\langle y\rangle^2\} = \langle y\rangle^2 - \langle y^2\rangle, \qquad \text{(B-9)}$$

$$\{\langle x\rangle\langle y\rangle\} = \langle x\rangle\langle y\rangle - \langle xy\rangle, \qquad \text{(B-10)}$$

$$\{\langle x\rangle\langle xy\rangle\} = \langle x\rangle\langle xy\rangle - \langle x^2 y\rangle, \qquad \text{(B-11)}$$

and

$$\{\langle y\rangle\langle xy\rangle\} = \langle y\rangle\langle xy\rangle - \langle xy^2\rangle. \qquad \text{(B-12)}$$

The parameters a, b, and c can then be expressed

$$a = \frac{\{\langle y\rangle\langle xy\rangle\}\{\langle x\rangle\langle y\rangle\} - \{\langle x\rangle\langle xy\rangle\}\{\langle y\rangle^2\}}{\{\langle x\rangle\langle y\rangle\}^2 - \{\langle x\rangle^2\}\{\langle y\rangle^2\}}, \qquad \text{(B-13)}$$

$$b = \frac{\{\langle y\rangle\langle xy\rangle\}\{\langle x\rangle^2\} - \{\langle x\rangle\langle xy\rangle\}\{\langle x\rangle\langle y\rangle\}}{\{\langle x\rangle\langle y\rangle\}^2 - \{\langle x\rangle^2\}\{\langle y\rangle^2\}}, \qquad \text{(B-14)}$$

and

$$c = \langle xy\rangle - a\langle x\rangle + b\langle y\rangle - ab. \qquad \text{(B-15)}$$

This particular method, which will be called Method 2 in the present paper, is similar to a method suggested by Meyer *et al.*[15] Since Methods 1 and 2 give essentially identical results in the range where the hyperbola approach can be used with any accuracy, and Method 2 is much simpler to use, we have chosen Method 2 as our standard method of least squares.

Fig. 20 illustrates the validity of the claim that Methods 1 and 2 are equivalent in the range of data where the rectangular hyperbola method can be of interest. In Fig. 20 we have used the same 2:1 rectangle considered in Figs. 17, 18, and 19, but now produced to t_{DA} = 1 prior to shut-in. In this figure the solid line represents asymptotes determined by Method, 1 and the dashed line asymptotes determined by Method 2. In these computations we have used intervals of length 0.2 log cycles. A similar comparison of the methods is obtained if we consider values of ρ. Note that the methods only start to diverge around α = 0.06, which is beyond the infinite-acting period for this reservoir.

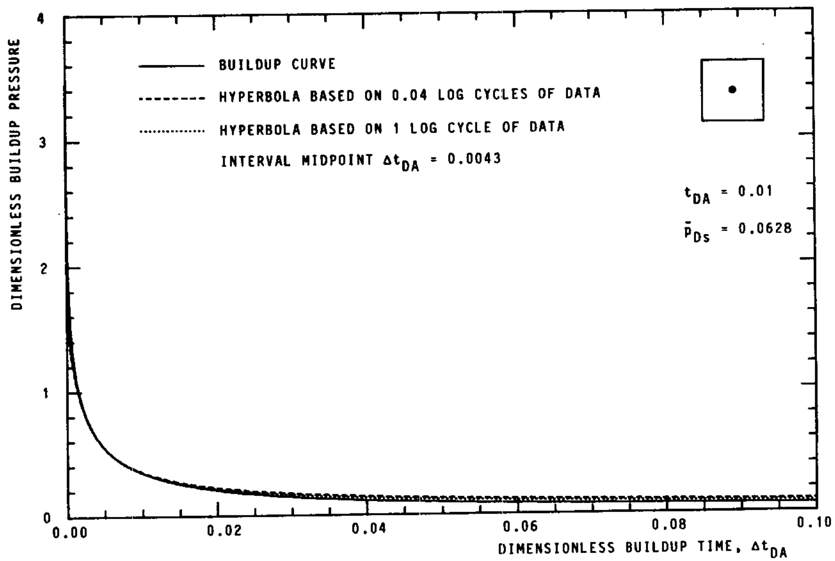

Fig. 1—Effect of interval length when hyperbolas are matched to buildup curves.

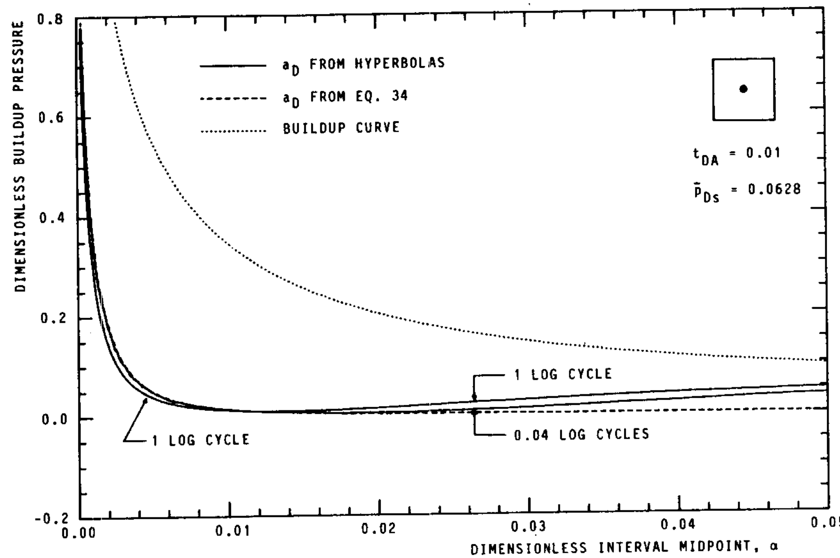

Fig. 2—Asymptotes from hyperbolas and Eq. 34 compared with buildup curve of an infinite-acting reservoir.

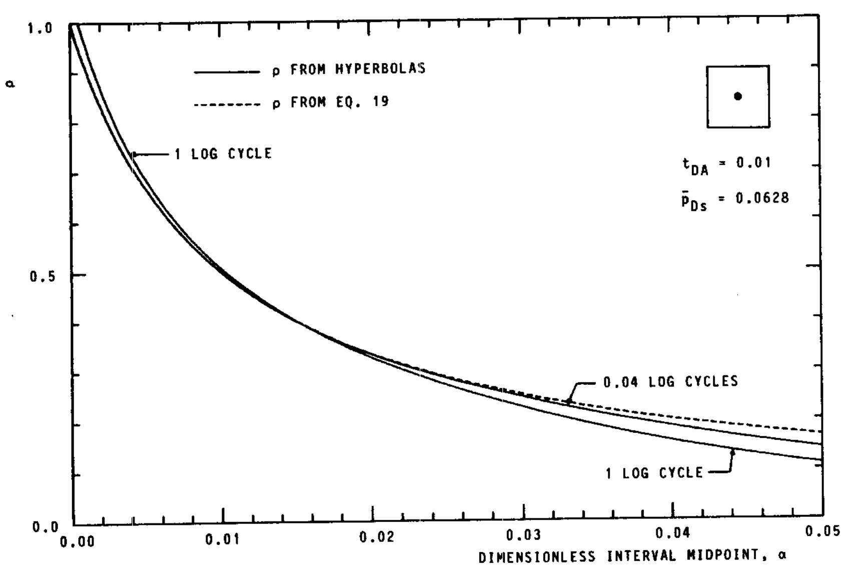

Fig. 3—Slope parameter ρ from hyperbolas and Eq. 19 for an infinite-acting reservoir.

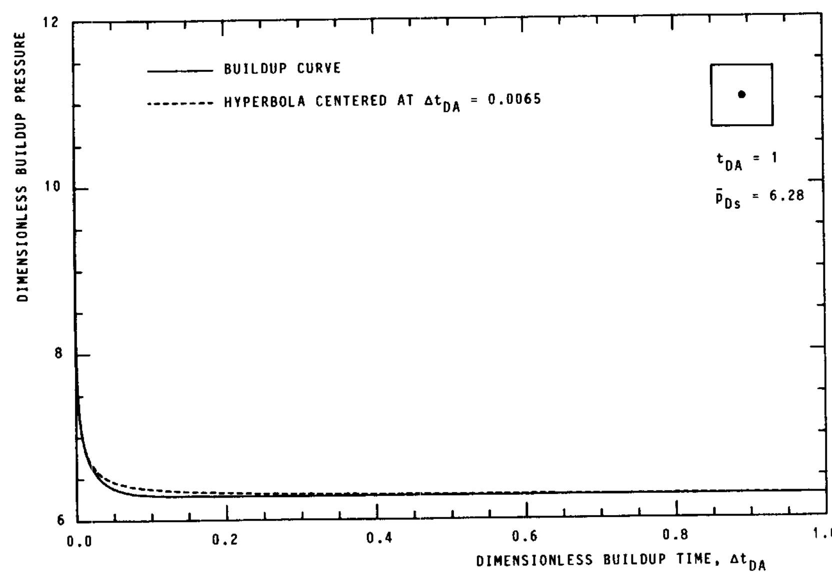

Fig. 4—Hyperbola matched to buildup curve of a closed square produced to pseudosteady state prior to shut-in.

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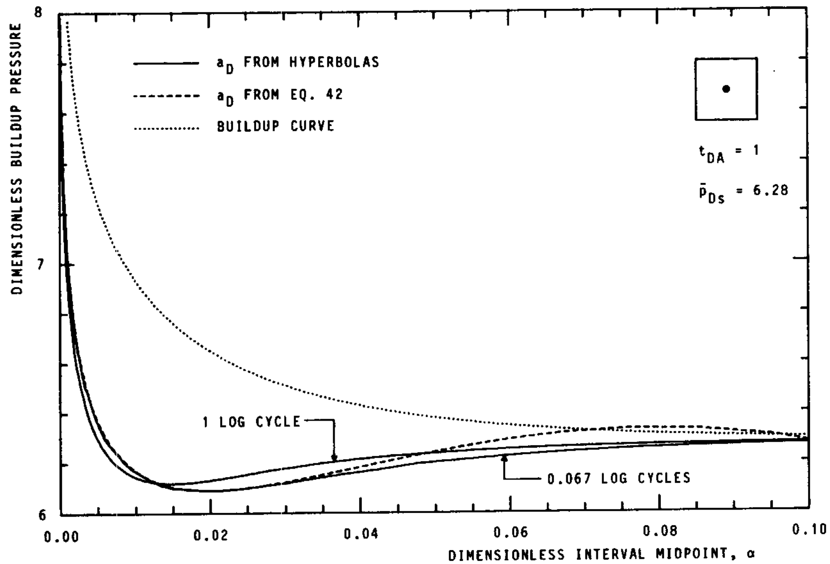

Fig. 5—Asymptotes from hyperbolas and Eq. 42 compared with buildup curve following pseudosteady-state flow for a closed square.

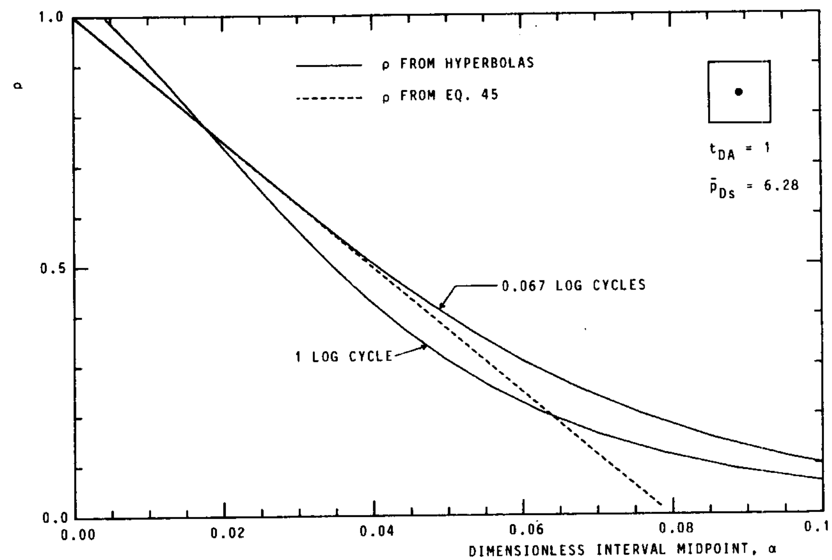

Fig. 6—Slope parameter ρ from hyperbolas and Eq. 45 for a closed square produced to pseudosteady state prior to shut-in.

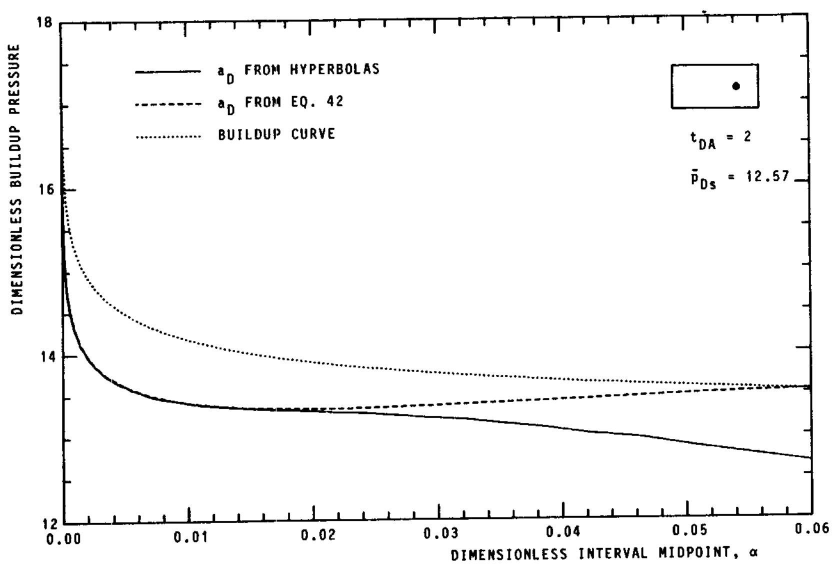

Fig. 7—Asymptotes from hyperbolas and Eq. 42 compared with buildup curve following pseudosteady-state flow for a closed 2:1 rectangle.

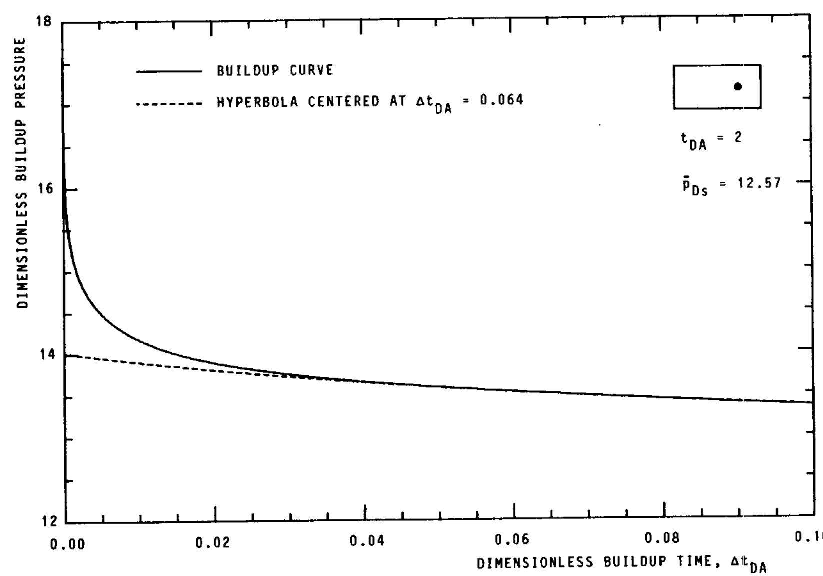

Fig. 8—Hyperbola with asymptote given by average pressure matched to buildup curve following pseudosteady-state flow for a closed 2:1 rectangle.

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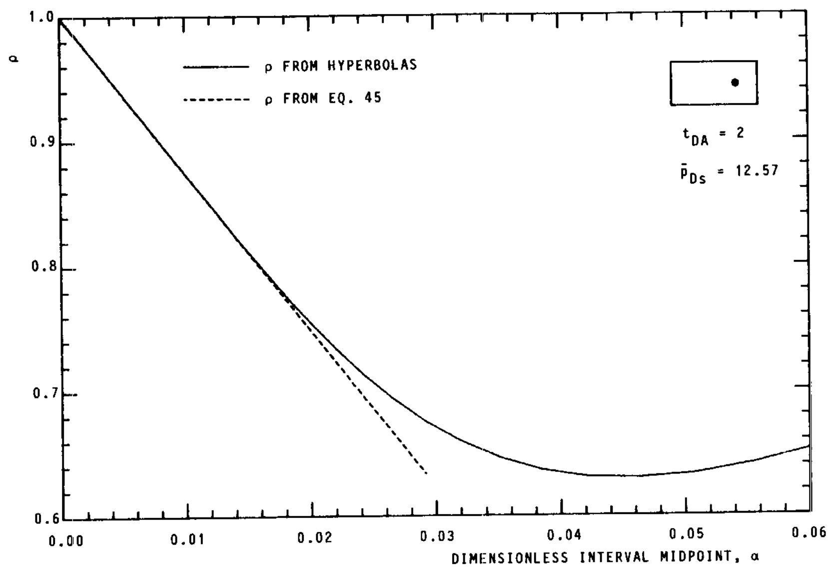

Fig. 9—Slope parameter ρ from hyperbolas and Eq. 45 following pseudosteady-state flow for a closed 2:1 rectangle.

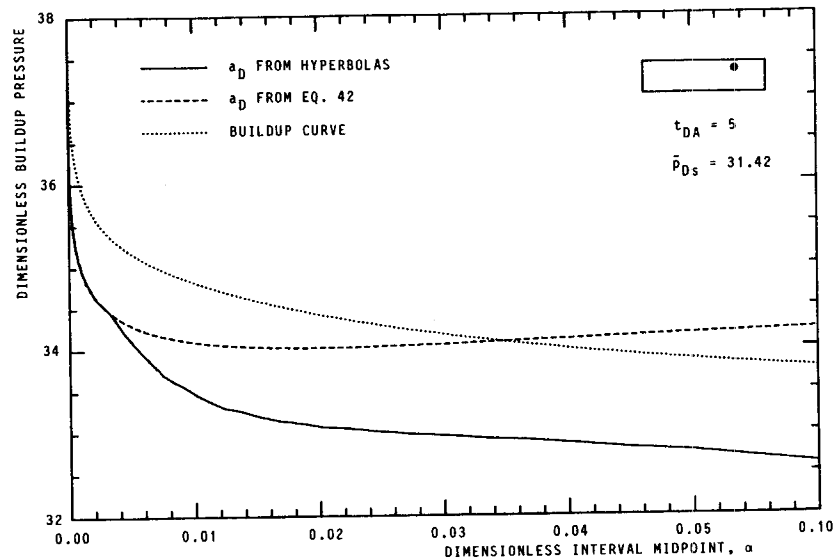

Fig. 10—Asymptotes from hyperbolas and Eq. 42 compared with buildup curve following pseudosteady-state flow for a closed 4:1 rectangle.

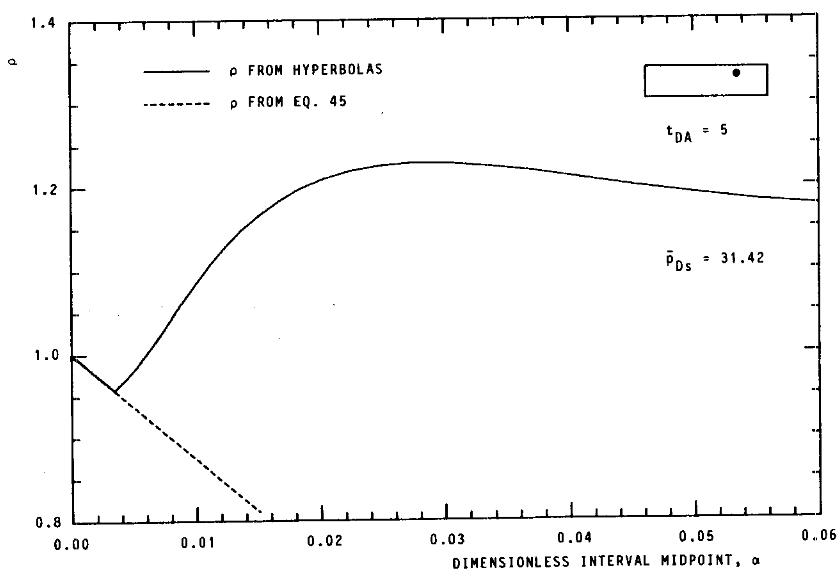

Fig. 11—Slope parameter ρ from hyperbolas and Eq. 45 following pseudosteady-state flow for a closed 4:1 rectangle.

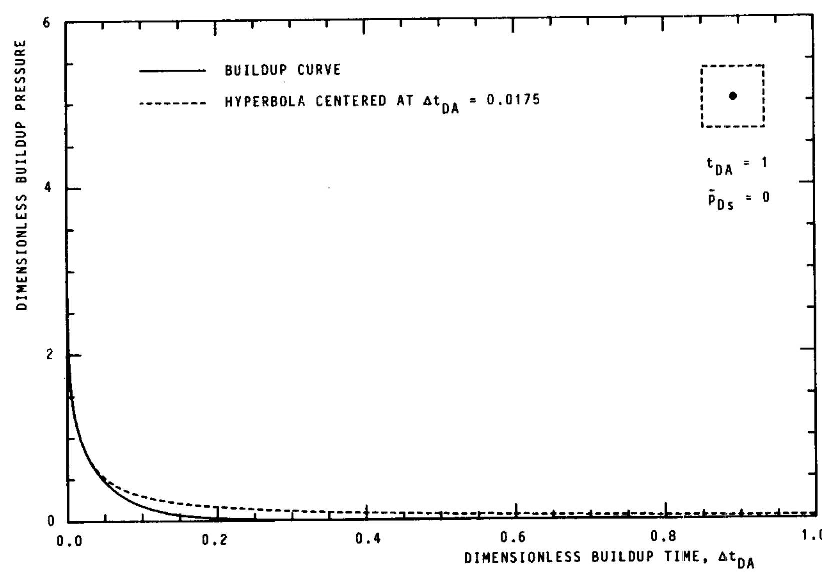

Fig. 12—Hyperbola matched to buildup curve of a constant-pressure square produced to steady state prior to shut-in.

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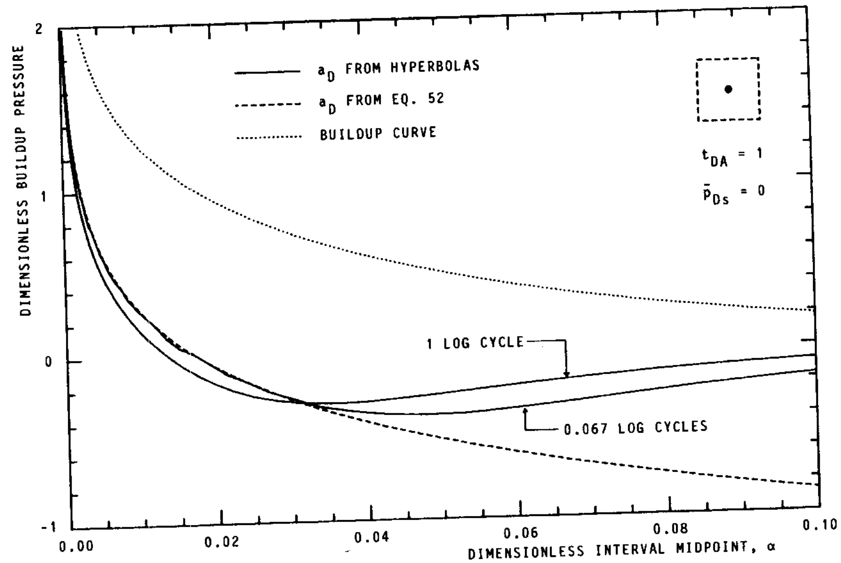

Fig. 13—Asymptotes from hyperbolas and Eq. 52 compared with buildup curve following steady-state flow for a constant-pressure square.

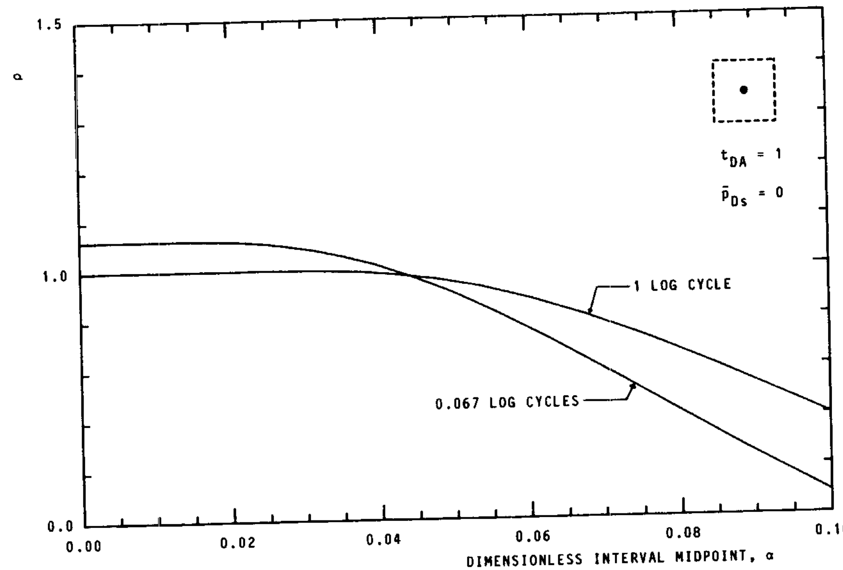

Fig. 14—Slope parameter ρ from hyperbolas for a constant-pressure square produced to steady state prior to shut-in.

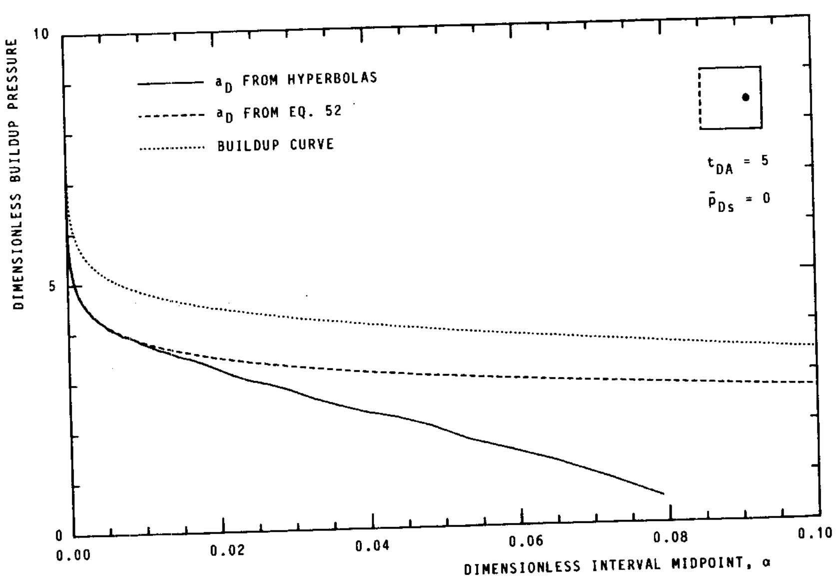

Fig. 15—Asymptotes from hyperbolas and Eq. 52 compared with buildup curve following steady-state flow for a square with one side at constant pressure.

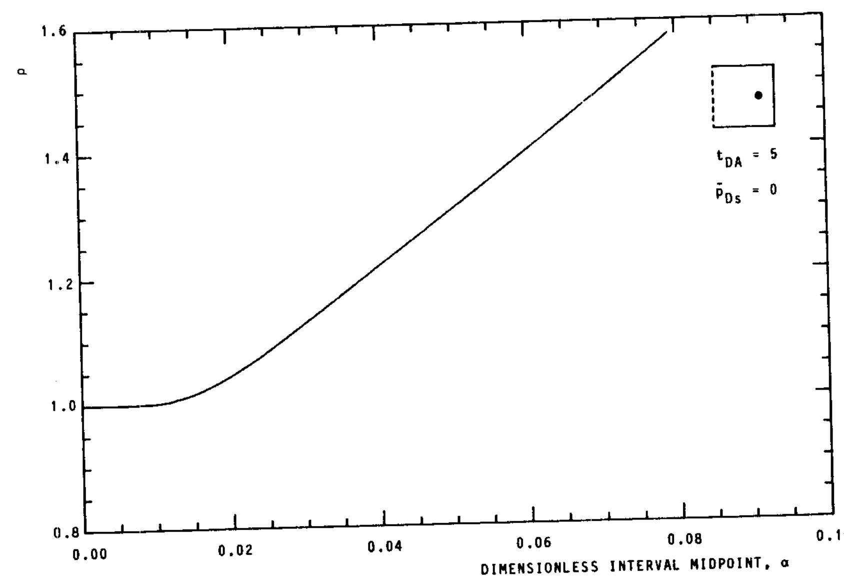

Fig. 16—Slope parameter ρ from hyperbolas following steady-state flow for a square with one side at constant pressure.

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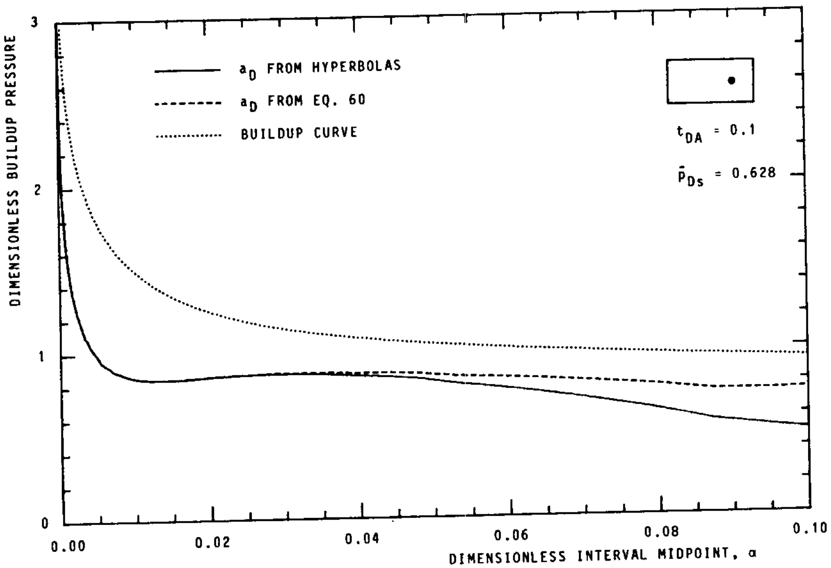

Fig. 17—Asymptotes from hyperbolas and Eq. 60 compared with buildup curve following nonstabilized flow for a closed 2:1 rectangle.

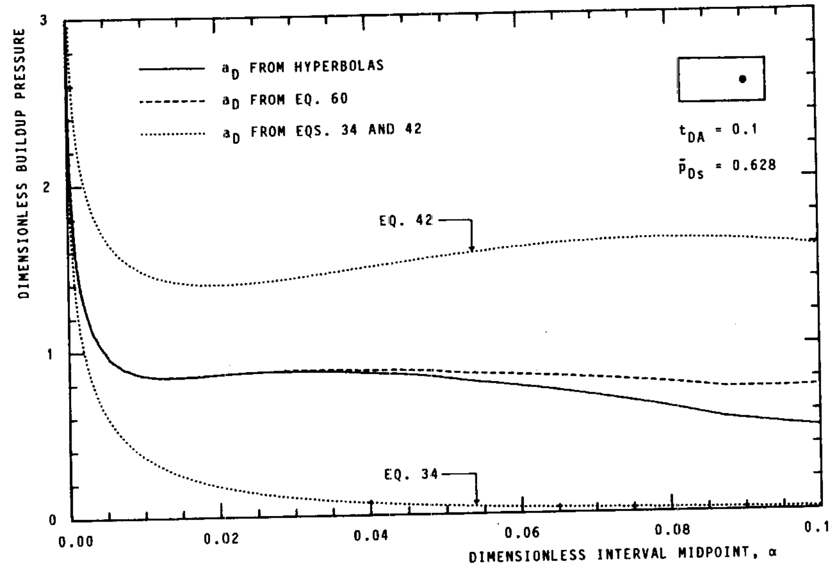

Fig. 18—Asymptotes from hyperbolas and Eqs. 34, 42, and 60 following nonstabilized flow for a closed 2:1 rectangle.

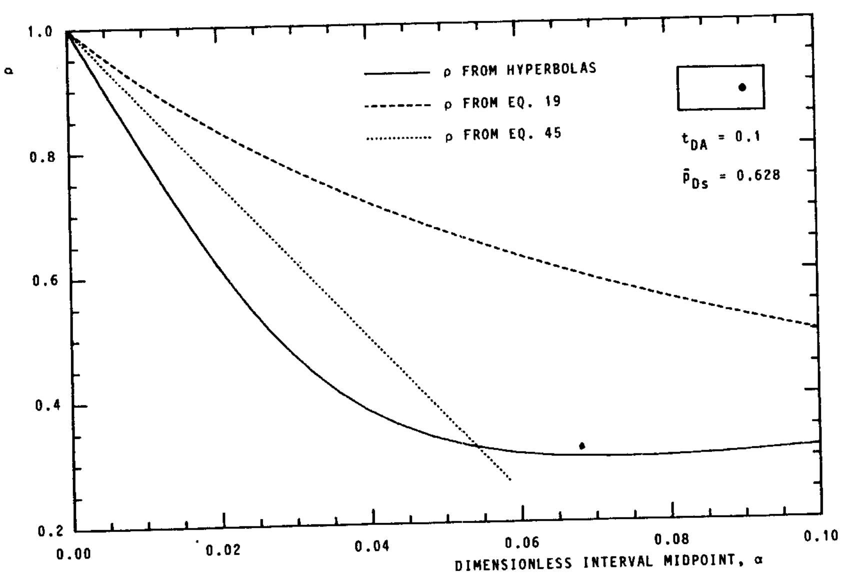

Fig. 19—Slope parameter ρ from hyperbolas and Eqs. 19 and 45 following nonstabilized flow for a closed 2:1 rectangle.

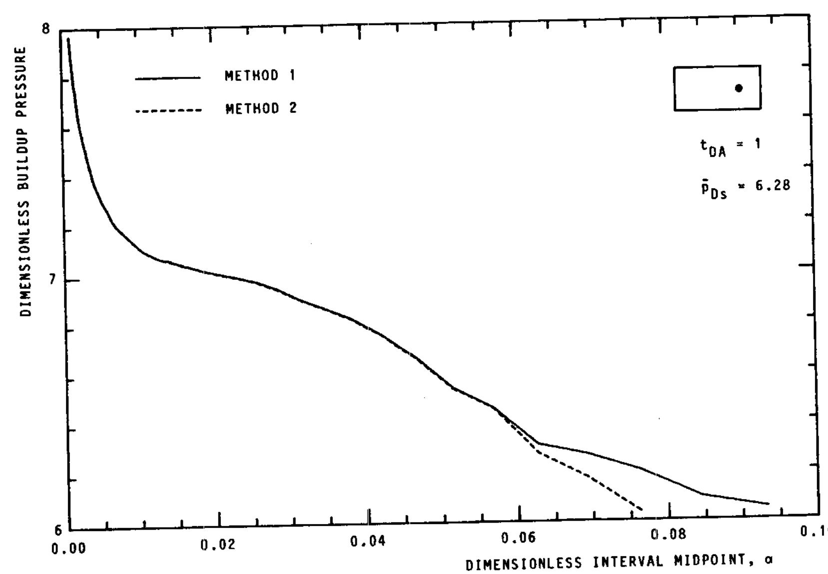

Fig. 20—A comparison of Methods 1 and 2 to determine asymptotes of hyperbolas matched to buildup data of a closed 2:1 rectangle.

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