# Relative Permeability and Capillar Pressure Concurrently Determined from Steady-State Flow Experiments

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#### Abstract

A new method is presented to interpret steady-state flow experiments for relative permeability and capillary pressure functions eliminating errors caused by the capillary end-effect. This is achieved by retaining the capillary term in the equations that are used to interpret the flow data.

The standard experimental procedure has to be extended to include variations in both total flowrate and the ratio of phase flowrates. Consistent values of saturation, relative permeability of each phase, and capillary pressure are then calculated at the inlet end.

Necessary modifications in laboratory procedures are discussed and the theoretical development is exemplified by numerical simulation of coreflood experiments.

# Introduction

During a steady-state procedure for measurement of relative permeability curves, the total flowrate of oil and water is usually kept constant while their ratio is changed at the inlet end of the core. After a change, it is necessary to wait until equilibrium in the core is reestablished, i.e., when both the pressure drop and the effluent flowrate ratio do not change with time. The individual flowrates and the pressure drop is then used to calculate the individual phase relative permeability values by Darcy's law, relating them to the average saturation in the core, determined by material balance.

The main inaccuracies of this method stem from the basic assumption that the capillary pressure can be neglected [1,2,3]. Actually, because of capillary effects, the saturation distribution along the core is nonuniform,

and the pressure drop is different in each phase. The capillary effects are difficult to avoid even if the total flowrate is high and for some rocks high flowrate cannot be reached for reasons like limited equipment capacity or stress that may cause rock damage.

In this paper a new steady-state technique is described that includes capillary effects. Both relative permeability and capillary pressure curves may be derived from the same experimental sequence.

The proposed experimental setup is not very much different from that of the conventional steady-state method. A special construction of the inlet endpiece is necessary, allowing complete separation of the flowing fluids outside the porous medium and measurement of the individual phase pressures. For a fixed fractional flow at the inlet, a number of steady-state experiments is required with varying total flowrate to include the capillary effect in the analysis of the data.

## Theory

The following standard equations describe one-dimensional, two-phase flow of immiscible, incompressible fluids in a porous medium,

$$u_{i} = -k\lambda_{i}\frac{\partial p_{i}}{\partial x}, \quad i = 1,2$$

$$u_{i} = -k\lambda_{i}\frac{\partial p_{1}}{\partial x} - k\lambda_{2}\frac{\partial p_{c}}{\partial x}, \quad (1)$$

$$p_{c}(S) = p_{2} - p_{1}.$$

From Eqs. (1) it follows that the expression for the velocity of the first phase is

$$u_1 = u_t f + k f \lambda_2 \frac{\partial p_c}{\partial x}.$$
 (2)

Conservation of mass gives

$$\frac{\partial u_1}{\partial x} + \phi \frac{\partial S_1}{\partial t} = 0.$$
(3)

Let us consider steady-state flow only. Then the saturation in the core is solely a function of the *x*-coordinate. Since  $\partial S_1 / \partial t = 0$ , integration of (3) then shows that the expression

$$u_1 = u_t F \tag{4}$$

is a constant. Here  $F = u_1 / u_t$  denotes the fixed fractional flow at the inlet. From Eqs. (2) and (4) it follows that

$$u_t(f-F) + kf\lambda_2 \frac{dp_c}{dx} = 0,$$
(5)

$$\begin{split} \frac{\Delta p_1 k}{u_t F} &= \int_0^L \frac{dx}{\lambda_1}, \quad \Delta p_1 = p_1(0) - p_1(L), \\ dx &= kf\lambda_2 \frac{dp_c}{u_t(F-f)}, \\ \frac{\Delta p_1 k}{u_t F} &= \int_0^L \frac{1}{\lambda_1} kf\lambda_2 \frac{dp_c}{u_t(F-f)}, \\ &= \frac{k}{u_t} \int_{p_c^0}^{p_c^L} \frac{(1-f)dp_c}{(F-f)}, \end{split}$$

where

$$p_c^0 = p_c |_{x=0}, \qquad p_c^L = p_c |_{x=L}$$

The expressions for phase pressure drops across the core are then

$$\Delta p_1 = F \int_{p_c^0}^{p_c^-} \frac{(1-f)dp_c}{(F-f)},$$
(6a)

and

$$\Delta p_2 = (1 - F) \int_{p_c^0}^{p_c^-} \frac{f d p_c}{(F - f)}.$$
 (6b)

The total velocity follows from Eq. (5) by integration,

$$u_{t} = -\frac{k}{L} \int_{p_{c}^{0}}^{p_{c}^{L}} \frac{f\lambda_{2}dp_{c}}{(f-F)},$$
(7)

and the average saturation is

$$S(F) = \frac{k}{u_t L} \int_{p_c^0}^{p_c^L} S \frac{f\lambda_2}{(F-f)} dp_c, \qquad (8)$$

with

$$S = \frac{1}{L} \int_{0}^{L} S(x) dx.$$

Eqs. (6), (7), and (8) relate the measurable quantities, i.e., pressure drop in each of the phases, total velocity, and average saturation in the core, to the unknown functions f,  $\lambda_2$ , and S. The last two equations have been considered in [4] for the particular case when F = 0 to develop a method for the interpretation of steady-state experiments.

The two main control parameters of the method are F and  $u_t$ . Let us consider the case when F is constant while  $u_t$  is varied. As explained below, the capillary pressure at the outlet of the core is constant. Hence, by differentiation of Eqs. (6)–(8) with respect to capillary pressure at the inlet end,  $p_c^0$ , one obtains

$$\frac{d\Delta p_1}{dp_c^0} = -\frac{F(1-f)}{(F-f)},\tag{9a}$$

$$\frac{d\Delta p_2}{dp_c^0} = -\frac{(1-F)f}{(F-f)},$$
(9b)

$$\frac{du_t}{dp_c^0} = \frac{k}{L} \frac{f\lambda_2}{(f-F)},\tag{10}$$

$$\frac{d(Su_t)}{dp_c^0} = -\frac{k}{L}S\frac{f\lambda_2}{(F-f)}.$$
(11)

From (10) and (11) it follows that

$$\frac{d(Su_t)}{du_t} = S(x=0), \tag{12}$$

and from (9) and (10) that

$$\frac{d\Delta p_1}{du_t} = \frac{F}{\lambda_1} \frac{L}{k}, \quad \frac{d\Delta p_2}{du_t} = \frac{(1-F)L}{k\lambda_2}, \quad (13a)$$

$$\lambda_1 = \frac{F}{\frac{d\Delta p_1}{du_t}} \frac{L}{k}, \qquad \lambda_2 = \frac{(1-F)}{\frac{d\Delta p_2}{du_t}} \frac{L}{k}. \quad (13b)$$

All the saturation-dependent quantities in Eqs. (13) are referred to the saturation at the inlet end, as determined by Eq. (12).

A number of possibilities to apply the formulae (12) and (13) is possible depending on what input information is available, i.e., whether the individual phase pressure drops are measured, and whether the capillary pressure is measured separately. The options are listed in Table 1.

Table 1: Input options and interpretation

$\Delta p$	$p_c$ known	$p_c$ unknown
None	Eq.(10)	S(x=0)
$\Delta p_1$	$\lambda_1(S), \ \lambda_2(S)$	$\lambda_1(S)$
$\Delta p_2$	$\lambda_1(S), \ \lambda_2(S)$	$\lambda_2(S)$
$\Delta p_1, \Delta p_2$	$\lambda_1(S), \ \lambda_2(S)$	$\lambda_1(S), \lambda_2(S), P_c$

# **Boundary Conditions**

Several practical difficulties may be envisioned when trying to apply the method. One of the main obstacles is how to measure the phase pressures. Ramakrishnan and Capiello [4] suggested to inject only the nonwetting phase at different rates in a core initially saturated with the wetting phase. Then F = 0 in Eqs. (13). The disadvantage is obvious: the relative permeability of the wetting phase cannot be determined. Also, only drainage curves can be measured.

The phase pressures may in principle be monitored in the porous medium itself by the technique of semipermeable pads [3]. However, the method is complicated and expensive and probably not viable for routine measurements.

Another method is to measure the phase pressures outside the core, in the tubing or grooves of the endpiece, provided that each phase pressure is continuous from the endpiece and into the core.

#### **Pressure Traverses**

Behavior of phase pressures across the core boundaries has been extensively discussed theoretically, see [5]–[8] and literature cited therein.

**Outlet End.** The capillary pressure outside the core in the receiving endpiece is assumed to be equal to zero.

If the flow process in the core is drainage and the capillary pressure curve is nonzero and positive for all saturation values, e.g., a water-wet system, pressure continuity at the outlet cannot be satisfied for both phases [7,8]. The saturation of the nonwetting phase at the core outlet corresponds to the lowest possible capillary pressure inside the core and the relative permeability of the nonwetting phase is close to zero. As explained in Ref. [5], the nonwetting phase pressure is

discontinuous and the wetting phase pressure is continuous. This is in agreement with the experiments of Richardson *et. al* [3] who state that the magnitude of the discontinuity is equal to the capillary pressure at the equilibrium nonwetting fluid saturation.

For an imbibition process, however, the capillary pressure curve is zero for some saturation, i.e., the endpoint of an spontaneous imbibition process. The outlet end saturation is fixed at this value, both phase pressures are continuous and the capillary pressure is zero and continuous across the boundary.

Experimentally, for a drainage process, a slight fluctuation in injection pressure, say, may shift the flow process at the outlet from drainage to imbibition, resulting in zero capillary pressure and continuity of both phases at the outlet.

Consequently, the capillary pressure at the outlet boundary of the core only depends on the properties of the relevant capillary pressure curve. It remains constant at different flowrates and may be zero or not, depending on the wettability of the core and type of displacement process.

**Inlet End.** With the saturation at the outlet boundary given as discussed above, the steady state saturation distribution in the core is defined by Eq. (5), so that the saturation in the core close to the inlet boundary,  $S_i^+$  and the corresponding capillary pressure  $p_c^+(S_i^+)$ , may uniquely be determined.

If the two phase pressures are equal on the outside of the inlet end, there will be a discontinuity of the wetting phase pressure going into the core, provided  $p_c^+(S_i^+)$  is nonzero. Otherwise, there would have been backflow of the nonwetting phase, contrary to the imposed boundary conditions of constant rate injection. However, if the two phases are injected into the core at different pressures through wetting and nonwetting membranes, both phase pressures will be continuous.

**Pressure Drop Across the Core.** Since the nonwetting phase pressure is continuous at the inlet and the capillary pressure is constant at the outlet, it follows that the total pressure drop measured outside porous medium corresponds to the pressure drop in the nonwetting phase plus a constant value equal to the capillary pressure at the outlet.

For an imbibition process, both phase pressures are continuous at the outlet end since the capillary pressure there is zero. At the inlet boundary, the wetting phase pressure is discontinuous. In this case, therefore, the pressure drop measured outside the core is equal to the pressure drop of the nonwetting phase through the core.

With the existing laboratory equipment, only the pressure drop outside the core is measurable in practice. An attempt to measure the individual phase pressure drops over the porous medium will generally give large errors because of the pressure discontinuities across the boundaries of the porous medium. In this situation, according to Table 1, only the phase mobility of the nonwetting phase may be determined if the capillary pressure curve is unknown *a priori*. If the capillary pressure curve is known, then both phase mobilities may be determined.

#### Modified Inlet Endpiece

To make the individual phase pressures measurable, the inlet endpiece should be modified to keep the two phases separated outside porous medium. It is suggested that the wetting phase (water) is injected into the core through a strongly water-wet material, e.g., semipermeable membrane. For a mixed-wet core it would also be necessary to have an oil-wet membrane between the core and the oil groove of the endpiece. This membrane prevents counter-current flow of the nonwetting phase at the inlet so that the water injection groove is filled with water only.

The use of water-wet and oil-wet micropore membranes at each end of a core has been reported by Longeron *et al.* [9]. We plan to develop an endpiece where part of the cross section is water-wet and connected to the water tubing and the rest is oil-wet and connected to the oil tubing, to enable measurement of individual phase pressures inside the core.

Numerical simulation with a coreflood simulator has confirmed the considerations presented here about boundary conditions. Also, it is important to have dispersed injection of each phase and uniform saturation in cross sections of the core since the theory is valid for one-dimensional flow.

# **Interpretation Procedure**

A Fortran program was developed to interpret laboratory data affected by errors by Eqs. (12) and (13) to determine the relative permeability and capillary pressure curves. The numerical method is a slipping window algorithm. To calculate derivatives, the data are smoothed by a least-square fit of a logarithmic function and the derivative is calculated analytically. The main control parameter is the length of the smoothening interval which is related to the error level. For larger errors, the interval has to be increased.

# Examples

A number of numerical experiments has been performed to test the interpretation procedure according to the following scheme: (1) simulate a multirate, steady-state experiment by a numerical coreflood simulator; (2) use the artificial data with or without addition of random errors to back-calculate the (input) relative permeability and capillary pressures curves.

#### Simulation Grid

A total of 72 blocks was used in the one-dimensional simulations. The first numerical block is the injection block with high k and low  $\phi$ . The pressure drop of the phases across the core is represented by the difference in pressure between the first core block (second numerical block) and the core outlet. Some grid refinement is used at the core inlet and outlet ends. The block lengths are

#### Core and Fluid Data

L = 20.0 cm; A = 10.64 cm<sup>2</sup>;  $\phi = 22\%$ ;  $k_O(S_{iw}) = 485$  mD;  $k_r$ : Corey type with exponents equal to 2.0;  $k_{rO}(S_{iw}) = k_{rW}(S_{Or}) = 1.0$ ;  $\mu_O = 1.06$  cp;  $\mu_W = 1.30$  cp. Two capillary pressure curves were used; a  $p_c$ -curve for a typical water-wet core, and one for a mixed-wet core. Hysteresis effects are not included.

#### Simulation of Multirate Steady-Steady Floods

Generally, starting at irreducible water saturation, oil and water are injected with stepwise constant rates according to a preset schedule of fractional flow values and total injection rates. For the examples presented here, the fractional flow is stepwise held constant while the total rate is increased in 20 steps. The rate-change schedule should be chosen such that the water saturation strictly increases at all positions along the core to avoid a mixture of hysteresis effects and subsequent difficulties with the interpretation. This implies that the ratechange schedule should be designed dependent on the wettability of the core sample.

For the water-wet case, Fig. 1 shows the fractional flow values (3), the rate schedule, the average water saturation, and the oil and water pressures at the inlet end. Fig. 2 gives the corresponding saturation profiles. Note that the water saturation profiles reveal a nonmonotonous development with possible mixed hysteresis effects. The experimental procedure must therefore be studied more in detail to find general guidelines to avoid this effect.

The corresponding data for the mixed-wet case are displayed in Figs. 3 and 4, now with 2 injection ratios.

## Interpretation

# Water-Wet Core

The calculated relative permeabilities of oil and water shown by filled and open circles in Fig. 5 are very close to the true values (simulator input) represented by solid lines. Also shown are the relative permeabilities of oil (filled squares) and water (open squares) calculated from Darcy's law, i.e., without account for capillary effects. One may observe large errors caused by the negligence of capillarity even for relatively large total rates corresponding to low water saturations.

If capillary effects are not properly accounted for, the interpretation errors become especially large for the wetting phase because of the error in pressure drop. As discussed above, the pressure drop measured in the tubing outside porous medium is for the nonwetting phase if the capillary pressure at the outlet is zero, as it is in this example. The relative error in the wetting phase pressure drop increases when the total rate decreases because of increasing dominance of capillary forces. The capillary pressure curve is also reproduced accurately, Fig. 6.

The sensitivity of the interpretation algorithm to measurement errors has been tested. Pressure drops and phase volumes were subjected to a 1% random error level and two smoothening intervals were tested. The results, not shown here, were quite satisfactory for both capillary pressure and relative permeabilities. The 1% error level, which may be regarded as realistic, leads to errors in the calculated relative permeabilities which is lower than the errors resulting from neglect of capillary effects even without measurement errors.

The Nsm-label in the figures is half the number of measurement points included in the smoothening interval.

#### Mixed-Wet Core.

The simulated results for the two fractional flow values of 1% and 99% of water are shown in Figs. 3 and 4. The calculated and the true relative permeabilities and capillary pressures are presented in Figs. 7 and 8 for the case of no errors introduced. The saturation interval is fairly well covered by just the two fractional flow values used.

When errors are artificially introduced, the results are qualitatively the same as for the water-wet core discussed above.

Note that for all rates and fractional flow values the saturation at the outlet end in Fig. 4 remains fixed at 0.5, the value where the input capillary pressure curve is zero, Fig. 8. Both phase pressures are therefore continuous at the outlet end for this case.

# Discussion

The new method has been demonstrated by numerical simulations of two examples. The design of an actual experiment would depend on rock and fluid properties, including wettability. Hysteresis effects, for instance, have been disregarded in this study. In general, a rate and fractional flow schedule should be chosen to give monotonously increasing or decreasing saturation change at any position along the core. Then the primary drainage and primary imbibition curves may be determined. Otherwise, different parts of the core may experience separate hysteresis loops or scanning curves with a composite overall effect that would be impossible to interpret.

Phase pressure measurement is the key to get necessary information for relative permeability and capillary pressure calculations from a multirate steady-state experiment. A feasible procedure is to use semipermeable membranes at the fluid inlet. The phases must be completely separated until they enter the core inlet face. Construction of such a inlet endpiece is the main challenge in the experimental setup.

The conventional steady-state method gives rise to considerable errors due to neglect of capillary endeffect. As this method is widely in use, an error analysis should be performed based the analytical derivation presented here, numerical simulations and existing experimental data. Methods for correction of the error can be developed, e.g., if the capillary pressure curve is measured separately, Table 1.

By the conventional steady-state technique, the largest inaccuracies due to capillary effects are observed close to the residual saturations. A combined and fast approach would be to use the multirate steady-state technique close to the residuals in combination with a pseudosteady-state technique in the intermediate saturation range. This combined procedure will not require any modification of laboratory equipment.

In general, for a water-wet core, neglect of capillary effects in the interpretation procedure will give a low residual oil saturation, Fig. 5, and a high value for a mixed-wet case.

# Conclusions

A new multirate steady-state method to determine relative permeability and capillary pressure curves from core flooding experiments has been developed.

The proposed experimental setup requires a special construction of the inlet endpiece, allowing complete separation of the flowing fluids outside the porous medium.

The experimental procedure consists of a number of conventional steady-state experiments with different fractional flow values and different total rates.

The method has been demonstrated by simulation experiments and its robustness by artificially induced random errors.

It is necessary to design the experiment such that all parts of the core follow the same hysteresis curve, primary drainage or primary imbition.

# Nomenclature

- A = cross sectional area of core
- f = fractional flow function
- F = fractional flow at the inlet end
- k = absolute permeability
- L = length of core
- p = pressure
- S =saturation
- t = time
- x = coordinate
- u = velocity
- $\phi$  porosity
- $\lambda$ -total mobility

#### Subscripts

- c = capillary
- i = fluid phases or irreducible or inlet
- o = oil
- r = relative
- t = total
- w = water

#### Superscripts

0 = @x = 0+ = inside the core @x = 0 --- = average L = @x = L

#### Operators

 $\Delta$  = difference

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Figure 1 - Simulated responses from multirate, steady-state flooding, water-wet case.



Figure 2 - Water saturation distribution from multirate, steady-state flooding, water-wet case.



Figure 3 - Simulated responses from multirate, steady-state flooding, mixed-wet case.



Figure 4 - Water saturation distribution from multirate, steady-state flooding, mixed-wet case.



Figure 5 - Relative permeabilities (RP); from theory in this paper; from Darcy's law with no corrections (NC); and true curves (T) from input data; water-wet case.



Figure 6 - Capillary pressure from theory and from input (True), water-wet case



Figure 7 - Relative permeabilities; from theory in this paper and true values (T) from simulator input.



Figure 8 - Capillary pressure from theory (points) and from simulator input (line); mixed-wet case.