Imbibition Relative Permeabilities from the Diffuse-Interface Theory

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Abstract

A new model for two-phase flow in porous media is presented, and an example of an application is given: the calculation of relative permeabilities for non-steady-state imbibition.

1 Introduction

A new model for two-phase flow in porous media has recently been presented in two papers [1, 2]. We shall here call it the “DI-model” (Diffuse-Interface). An overview is given in section 2 and an example of its use is given in section 3.

The model is based on the diffuse interface model of fluid mechanics, where the two phases are manifestations of one and the same fluid, the transition from one phase to the other being taken care of by an equation of state of the van der Waals type. There is just one set of balance equations, in contradistinction to the traditional two-continuum model of two-phase flow, where each phase is considered as a separate fluid.

The model is at present restricted to fluids of one chemical component. Most of the work done with it has been directed towards establishing a “dialogue” with the traditional two-continuum model for two-phase flow.

On the one hand, the DI-model can provide the traditional model with information about relative permeabilities. This is so because the DI-model does not use relative permeabilities and is thus capable of making statements about them. In fact, expressions for relative permeabilities have been proposed [2], giving them in terms of the thermodynamical properties of the fluid, the wetting properties of the rock (as embodied in the capillary pressure), and some parameters.

On the other hand, the traditional two-continuum model has provided the DI-model with the means to incorporate experimentally obtained information about wetting, through capillary pressure versus saturation correlations [2].

In the present paper we continue the dialogue between the two models by introducing, in the DI-model, information leading to a liquid-wet rock. We then use the

DI-model with this additional information to calculate relative permeabilities for a non-steady-state imbibition flow.

2 Description of the model

The model assumes that temperature is a constant, and considers a fluid consisting of only one chemical component, with an equation of state of the van der Waals type, i.e., including the possibility of a phase transition. At the pore level, the two phases (liquid and vapor) are treated as a single fluid with variable density, according to the diffuse-interface model [1]. The mass, momentum, and energy balance equations are upscaled from the pore level to the macroscopic level (also called the Darcy level), by using the Marle [1] averaging technique. The upscaled equation describing two-phase flow in porous media, involves one dependent variable, R, the upscaled fluid density, and four independent variables, position $\mathbf{x}$ and time t. This equation is

$$\frac{\partial R}{\partial t} + \boldsymbol{\nabla} \cdot (R\mathbf{V}) = 0, \tag{1}$$

where $\mathbf{V}$ is the Darcy-level velocity:

$$\mathbf{V} = -\frac{KR}{\phi\eta}\boldsymbol{\nabla}\left(\frac{d\Psi}{dR} - \Lambda\nabla^2 R + G\right). \tag{2}$$

Here $\Psi(R)$ is the total Helmholtz free energy of the bulk fluid, and

$$G = \mathbf{g}\cdot\mathbf{x}, \tag{3}$$

is the gravitational potential energy. K is the absolute permeability, Λ a constant, ϕ the porosity, and η the fluid viscosity. Note that the flow equation, obtained by combining equations (1) and (2),

$$\frac{\partial R}{\partial t} = \boldsymbol{\nabla}\cdot\left(\frac{KR^2}{\phi\eta}\boldsymbol{\nabla}\left(\frac{\partial\Psi}{\partial R} - \Lambda\nabla^2 R + G\right)\right), \tag{4}$$

is (when G is identically zero) the Cahn-Hilliard equation.

The total Helmholtz free energy of the bulk fluid is of the form

$$\Psi(R) = W(R) + I(R) - \bar{P} \tag{5}$$

where $\bar{P}$ is a constant (the pressure at equilibrium). W is a function with two minima, occuring at two distinct values, $R = R_v$ and $R = R_l$, such that $W(R_v) = W(R_l) = 0$. R_v and R_l are the densities of the pure vapor and liquid phases.

W can be obtained from a known equation of state by

$$W(R) = P^b(R_v)\left(1 - \frac{R}{R_v}\right) + R\int_{R_v}^{R} P^b(R')\frac{dR'}{R'^2}. \tag{6}$$

Alternatively, one can use a simple W-function [1, 2] such as

$$W(R) = (P_c/R_c^4)(R - R_v)^2(R - R_l)^2, \tag{7}$$

where P_c and $R_c \equiv (R_l + R_v)/2$ are the pressure and density at the critical point. Equation (7) is behind the calculations presented in this paper.

I has been shown to be related to the empirical capillary pressure correlation P^c [2] through

$$\frac{d^2 I}{dR^2} = \frac{1}{R}\frac{dP^c}{dR}. \tag{8}$$

A well-posed problem is obtained by supplying an initial condition $R(\mathbf{x}, 0) = F(\mathbf{x})$, and a boundary condition. The boundary conditions which are relevant to one-dimensional reservoir studies are of the following two types

$$(a)\left\{\begin{array}{l} R = \alpha \\ \frac{\partial R}{\partial x} = 0, \end{array}\right. \qquad (b)\left\{\begin{array}{l} V = G \\ \frac{\partial R}{\partial x} = 0. \end{array}\right.$$

where α is a constant and G a function of x, R, and $\partial R/\partial x$.

A boundary condition on R in the present model is equivalent, because of the equation of state, to a condition on the pressure. The boundary conditions involving $\partial R/\partial x$ are new to this model and have been discussed in a previous publication [1].

Relative permeabilities do not exist in the model. The wetting properties of the rock are entered by means of the I-function.

The determination of the I-function follows a method given in a previous publication [2], and adapted to the case of a liquid wet rock. It is based on equation (8) relating I to capillary pressure. This function is here assumed to be of the form

$$P^c = C\left(\frac{1 - S_{lr}}{S_l - S_{lr}}\right)^a, \tag{9}$$

where S_l and S_{lr}, are the liquid saturation and the residual liquid saturation. C is the entry pressure and $1/a$ is the pore size distribution index. Saturation is interpreted as follows:

$$S_l = \frac{R - R_v}{R_l - R_v}, \tag{10}$$

so that

$$S_{lr} = \frac{R_v^* - R_v}{R_l - R_v} \quad \Longleftrightarrow \quad R_v^* = R_v + (R_l - R_v)S_{lr}. \tag{11}$$

R_v^* is the density of vapor with irreducible liquid. I is determined in such a way that $W + I$ has two minima of value zero, occuring at $R = R_v^*$ and $R = R_l$. In addition, one has to introduce I^s, a “smoothed out ” I, to avoid the singularity which is due to the singularity in P^c at $S_l = S_{lr}$.

The previously published expressions for the relative permeabilities [2] are

$$k_{rl} = \frac{S_l^*}{1 + \dfrac{\gamma(1 - S_l^*)}{\tilde{g}\tilde{R}_l}}, \qquad k_{rv} = \frac{1 - S_l^*}{1 + \dfrac{\gamma(S_l^*)}{\tilde{g}\tilde{R}_v^*}}, \tag{12}$$

where

$$S_l^* = \frac{\tilde{R} - \tilde{R}_v^*}{\tilde{R}_l - \tilde{R}_v^*}, \qquad \gamma(S_l^*) = -\left[\frac{d\tilde{P}^c}{d\tilde{R}}\sqrt{\frac{2}{\tilde{\Lambda}}[\tilde{W}(\tilde{R}) + \tilde{I}^s(\tilde{R})]}\right]_{\tilde{R}=\tilde{R}_l-(\tilde{R}_l-\tilde{R}_v^*)S_l^*},$$

where the tildes denote dimensionless quantities (see the Appendix). The expressions above were derived for steady-state flow.

3 Relative permeabilities for imbibition

The model has been used, in two earlier publications [1, 2], to investigate the relative permeability concept.

In the first reference [1], a steady state ganglionic flow of two phases was studied and it was shown that the generalized two-continuum model with viscous coupling is compatible with the present model.

In the second reference [2] we looked at a nearly steady state drainage flow in a vapor wet rock. We derived the above theoretical expressions for relative permeabilities, and we showed that they are in good agreement with the Corey-Burdine expressions.

Here we present results from a non steady-state imbibition flow. The calculations have been performed in one space dimension and the results are presented in terms of dimensionless quantities (see the Appendix). The unit of length is such that $0 \le \tilde{x} \le 1$ and we have taken the $\tilde{x}$-axis to point downwards so that $\tilde{g}$ is negative.

The constants have been chosen as follows:

$$\tilde{R}_l = 1.6, \quad \tilde{R}_v = 0.4, \quad S_{lr} = 0.4, \tag{13}$$

$$\tilde{\Lambda} = 0.01, \quad \tilde{g} = -0.5, \quad \eta_v/\eta_l = 0.1. \tag{14}$$

$$a_l = 1.00, \quad a_v = 1.40. \tag{15}$$

(In (14), η_l and η_v are the viscosities of the liquid with density R_l and the vapor with density R_v.) The two minima of the $\tilde{W} + \tilde{I}^s$-function occur at $\tilde{R}_v^* = 0.88$ and $\tilde{R}_l = 1.60$. These are the densities of the vapor and liquid phases supported by this $\tilde{W} + \tilde{I}^s$-function.

We consider imbibition in a one-dimensional liquid-wetting porous medium. We want the flow to be non-steady state and achieve this by arranging imbibition to take place at diminishing velocities, leading to a static equilibrium. Specifically, we consider a medium which is initially filled mostly with vapor overlying a thin layer of liquid, with capillary forces larger that gravity forces so that imbibition takes place, the amount of vapor decreasing at the top and the amount of liquid increasing at the bottom. We fix the boundary conditions, as explained below, so that imbibition approaches, as $\tilde{t} \to \infty$, a static final state with mostly liquid underlying a thin layer of vapor.

The mathematical formulation is as follows.

$\tilde{R}(\tilde{x}, 0)$ is given as a momotonically increasing function of $\tilde{x}$, where $\tilde{R}$ stays close to $\tilde{R}_0$ for most values of $\tilde{x} < 1$, then rises rapidly to $\tilde{R}_1$, reaching $\tilde{R}_1$ at $\tilde{x} = 1$. Here $\tilde{R}_0 \approx \tilde{R}_v^*$ and $\tilde{R}_1 \approx \tilde{R}_l^*$. The determination of the exact values of $\tilde{R}_0$ and $\tilde{R}_1$ is given below.

The pressure is kept constant at both ends, which, in the present model, means imposing boundary conditions

$$\begin{aligned} \tilde{R}(0, \tilde{t}) &= \tilde{R}_0, \qquad & \left.\frac{\partial \tilde{R}}{\partial \tilde{x}}\right|_{\tilde{x}=0, \forall \tilde{t}} &= 0, \\ \tilde{R}(1, \tilde{t}) &= \tilde{R}_1, \qquad & \left.\frac{\partial \tilde{R}}{\partial \tilde{x}}\right|_{\tilde{x}=1, \forall \tilde{t}} &= 0. \end{aligned} \tag{16}$$

The solution of equation (4) will represent imbibition as described above if the values of $\tilde{R}_0$ and $\tilde{R}_1$ allow the existence of the state of equilibrium specified. To make sure that this is the case we first solve this equation with no-flow boundary conditions at top and bottom and with $\tilde{R}(\tilde{x}, 0)$ equal to a constant C. By trial and error we find a value of C that leads, when $\tilde{t} \to \infty$, to a state that can reasonably be described as "mostly liquid underlying a thin layer of vapor." With the numerical values given in (13)–(15) we found $\tilde{R}_0 = 0.8280$ and $\tilde{R}_1 = 1.6037$.

Figures 1 and 2 illustrate the solution of the imbibition problem: Figure 1 is a plot of the density, Figure 2 is a plot of the fluid velocity, both versus $\tilde{x}$, for values of $\tilde{t}$ in the range $[0, 0.3]$. It will be noted that flow is of the countercurrent type.

"Experimental" relative permeabilites are calculated from the results of the calculation and compared to the theoretical formulas given above (equations (12)). To get the "experimental" values we start with the definitions in terms of dimensionless quantities [2]:

$$\tilde{V}_l^* = \frac{k_{rl(exp)}}{\tilde{\eta}(\tilde{R}_l)} \left(-\frac{\partial \tilde{p}_l^*}{\partial \tilde{x}} + \tilde{g}\tilde{R}_l \right), \tag{17}$$

$$\tilde{V}_v^* = \frac{k_{rv(exp)}}{\tilde{\eta}(\tilde{R}_v^*)} \left(-\frac{\partial \tilde{p}_v^*}{\partial \tilde{x}} + \tilde{g}\tilde{R}_v^* \right). \tag{18}$$

These formulas mean that we look at the fluid in the transition region (i.e., the region where $\tilde{R}_v^* < \tilde{R} < \tilde{R}_l$) as a mixture of two phases, each phase having a density singled out by a minimum of the function $\tilde{W} + \tilde{I}^s$. $\tilde{V}_a^*$ is the velocity of phase a, and $\tilde{p}_a^*$ is the pressure in phase a ($a = l, v$). The present model does not provide these quantities, and we know of no rigorous derivation. To obtain them, we proceed by adaptng the method described previoulsy [2] to the liquid-wet case. With this in mind we introduce the following notations for the velocities and momenta at, respectively, $S_l^* = 0$ and $S_l^* = 1$:

$$\tilde{V}_0 = \tilde{V}|_{S_l^*=0}, \quad \tilde{V}_1 = \tilde{V}|_{S_l^*=1}, \tag{19}$$

$$\tilde{\Gamma}_0 = \tilde{\Gamma}|_{S_l^*=0}, \quad \tilde{\Gamma}_1 = \tilde{\Gamma}|_{S_l^*=1} \tag{20}$$

(where $\tilde{\Gamma} = \tilde{R}\tilde{V}$). It is now postulated that [2]

$$\tilde{V}_l^* = \frac{\tilde{\Gamma}_0 - \tilde{\Gamma}}{\tilde{\Gamma}_0 - \tilde{\Gamma}_1}\,\tilde{V}_1, \qquad \tilde{V}_v^* = \frac{\tilde{\Gamma} - \tilde{\Gamma}_1}{\tilde{\Gamma}_0 - \tilde{\Gamma}_1}\,\tilde{V}_0, \tag{21}$$

where the right-hand sides are obtained by using the solution to the differential equation of the model. It is further postulated that

$$\frac{\partial \tilde{p}_l^*}{\partial \tilde{x}} = -\gamma_{exp}(1 - S_l^*), \qquad \frac{\partial \tilde{p}_v^*}{\partial \tilde{x}} = -\gamma_{exp}(S_l^*). \tag{22}$$

where

$$\gamma_{exp}(S_l^*) = \left[\frac{d\tilde{P}^{cs}}{d\tilde{R}}\frac{\partial \tilde{R}}{\partial \tilde{x}}\right]_{1-S_l^*}. \tag{23}$$

In this expression, the derivative of $\tilde{P}^{cs}$ is given by equation (8), written with dimensionless quantities and with $\tilde{I}^s$ instead of I, while the partial derivative of $\tilde{R}$ is obtained by using the solution of the differential equation.

Thus, at chosen time steps, the solution of the differential equation is used to calculate $k_{rl(exp)}$ and $k_{rv(exp)}$ at values of $\tilde{x}$ where the density $\tilde{R}$ is intermediate between $\tilde{R}_v^*$ and $\tilde{R}_l$. These values are shown in figure 3: it is seen that a large number of relative permeabilities are larger than 1. A closer look reveals that this is characteristic for short times and is an indication that equations (17) and (18) are too simple to describe the transient behavior of the flow. However, when the experimental values are normalized by the end-point values, i.e., when plotting

$$\frac{k_{rl(exp)}}{k_{rl(exp)} \text{ at } S_l^* = 1} \quad \text{and} \quad \frac{k_{rv(exp)}}{k_{rv(exp)} \text{ at } S_l^* = 0}$$

versus S_l^* for given $\tilde{t}$, all points gather remarkably well around the theoretical curves given by equations (12). See figure 4.

Figure 3 also shows that the relative permeabilities become gradually flatter as the fluid decelerates to zero velocities. We have plotted the end-point relative permeabilities versus the fluid velocity: see figures 5 and 6. The straight lines shown are the least square fits. Specifically, it is found that

$$k_{rl(exp)(S_l^*=1)} = 3.891\,\tilde{V}_{(S_l^*=1)} + 0.004, \tag{24}$$

$$k_{rv(exp)(S_l^*=0)} = 1.822\,\tilde{V}_{(S_l^*=0)} - 0.017. \tag{25}$$

4 Conclusion

Relative permeabilities, normalized by their values at the end-point saturations are in fairly good agreement with theoretical expressions written in terms of the thermodynamical and wetting properties of the fluid. The relative permeabilities at the end-point saturations are not necessarily smaller than 1, but there is a linear relation between them and the fluid velocities at these saturations. The next step in the investigation of the relative permeability concept is to find out if the coefficients in the linear relation are independent of the flow type.

Appendix: dimensionless quantities

All dimensionless densities are equal to the corresponding dimensional densities divided by the critical density R_c. All dimensionless pressures are equal to the corresponding pressures divided by the critical pressure P_c. All dimensionless potentials $(\tilde{I}^s, \tilde{W})$ are equal to the corresponding dimensional potentials divided by P_c. We have used the simple expression (7) so that $R_c = (R_l + R_v)/2$.

L being a unit of length, the other dimensionless quantities used in the text are:

$$\begin{aligned}
\tilde{g} &= (R_c L/P_c) g \\
\tilde{t} &= (K P_c/(\phi \eta_c L^2)) t \\
\tilde{V} &= (\phi \eta_c L/(K P_c)) V \\
\tilde{x} &= x/L \\
\tilde{\eta}(\tilde{R}) &= \eta/\eta_c = (\eta_v/\eta_l)^r, \quad r = (\tilde{R}_l + \tilde{R}_v - 2\tilde{R})/(2(\tilde{R}_l - \tilde{R}_v)) \\
\tilde{\Lambda} &= (R_c^2/(P_c L^2)) \Lambda
\end{aligned}$$

References

[1] Papatzacos, P.: "Macroscopic two-phase flow in porous media assuming the diffuse-interface model at pore level," *Transport in porous media* (2002) **49**, 139–174.

[2] Papatzacos, P., and Skjaeveland, S.M.: "Relative Permeability from Capillary Pressure," *Paper SPE77540, Presented at the SPE Annual Technical Conference and Exhibition, San Antonio, Texas, 29 September–2 October 2002*.

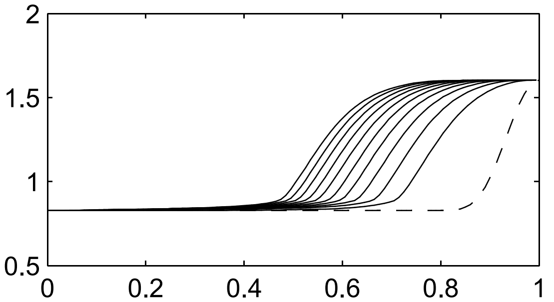

Figure 1: Dimensionless density versus dimensionless x-coordinate, for times 0.0 (broken curve) to 0.3 by steps of 0.03.

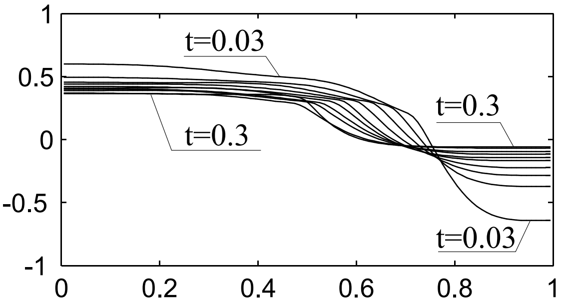

Figure 2: Dimensionless velocity versus dimensionless x-coordinate, for times 0.03 to 0.3 by steps of 0.03.

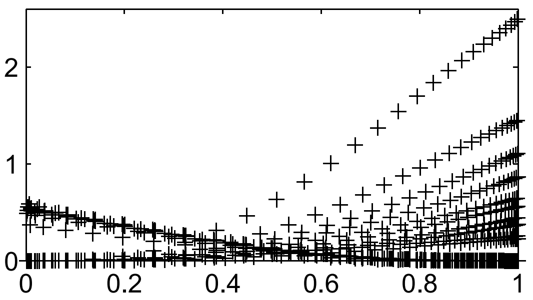

Figure 3: ``Experimental'' relative permeabilities versus normalized liquid saturation for the times of Figure 2

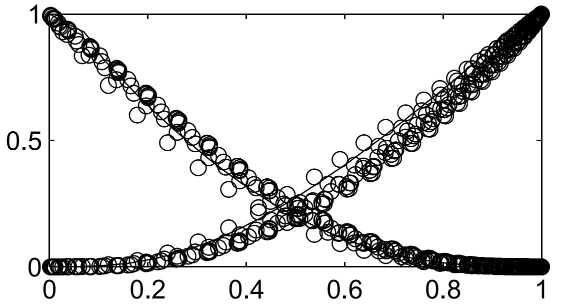

Figure 4: Relative permeabilities: the circles are the values shown in Figure 3, normalized by the end-point values. The lines show the theoretical curves.

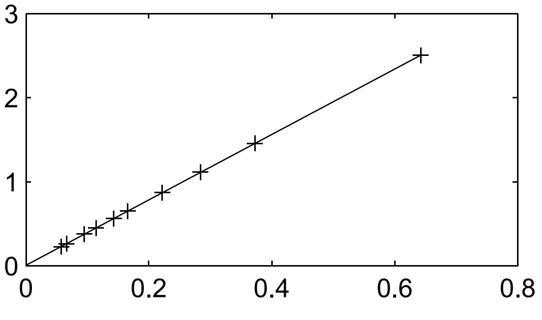

Figure 5: End-point liquid relative permeability versus dimensionless liquid velocity. The straight line is a least squares fit.

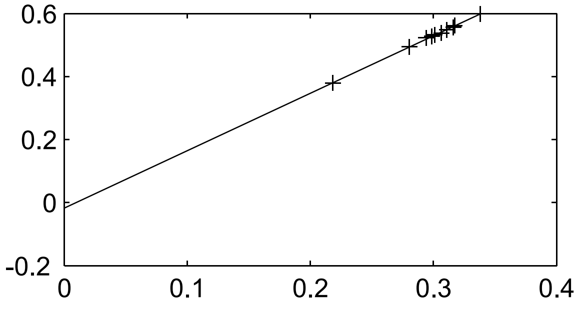

Figure 6: End-point vapor relative permeability versus dimensionless vapor velocity. The straight line is a least squares fit.