PROBLEM # 1

Calculate the formation compressibility at p = 5000 psig from the following experimental data.

Pore Volume (cm^3)	Confining Pressure (psig)
50.0	3000
48.5	4000
47.8	5000
47.2	6000
47.0	7000

THEORY

$$c_f = \frac{1}{V_p}\frac{\partial V_p}{\partial p} = -\frac{1}{V_p}\frac{\partial V_p}{\partial p_{confining}}, \quad \frac{dV}{dp}(5000) \approx \frac{V(6000) - V(4000)}{6000 - 4000}$$

SOLUTION

1) $\frac{dV}{dp_{confining}} \approx \frac{47.2 - 48.5}{2000} = -6.5 \times 10^{-4}\ cm^3/psi$

NOTE: this is an average of the forward & backward difference quotients and is more accurate!

FWD: $\frac{dv}{dp} \approx \frac{47.2 - 47.8}{1000} = -6 \times 10^{-4}\ cm^3/psi$

BWD: $\frac{dv}{dp} \approx \frac{47.8 - 48.5}{1000} = -7 \times 10^{-4}\ cm^3/psi$

2) $c_f = -\frac{1}{V_p}\frac{dV_p}{dp_{confining}} = -\frac{1}{47.8\ cm^3}(-6.5 \times 10^{-4}\ cm^3/psi)$

$= \underline{1.36 \times 10^{-5}\ psi^{-1}}$

CONCLUSION

$c_f = 1.36 \times 10^{-5}\ psi^{-1}$

PROBLEM # 2

A reservoir covers 12,000 acres and is 20 feet thick. The formation has initial porosity of 0.25. The pore volume compressibility of the reservoir rock is 3.0×10^{-6} psi^{-1}. Calculate the change in reservoir pore volume (in barrels) if the reservoir pressure decreases by 2,000 psi. (1 acre·ft = 7758 barrels).

THEORY

$$V_p = 7758\left(\frac{bbls}{ac \cdot ft}\right)A(ac)h(ft)\phi, \quad c_f = \frac{1}{V_p}\frac{\partial V_p}{\partial p}$$

SOLUTION

1) $V_{pi} = 7758\,A h \phi$

$= (7758)(12{,}000)(20)(0.25) = 4.655 \times 10^8\ rb$

2) Assumption

c_f is constant

3) separate variables & integrate

$$c_f \int_{p_i}^{p_i-2000} dp = \int_{V_{pi}}^{V_p} \frac{dV_p}{V_p} = \ln\frac{V_p}{V_{pi}} \Rightarrow V_p = V_{pi}e^{-2000c_f}$$

4) Solve for change in pore volume

$\Delta V_p = V_p - V_{pi}$

$= V_{pi}e^{-2000c_f} - V_{pi}$

$= V_{pi}\left(e^{-(2000)(3\times10^{-6})} - 1\right)$

$= 4.655\times10^8\left(e^{-0.006} - 1\right) = \underline{\underline{-2.785\times10^6\ rb}}$

CONCLUSION

V_{pi} decreases $2.785\times10^6\ rb$

PROBLEM # 3

Use the generalized form of Darcy's equation to derive the flow equation for ~~linear dipping flow~~ spherical flow of an incompressible liquid in a hemispherical shaped reservoir with a well that bottoms at the center of the sphere. Neglect the gravity term (assume dz/ds = 0). Let the radius of the sphere and well be r_e and r_w, respectively.

THEORY

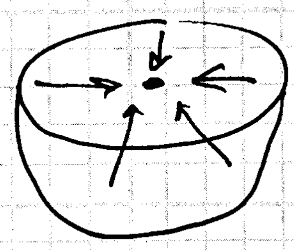
$$\frac{q_s}{A} = -\frac{k}{\mu}\left(\frac{dp}{ds} - \rho g \frac{dz}{ds}\right), \quad SA_{sphere} = 4\pi r^2$$

SOLUTION

1) Assumptions

- Darcy's Law (Darcy flow, SS flow, non-reactive fluid, single phase flow (S=1))
- PVT (incompressible liquid, isothermal conditions)
- Geometry (spherical flow, $ds = -dr$, $A = 2\pi r^2$, $\frac{dz}{ds} = 0$) $r_w \rightarrow$ just penetrates hemisphere

SKETCH



2) $$V_s = -\frac{k}{\mu}\left(\frac{dp}{ds} - \frac{\rho g}{c}\frac{dz}{ds}\right) = \frac{q}{A}$$

$$q_r = 2\pi r^2 \frac{k}{\mu}\frac{dp}{dr}$$

3) separate variables & integrate

$$q_r \int_{r_w}^{r_e} \frac{dr}{r^2} = \frac{2\pi k}{\mu}\int_{p_w}^{p_e} dp$$

$$q_r\left(-\frac{1}{r_e} + \frac{1}{r_w}\right) = \frac{2\pi k}{\mu}(p_e - p_w) \Rightarrow q_r = \frac{2\pi r_w k\,(p_e - p_w)}{\mu\left(1 - \frac{r_w}{r_e}\right)} \quad \square$$

CONCLUSION

$$q_r = \frac{2\pi r_w k\,(p_e - p_w)}{\mu\left(1 - \frac{r_w}{r_e}\right)} \Rightarrow q_r \propto \text{circumference of well bore}$$

$\propto k/\mu$

$\propto p_e - p_w$

if $r_w \ll r_e$

PROBLEM # 4

Show that the flow equation for linear flow of an incompressible liquid in a dipping bed is

$$q = \frac{kA}{\mu}\left[\frac{\Delta p}{L} + \frac{\rho g \sin\theta}{D}\right]$$, where $D = 1.01325 \times 10^6$ and θ is the dip angle.

THEORY

$$\frac{q_s}{A} = -\frac{k}{\mu}\left(\frac{dp}{ds} - \rho g \frac{dz}{ds}\right), \quad z^+ \downarrow, \theta = \sin^{-1}\left(z/L\right)$$

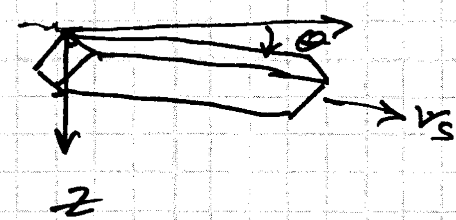
SOLUTION

1) Assumptions

- Darcy's Law (Darcy flow, SS flow, non-reactive fluid, single phase flow) — s = 1
- PVT (incompressible liquid, isothermal conditions)
- Geometry (linear flow in dipping bed, A constant)

2) $v_s = -\frac{k}{\mu}\left(\frac{dp}{ds} - \frac{\rho g}{c}\frac{dz}{ds}\right) = \frac{q}{A}$

$$\frac{q}{A} - \frac{k}{\mu}\frac{\rho g}{c}\frac{dz}{ds} = -\frac{k}{\mu}\frac{dp}{ds}$$



SKETCH

3) separate variables & integrate

$$\frac{q}{A}\int_0^L ds - \frac{k}{\mu}\frac{\rho g}{c}\int_0^z dz = -\frac{k}{\mu}\int_{p_1}^{p_2} dp$$

$$\frac{q}{A}L - \frac{k}{\mu}\frac{\rho g z}{c} = \frac{k}{\mu}(p_1 - p_2) \Rightarrow \frac{q}{A} = \frac{k}{\mu}\left(\frac{p_1 - p_2}{L} + \frac{\rho g}{c}\frac{z}{L}\right)$$

CONCUSION

$$q = \frac{kA}{\mu}\left[\frac{\Delta p}{L} + \frac{\rho g}{D}\sin\theta\right]$$ where $\Delta p = p_1 - p_2$

$D = c = 1.01325 \times 10^6 \frac{dyn/cm^2}{atm}$

PROBLEM # 5

Show that the flow equation for vertical flow of an incompressible liquid upward with head, h, is given by Eq. 2-24, p.75 of ABW.

THEORY

$$\frac{q_s}{A} = -\frac{k}{\mu}\left(\frac{dp}{ds} - \rho g \frac{dz}{ds}\right)$$

SOLUTION

1) Assumptions

- Darcy's Law applies (Darcy flow, ss flow, non-reactive fluid, single phase flow) $\longrightarrow s = 1$
- PVT (incompressible liquid, isothermal conditions)
- Geometry (vertical flow upward, A constant) $dz = -ds$

SKETCH

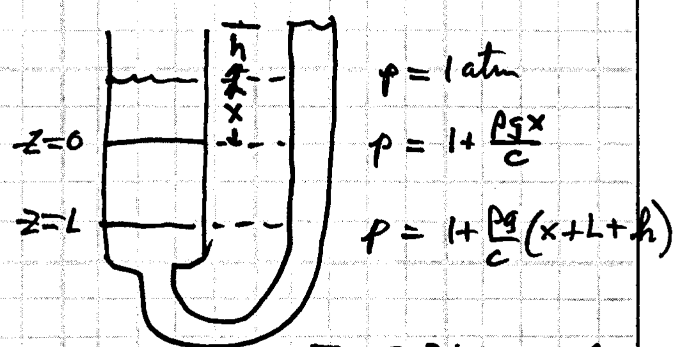


Fig 2-31, p74 of ABW

$$2)\ v_s = -\frac{k}{\mu}\left(\frac{dp}{ds} - \frac{\rho g}{c}\frac{dz}{ds}\right)$$

$$= -\frac{k}{\mu}\left(-\frac{dp}{dz} + \frac{\rho g}{c}\right) = \frac{q}{A}$$

$$= \frac{k}{\mu}\frac{dp}{dz} - \frac{k}{\mu}\frac{\rho g}{c}$$

3) Separate variables & integrate

$$\left(\frac{q}{A} + \frac{k}{\mu}\frac{\rho g}{c}\right)\int_L^0 dz = \frac{k}{\mu}\int_{1+\frac{\rho g}{c}(x+L+h)}^{1+\frac{\rho g x}{c}} dp$$

$$-\left(\frac{q}{A} + \frac{k}{\mu}\frac{\rho g}{c}\right)L = -\frac{k}{\mu}\left(\frac{\rho g L}{c} + \frac{\rho g}{c}h\right) \Rightarrow \frac{q}{A} = \frac{k}{\mu L}\left(\frac{\rho g h}{c}\right) \quad \square$$

CONCLUSION

$$\frac{q}{A} = \frac{k}{\mu L}\left(\frac{\rho g h}{c}\right)$$ which is Eq 2-24, p 75 of ABW.