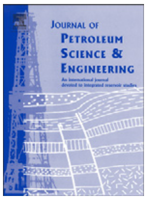
Journal of Petroleum Science and Engineering 71 (2010) 19–22

Contents lists available at ScienceDirect

Journal of Petroleum Science and Engineering

journal homepage: www.elsevier.com/locate/petrol



A single-parameter fit correlation for estimation of oil recovery from fractured water-wet reservoirs

Dag Chun Standnes *

Statoil ASA Bergen, Sandslihaugen 30, 5020 Bergen, Norway

ARTICLE INFO

Article history:
Received 18 March 2009
Accepted 14 December 2009

Keywords:
spontaneous imbibition
correlation
Aronofsky model
oil recovery
single matrix block

ABSTRACT

Spontaneous imbibition of water into the matrix blocks due to capillary forces is an important recovery mechanism for recovery of oil from fractured water-wet reservoirs. Such systems are usually modeled as a dual porosity system where the exchange rate of oil and water is described by so called transfer functions. A widely used correlation for predicting oil recovery from an individual matrix block is the Aronofsky model. In general this exponential decay rate function under predict early time recovery and overestimates late time oil recovery. Based on recent advances solving the Washburn equation, a new correlation for describing oil recovery vs. time for single matrix blocks is described. The correlation improves the fit to spontaneous imbibition data published in the literature. The correlation containing the Lambert's *W* function keeps the simplicity of the Aronofsky model as only one fit parameter is needed in order to predict oil recovery vs. time. Optimal fit to experimental data is obtained with a fit parameter value of 0.05 for the Aronofsky correlation whereas the value of 0.135 should be used in the improved correlation case.

1. Introduction

Spontaneous imbibition of water into the matrix blocks is regarded as a very important driving mechanism for oil recovery in water-wet dual porosity media. Water is sucked into the matrix blocks from the fracture system by capillary forces and oil is expelled. The oil production rate from the matrix blocks in such reservoirs is usually described by a so-called transfer function describing the rate by which is expelled from the matrix blocks. One of the most widely used transfer function correlations is the well known Aronofsky correlation reading:

$$R = R_{\max}\cdot(1-e^{-\omega t}) \tag{1}$$

where R is oil recovery as a function of time, $R_{\max}$ is maximum oil recovery, ω *is* fit parameter and t is the imbibition time.

Aronofsky et al. (1958) derived Eq. (1) based on an abstract model under the assumptions: (A) oil recovery is a continuous monotonic function of time and it converges to some finite limit (B) none of the properties which determine the rate of convergence change sufficiently during the process to affect this rate or the limit. They did not consider the physics of the oil displacement process in this derivation. More complicated and sophistical expressions including multi-exponential expressions have also been used in order to improve the fit to experimental data (Civan, 1998; Kazemi et al., 1992; Gupta and Civan, 1994).

The advantage of Eq. (1), however, is its simplicity because only one parameter needs to be adjusted to fit experimental data. Ma et al. (1997) proposed a modified version of Eq. (1) reading:

$$\frac{R}{R_{\max}} = (1-e^{-\Omega t_D}) \tag{2}$$

$$t_D = t\sqrt{\frac{k}{\phi}}\frac{\sigma}{\sqrt{\mu_w\mu_o}}\frac{1}{L_C^2} \tag{3}$$

where t_D is dimensionless time, σ is interfacial tension, k absolute permeability, ϕ porosity, μ_w, μ_o water and oil viscosity and L_C characteristic length:

$$L_C^2 = \frac{V_b}{\sum_{i=1}^{n}\frac{A_i}{l_i}} \tag{4}$$

V_b	Bulk volume of rock sample (m^3)
A_i	Area of the i-th imbibition surface (m^2)
l_i	Length from the i-th imbibition surface to the no-flow boundary (m)

Eq. (2) was shown to fit several imbibition experiments performed on strongly water-wet rock samples with different geometry and fluid properties (Ma et al., 1997). The fit parameter should be approximately

---

* Tel.: +47 48080134.
E-mail address: dcs@statoil.com.

0920-4105/$ – see front matter © 2009 Elsevier B.V. All rights reserved.
doi:10.1016/j.petrol.2009.12.008

$\Omega = 0.05$ in order to match the experimental results in an optimal way. The work reported here will focus on an improved single-parameter fit correlation for prediction of oil recovery from fractured water-wet reservoirs. The application of correlations has been shown to be very efficient in terms of computer time when dealing with field scale dual-porosity modeling (Kazemi et al., 1992).

2. The fit parameter model

2.1. Description of correlation for prediction of oil recovery vs. time

The standard correlation to be used when predicting oil recovery from water-wet matrix blocks is the Aronofsky correlation (Eq. (1)). Several researchers have, however, observed that this correlation usually underestimates early time recovery and over predict late time recovery. Fries and Dreyer (2008) have recently studied flow through capillary tubes and showed that it is in fact possible to explicitly solve the Washburn equation (Washburn, 1921) for vertical flow including gravity with respect to height. The solution is given as:

$$h(t) = \frac{a}{b}\left[1 + W(-e^{-1-\frac{b^2 t}{a}})\right] \tag{5}$$

where,

$$a = \frac{2\sigma\cos\theta}{\phi\mu_w}\frac{k}{r} \tag{6}$$

$$b = \frac{\rho g k}{\phi\mu_w} \tag{7}$$

r is tube radius, ρ is water-phase density, θ is contact angle and g is acceleration due to gravity. $W(x)$ is the Lambert's W function defined by an inverse exponential function:

$$x = W(x)e^{W(x)} \tag{8}$$

Dividing Eq. (5) by L the length of the capillary tube gives the fraction of the tube filled (e.g., saturation), because a/bL is effectively the capillary rise to the gravity head. Normalized oil recovery as a fraction of recoverable oil vs. time is then given by:

$$\frac{R(t)}{R_{\max}} = \left[1 + W(-e^{-1-\frac{b^2 t}{a}})\right] \tag{9}$$

Introducing $\alpha = \frac{b^2}{a}$ as a pure fit parameter gives:

$$\frac{R(t)}{R_{\max}} = 1 + W(-e^{-1-\alpha t}) \tag{10}$$

Eq. (10) can be used to fit experimental data adjusting α to the appropriate value. Since Eq. (5) was derived taking into account gravity forces, the correlation in Eq. (10) should probably fit experimental data more accurately than Eq. (1) especially for late times because gravity forces will always be important when approaching maximum oil recovery due to significant decrease in the capillary forces.

2.2. Lambert's W function

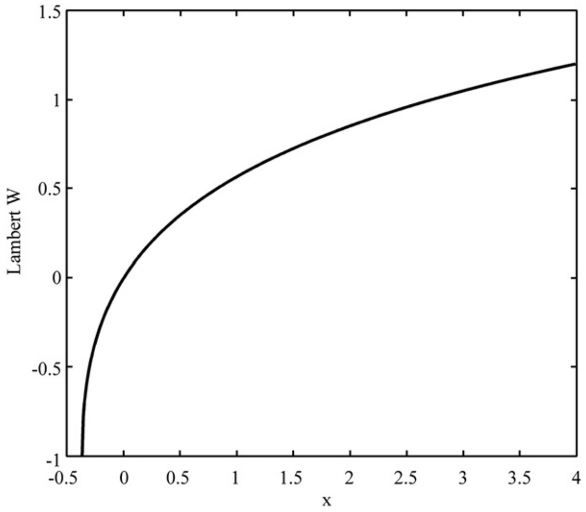


Fig. 1. The Lambert's W function plotted in the range, $-e^{-1} \leq x \leq 4$.

The Lambert's W function is in general a complex function taking values in the Gauss plane. The function has real values for values greater than $-e^{-1}$. Fig. 1 shows a plot of Lambert's W function for $x \geq -e^{-1}$. The relevant range to be used in Eq. (10) is ranging from $-1/e$ to zero (t in the range from zero to infinity). Lambert's W function can be plotted in standard commercial mathematical programs (also referred to as the Omega function, $W[x]$ and ProductLog$[x]$). Spreadsheet manipulations are also possible using the following expression given by Fries and Dreyer (2008):

$$W(x) \approx -1 + \frac{\sqrt{2 + 2ex}}{1 + \frac{4.13501\sqrt{2 + 2ex}}{12.7036 + \sqrt{2 + 2ex}}}, \quad -e^{-1} \leq x \leq 0 \tag{11}$$

e is Eulers number (2.718282…). The maximum relative error in the range $-e^{-1} \leq x \leq 0$ was 0.1%.

3. Results and discussions

3.1. Comparison between Aronofsky and improved correlation

Fig. 2 shows oil recovery fraction of recoverable oil vs. dimensionless time represented by the Aronofsky model plotted together with the improved correlation such that they are equal at 0.5 of recoverable oil. The Aronofsky model predicts lower oil recovery at early times and higher rates at late times. Fig. 3 depicts oil recovery fraction of recoverable oil vs. time for the Aronofsky model together with experimental data points generated by performing spontaneous imbibition tests on a wide variety of rock sample types, shapes and fluid systems (Mattax and Kyte, 1962; Hamon and Vidal, 1986; Zhang et al., 1996; Tavassoli et al., 2005). By comparing the plots in Figs. 2 and 3 it can clearly be seen that the correlation based on the Lambert's W function is superior to the Aronofsky correlation. The correlation

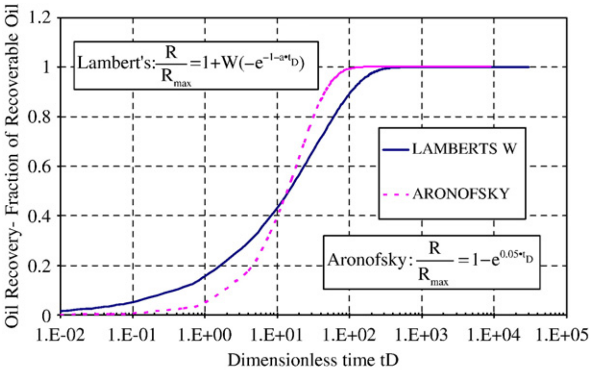


Fig. 2. The Aronofsky model and the improved correlation based on the Lambert's W function. The fit parameter in the Aronofsky model is $\Omega = 0.05$ which has been shown to give optimal correspondence to experimental data (Ma et al., 1997). The α fit parameter ($= 0.0135$) in the improved correlation has been adjusted so that the two plots are equal at 0.5 recoverable oil.

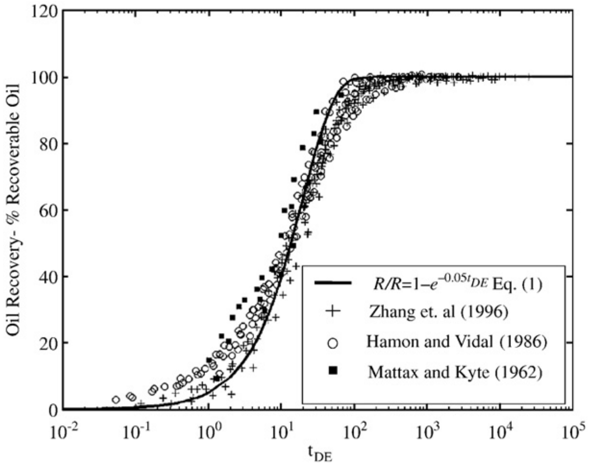


Fig. 3. Oil recovery as % of recoverable oil vs. dimensionless time for the Aronofsky model and experimental data generated by Mattax and Kyte (1962), Hamon and Vidal (1986) and Zhang et al. (1996) (plot taken from Tavassoli et al. (2005)). Compared to the functions in Fig. 2 it can easily be seen that the correlation based on Lambert's *W* function fits the experimental data significantly better than the Aronofsky model.

based on Lambert's *W* function has similar to the Aronofsky correlation only one fit parameter but tends to fit experimental data significantly more accurate. It is important to notice that the Aronofsky match cannot be improved by just changing the fit parameter value. Since omega is a multiplier of time it will not change the curve shape but just shift the curve along the time axis.

Eq. (3) has been shown to account properly for large differences in rock sample shape, size and differences in fluid viscosities and interfacial tension. Estimates of oil recovery vs. time can then be generated for variations in these parameters if experimental data for oil recovery vs. time exists for one complete set of the parameters included in Eq. (3). Table 1 shows input parameters for typical laboratory scale imbibition experiments. Oil recoveries vs. imbibition time corresponding to the input parameters in Table 1 are generated with the following equation:

$$\frac{R(t)}{R_{\max}} = 1 + W\left(-e^{-1-\alpha t_D}\right) \tag{12}$$

and plotted in Fig. 4 using both the Aronofsky and the improved correlation. Large differences in predicted oil recovery are seen indicating the importance of using the improved correlation when estimating oil recovery from water-wet single matrix blocks.

4. Conclusion

The following conclusions can be drawn from this work:

- A simple one-parameter fit correlation for predicting oil recovery vs. time from water-wet matrix block has been described

Table 1
Data for Basecase and two other cases with differences in fluid viscosities.

Data for hypothetical core sample/fluid systems	
Diameter	3.8 cm
Height	7 cm
Porosity	0.25
Permeability	500 mD
Oil–water interfacial tension	30 mN/m
Characteristic length	0.000157 m^2
Oil viscosity	1.00 mPa s
Water viscosity	1.00 mPa s

Basecase (oil viscosity = 10 mPas).
Basecase (water viscosity = 10 mPas and oil viscosity = 10 mPas).

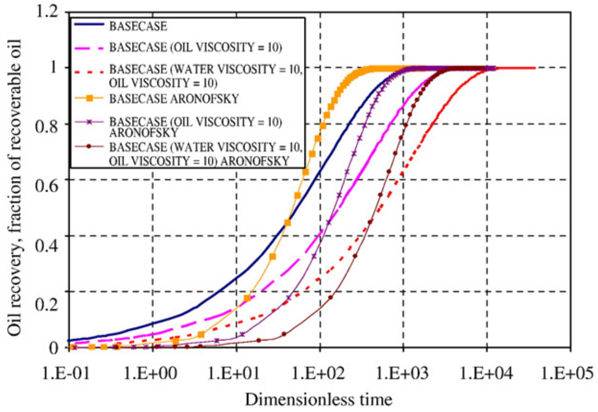


Fig. 4. Spontaneous imbibition data for imbibition of water into generic core/fluid systems generated using both the Aronofsky and the improved correlation. The cases shown for both models are Basecase (both water and oil viscosities = 1.0 mPas), Basecase with only oil viscosity changed to 10 mPas and Basecase with both oil and water viscosities changed to 10 mPas.

- The correlation improves the fit to experimental data compared to the standard Aronofsky exponential decay correlation
- The improved correlation is as simple as the Aronofsky in terms of adjustable parameters. Only one parameter needs to be adjusted to fit experimental spontaneous imbibition data
- The Aronofsky model has optimal fit to experimental data for $\Omega = 0.05$ while optimal fit to the same data is obtained with $\alpha = 0.135$ for the improved correlation

Nomenclature

$a = \frac{2\sigma\cos\theta}{\varphi\mu_w}\frac{k}{r}$	(L^2/T)
A_i	Area of the i-th imbibition surface (L^2)
$b = \frac{\rho g k}{\varphi\mu_w}$	(L/T)
e	Eulers number (2.718282…).
g	Acceleration due to gravity (L/T^2)
h	Height (L)
k	Absolute permeability (L^2)
L	Length of the capillary tube (L)
L_C	Characteristic length (L)
l_i	Length from the i-th imbibition surface to the no-flow boundary (L)
R	Oil recovery as a function of time (L^3)
$R_{\max}$	Maximum oil recovery obtained by spontaneous imbibition (L^3)
r	Radius of the capillary tube (L)
t	Imbibition time (T)
t_D	Dimensionless time (−)
V_b	Bulk volume of rock sample (L^3)
$W(x)$	Lambert's *W* function
x	Function argument
α	Fit parameter in the improved correlation (dimensionless)
σ	Oil–water interfacial tension (M/T^2)
ϕ	Fractional porosity
μ_w	Water viscosity (M/LT)
μ_o	Oil viscosity (M/LT)
θ	Contact angle
ω	Fit parameter (T^{-1})
Ω	Fit parameter (dimensionless)

Acknowledgements

The author thanks Statoil for the permission to publish this paper and professor M. Blunt for providing the plot shown in Fig. 3.

The assistance from Liping Yu Ph.D. in preparing Fig. 1 is also highly appreciated.

References

Aronofsky, J.S., Massé, L., Natanson, S.G., 1958. A model for the mechanism of oil recovery from the porous matrix due to water invasion in fractured reservoirs. Petroleum Trans, AIME 213, 17–19.

Civan, F., 1998. Quadrature solution for waterflooding of naturally fractured reservoirs. SPE REE 141–147 April.

Fries, N., Dreyer, M., 2008. An analytical solution of capillary rise restrained by gravity. J Colloid Interface Sci 320, 259–263.

Gupta, A., Civan, F., 1994. An improved model for laboratory measurement of matrix to fracture transfer function parameters in immicible displacement. Paper SPE 28929 presented at the SPE 69th Annual Technical Conference and Exhibition held in New Orleans, LA, U.S.A., Sept. 25–28.

Hamon, G., Vidal, J., 1986. Scaling-up the capillary imbibition process from laboratory experiments on homogeneous and heterogeneous samples. Paper SPE 15852 presented at the 1986 European Petroleum Conf. of SPE held in London, UK.

Kazemi, H., Gilman, J.R., Eisharkawy, A.M., 1992. Analytical and numerical solution of oil recovery from fractured reservoirs with empirical transfer functions. SPE RE 7 (2), 219–227 May.

Ma, S., Morrow, N.R., Zhang, X., 1997. Generalized scaling of spontaneous imbibition data for strongly water-wet systems. J Petrol Sci Eng 18, 165–178.

Mattax, C.C., Kyte, J.R., 1962. Imbibition oil recovery from fractured, water drive reservoirs. Soc Pet Eng J 177–184 June.

Tavassoli, Z., Zimmerman, R.W., Blunt, M.J., 2005. Analytical analysis for oil recovery during counter-current imbibition in strongly water-wet systems. Transp Porous Med 58, 173–189.

Washburn, E.W., 1921. The dynamics of capillary flow. Phys Rev 17, 273–283.

Zhang, X., Morrow, N.R., Ma, S., 1996. Experimental verification of a modified scaling group for spontaneous imbibition. SPE RE 11, 280–285.