

Scaling Group for Spontaneous Imbibition Including Gravity

Dag Chun Standnes*

Statoil ASA Bergen Sandslihaugen 30, 5020 Bergen, Norway

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Spontaneous imbibition of water into the matrix blocks due to capillary forces is an important recovery mechanism for recovery of oil from fractured water-wet reservoirs. Scaling groups is one important way to estimate oil recovery from such types of reservoirs. This paper describes a new scaling group for the estimation of oil recovery from single matrix blocks which also accounts for gravity effects. The group is derived on the basis of the analytical solution of the Washburn equation accounting for gravity effects using the Lambert's W function. The new scaling law was tested using numerical simulation varying the core permeability and the size of the core sample. The new scaling law was able to perfectly scale these data into one unique single curve describing oil recovery as a fraction of recoverable oil versus a new dimensionless time introduced in this research.

1. Introduction

Spontaneous imbibition of water into the matrix blocks is regarded as a very important driving mechanism for oil recovery in water-wet fractured reservoirs. Water is sucked into the matrix blocks from the fracture system by capillary forces, and oil is expelled. The oil production rate from the matrix blocks in such reservoirs can be estimated using a so-called scaling equation. The most famous scaling equation is the expression introduced by Mattax and Kyte in 1962.¹ The equation reads

$$t_{\text{DMK}} = t \sqrt{\frac{k}{\phi} \cdot \frac{\sigma}{\mu_w} \frac{1}{L^2}} \quad (1)$$

where, t_{DMK} is dimensionless time introduced by Mattax and Kyte, σ is interfacial tension, k is the absolute permeability, ϕ is porosity, μ_w is water viscosity, and L is a characteristic length. Predicting oil recovery for variations in the rock-fluid quantities in eq 1 is then possible since oil recovery is a unique function of dimensionless time. The Mattax and Kyte equation was derived on the basis of the following assumptions: (1) The shape of the model must be identical to that of the matrix block. (2) The reservoir water-to-oil viscosity ratio must be duplicated in the laboratory tests. (3) Initial fluid distribution in the reservoir matrix block and the pattern of water movement in the surrounding fractures must be duplicated in the laboratory tests. (4) The relative permeability functions must be the same for the matrix block and the laboratory model. (5) The capillary pressure functions for the matrix block and the laboratory model must be related by direct proportionality. (6) Gravity effects are neglected. Much work has been performed in order to relax some of the conditions described above. On the basis of a comprehensive set of empirical data,

the viscosity term has been modified to include the oil viscosity μ_o such that the viscosity term reads²

$$\mu_g = \sqrt{\mu_w \mu_o} \quad (2)$$

The characteristic length term has furthermore been modified by Zhang et al.³ to account for differences in shape and boundary conditions:

$$L_c^2 = \frac{V_b}{\sum_{i=1}^n \frac{A_i}{l_i}} \quad (3)$$

where V_b is the bulk volume of the rock sample, A_i is the area of the i th imbibition surface, and l_i is the length from the i th imbibition surface to the no-flow boundary.

One major challenge with eq 1 is the condition that gravity forces should be neglected. The aim of this paper is to introduce a new scaling group based on recent advances solving the Washburn equation,⁴ including gravity effects, analytically. Several authors have addressed the challenge of including gravity in a general scaling equation because it is very important when the gravity forces become comparable to the capillary forces due to high water saturation in the matrix blocks or the introduction of surface-active agents.

Xie and Morrow⁵ proposed that the general scaling law including gravity forces can be expressed as

$$t_{\text{DXM}} = \frac{k \left(P_c f(\theta) + \frac{\Delta \rho g L_c^2}{L_H} \right)}{\phi \sqrt{\mu_w \mu_o} L_c^2} t \quad (4)$$

Here, P_c is the capillary pressure, $f(\theta)$ is a wettability function where θ is the contact angle, g is acceleration due to gravity, $\Delta \rho$ is the density difference between the fluid phases, t_{DXM} is a dimensionless time from the work of Xie and Morrow, and L_H

*Author e-mail: dcs@statoil.com.

(1) Mattax, C. C.; Kyte, J. R. *Soc. Pet. Eng. J.* **1962**, June, 177–184.

(2) Ma, S.; Morrow, N. R.; Zhang, X. *J. Pet. Sci. Eng.* **1997**, 18, 165–178.

(3) Zhang, X.; Morrow, N. R.; Ma, S. *SPE RE* **1996**, 11, 280–285.

(4) Washburn, E. W. *Phys. Rev.* **1921**, 17, 273–283.

(5) Xie, X.; Morrow, N. R. Paper 2026 presented at the Society of Core Analysts Symposium held in Abu Dhabi, Oct. 18–22, 2000.

is the vertical height of the core sample. Li and Horne⁶ derived another scaling group which was used to scale the data set generated by Schechter et al.⁷ using model fluid systems and large core samples. Li and Horne's expression reads

$$t_{DLH} = c^2 \frac{kk_{re}^* P_c^* (S_{wf} - S_{wi})}{\phi \mu_w L_c^2} t \quad (5)$$

where S_{wf} is the water saturation behind the imbibition front, S_{wi} is the initial water saturation, k_{re}^* is the relative permeability pseudofunction associated with k_{rw}^* and k_{ro}^* , μ_w is the effective viscosity, c is the ratio between capillary and gravity forces, P_c^* is the capillary pressure at S_{wf} , and t_{DLH} is dimensionless time from the work of Li and Horne. They managed to improve the scaling results considerably by applying eq 5 to scale the Schechter et al.⁷ data set as compared to eq 1 modified with eqs 2 and 3. They attributed the improvement to the fact that eq 5 also accounts for gravity forces which become important for the cases where low oil–water interfacial tension values were applied.

Babadagli⁸ and Hatiboglu and Babadagli⁹ investigated different scaling expressions under conditions where gravity forces were important for the displacement process. Babadagli⁸ was not able to scale spontaneous imbibition data from Berea sandstone cores for both high and low interfacial tension values using eq 1 modified with eqs 2 and 3. A fit parameter referred to as the boundary condition factor was introduced in order to obtain a reasonably good scaling match for the cases where the oil–water interfacial tension was reduced using surfactants. Tavassoli et al.¹⁰ arrived at an expression similar to the implicit expression relating imbibition height and time given by Washburn.⁴ They performed numerical simulations in order to analyze the fluid flow behavior when gravity forces were dominant. The research topic of spontaneous imbibition has been reviewed by Morrow and Mason.¹¹

2. Scaling groups

2.1. Derivation of Mattax and Kyte Scaling Group from the Solution of the Washburn Equation. There are many ways of deriving the Mattax and Kyte equation, which is basically the ratio of viscous to capillary forces. In this section, the equation will be derived from the special solution of the Washburn equation neglecting gravity. The case where gravity is included will be treated in the next section. Neglecting the gravity forces, Washburn showed that the capillary rise of a liquid in a capillary is given by

$$h^2 = \frac{\sigma R \cos \theta}{2\mu_w} t \quad (6)$$

where R is the tube radius, θ is the contact angle, σ is the oil–water IFT, t is the imbibition time, and h is the capillary rise. Dividing this equation by the total height of the tube L squared yields

$$\left(\frac{h}{L}\right)^2 = \frac{\sigma R \cos \theta}{2\mu_w L^2} t \quad (7)$$

(6) Li, K.; Horne, R. N. *SPE Paper*, 2002, No. 77544.
 (7) Schechter, D. S.; Zhou, D.; Orr, F. M., Jr. *J. Pet. Sci. Eng.* **1994**, *11*, 283–300.
 (8) Babadagli, T. *S. Pet. Eng. J.* **2001**, *6* (4), 465–478.
 (9) Hatiboglu, C. U.; Babadagli, T. *J. Pet. Sci. Eng.* **2007**, *59*, 106–122.
 (10) Tavassoli, Z.; Zimmerman, R. W.; Blunt, M. J. *J. Pet. Sci. Eng.* **2005**, *48*, 94–104.
 (11) Morrow, N. R.; Mason, G. *Curr. Opin. Colloid Interface Sci.* **2001**, *6*, 321–337.

When saturation S (S here equals h/L) of the imbibing fluid as the fraction of the tube filled and the Leverett's microscopic radius are introduced

$$R = \sqrt{\frac{8k}{\phi}} \quad (8)$$

yields,

$$S^2 = \sqrt{\frac{2k}{\phi}} \times \frac{\sigma \cos \theta}{\mu_w L^2} t \quad (9)$$

Spontaneous imbibition can therefore be scaled by a dimensionless time t_{DMK} given by (assuming strongly water-wet conditions implying that $\cos \theta$ can be set to unity and neglecting the square-root factor (only a multiplicative factor))

$$t_{DMK} = \sqrt{\frac{k}{\phi}} \times \frac{\sigma}{\mu_w L^2} t \quad (10)$$

which is equal to Mattax and Kyte's scaling group expression eq 1.

2.2. Derivation of New Scaling Group from the Solution of the Washburn Equation Including Gravity. The Washburn equation describes movement of fluids through tubes. Fries and Dreyer¹² have recently studied flow through capillary tubes and showed that it is in fact possible to explicitly solve the Washburn equation for vertical flow including gravity with respect to height. The solution is given as

$$h(t) = \frac{a}{b} [1 + W(-e^{-1-b^2 t/a})] \quad (11)$$

where

$$a = \frac{2\sigma \cos \theta}{\phi \mu_w} \frac{k}{R} \quad (12)$$

$$b = \frac{\rho_w g k}{\phi \mu_w} \quad (13)$$

where ρ_w is the water-phase density. $W(x)$ is Lambert's W function¹² defined by an inverse exponential function:

$$x = W(x) e^{W(x)} \quad (14)$$

Similarly as in the previous section, dividing eq 11 by L , the length of the capillary tube, gives the fraction of the tube filled (e.g., saturation) because $a/(bL)$ is effectively the capillary rise to the gravity head. Normalized oil recovery as a fraction of recoverable oil versus time is then given by ($a/(bL) = 1$):

$$R(t) = [1 + W(-e^{-1-b^2 t/a})] \quad (15)$$

Hence, spontaneous imbibition of water into porous media scales with a dimensionless time t_{D1} given by ($b^2/a = a/L^2$):

$$t_{D1} = 1 + W(-e^{-1-\frac{a,t}{L^2}}) \quad (16)$$

And since

$$\frac{b^2}{a} = \frac{a}{L^2} = \sqrt{\frac{k}{2\phi}} \times \frac{\sigma}{\mu_w L^2} \quad (17)$$

(12) Fries, N.; Dreyer, M. *J. Colloid Interface Sci.* **2008**, *320*, 259–263.

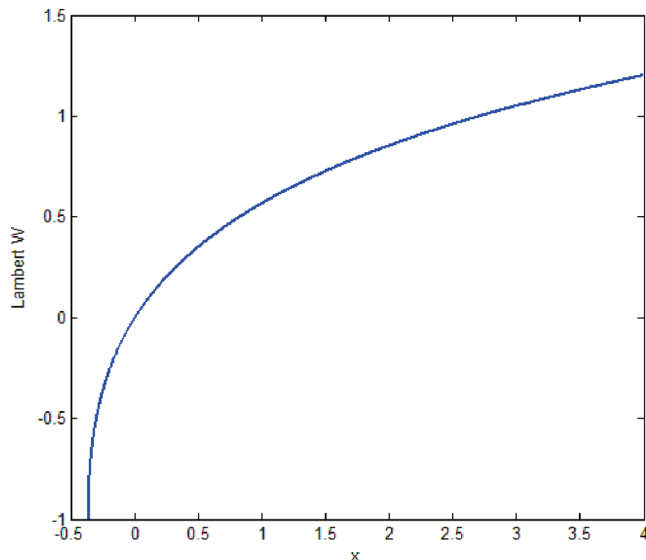


Figure 1. Lambert’s W function plotted in the range $-1/e$ to 4 using MATLAB.

eq 16 becomes

$$t_{D1} = 1 + W(-e^{-1-t} \sqrt{\frac{k}{2\phi} \frac{\sigma}{\mu L^2}}) \quad (18)$$

Finally, generalizing this equation to also include the modified viscosity and characteristic length terms yields the new dimensionless time expression:

$$t_{D_NEW} = 1 + W(-e^{-1-t} \sqrt{\frac{k}{2\phi} \frac{\sigma}{\mu_g L_c^2}}) \quad (19)$$

Plotting oil recovery as fraction of the recoverable oil versus t_{D_NEW} based on experimental data should then give a universal curve which can be used to estimate the oil recovery fraction of recoverable oil versus time also for cases where gravity forces have significant impact on the fluid flow.

2.3. Lambert’s W Function. Lambert’s W function¹² is in general a complex function taking values in the Gauss plane. The function has real values for values greater than $-e^{-1}$. Figure 1 shows a plot of Lambert’s W function for $x \geq -e^{-1}$ using MATLAB.¹³ The relevant range to be used in eq 14 is ranging from $-1/e$ to zero (t in the range from zero to infinity). Lambert’s W function can be plotted in standard commercial mathematical programs (also referred to as the Omega function, $W[x]$, and $\text{ProductLog}[x]$). Spreadsheet manipulations are also possible using the following expression given by Fries and Dreyer:¹²

$$W(x) \approx -1 + \frac{\sqrt{2+2ex}}{4.13501\sqrt{2+2ex} + 12.7036 + \sqrt{2+2ex}}, \quad -e^{-1} \leq x \leq 0 \quad (20)$$

where e is Eulers number (2.718282...). The maximum relative error in the range $-e^{-1} \leq x \leq 0$ was 0.1%.

3. Modeling Oil Recovery versus Time

In order to check the new scaling law, numerical simulations were performed using the commercial reservoir simulator ECLIPSE 100.¹⁴ The grid was Cartesian with dimensions

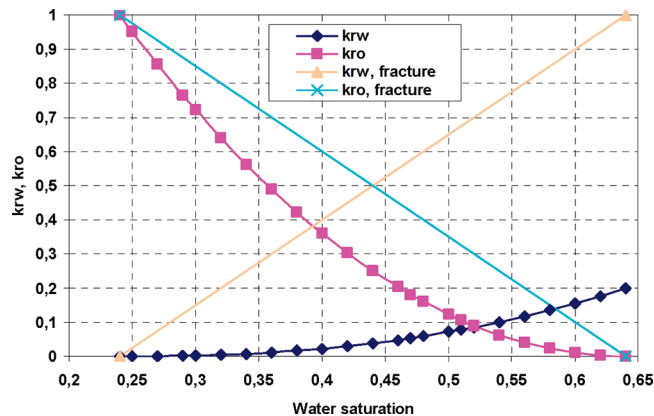


Figure 2. Relative permeability curves as input to the numerical simulations.

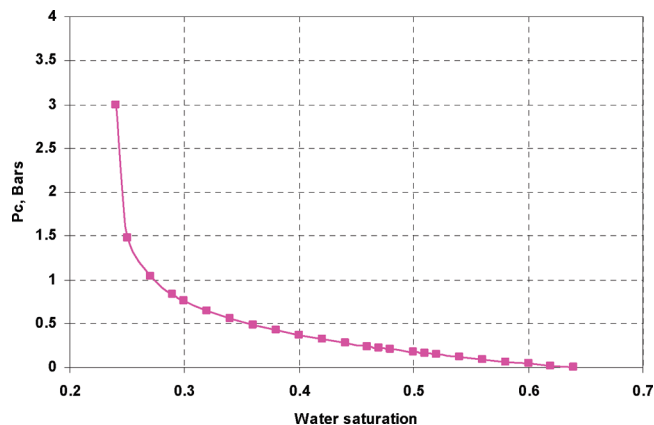


Figure 3. Capillary pressure curve as input to the numerical simulations.

$80 \times 80 \times 80$. The rock sample was represented by a small cube with dimensions $20 \times 20 \times 20$ (each grid block is usually 0.25 cm (in one case 0.5 cm)). The rock sample was considered to be homogeneous with a permeability and porosity of $k_x = k_y = k_z = 10$ mD and $\phi = 0.25$, respectively (some cases also had higher permeabilities). The outermost grid blocks represented the surrounding water phase had high porosity (0.99) and very high permeability $k_x = k_y = k_z = 1\,000\,000$ mD. All grid blocks outside the grid blocks representing the rock sample had water saturation initially equal to unity. Relative permeability to oil and water varied linearly with water saturation, and capillary pressure was equal to zero in this region.

3.1. Description of Flow Functions. Water and oil relative permeability are assumed to be properly represented by modified Corey functions described by Tweheyo et al.:¹⁵

$$k_{rw} = k_{rwe} \left[\frac{S_w - S_{wi}}{1 - S_{or} - S_{wi}} \right]^{n_w} \quad (21)$$

$$k_{ro} = k_{roe} \left[\frac{1 - S_{or} - S_w}{1 - S_{or} - S_{wi}} \right]^{n_o} \quad (22)$$

where S_{wi} is the initial water saturation, S_w is the water saturation, S_{or} is the residual oil saturation, k_{roe} is the end

(13) MATLAB 7.6.0 (R2008a); MathWorks: Natick, MA.
 (14) ECLIPSE 100 Reference Manual, versions 2008.1, 2008; Schlumberger GeoQuest: Houston, TX, 2008.

(15) Tveheyo, M. T.; Talukdar, M. S.; Torseter, O.; Vafaeinezhad, Y. Paper presented at the 11th Oil, Gas and Petrochemical Congress & Exhibition held in Teheran, Iran, 2001.

Table 1. Input Data to Numerical Simulations of Spontaneous Imbibition for Different Tests

test #	water viscosity, mpas	oil viscosity, mpas	cube dimensions, cm ³	L _c , m	IFT, mN/m	permeability, mD
1	2	1	10 × 10 × 10	0.0289	40	10
2	2	1	5 × 5 × 5	0.0144	40	10
3	2	1	10 × 10 × 10	0.0289	40	1000
4	2	1	10 × 10 × 10	0.0289	40	100
5	2	1	10 × 10 × 10	0.0289	40	10

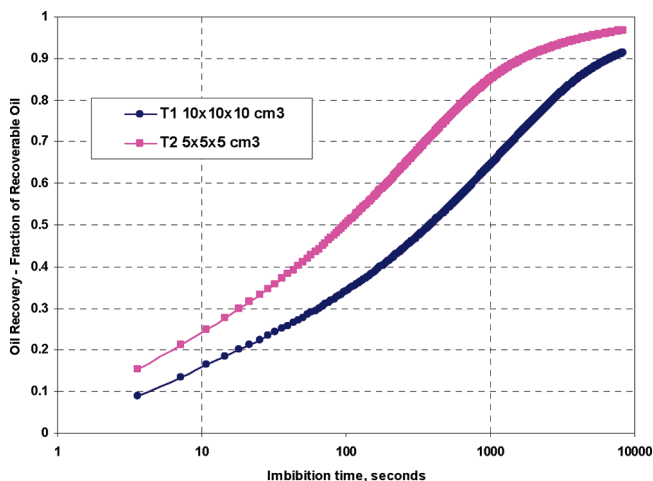


Figure 4. Oil recovery fraction of recoverable due to spontaneous imbibition of water into cubes of different sizes. Water viscosity equal to 2 cP; oil viscosity, 1 cP; permeability, 10 mD.

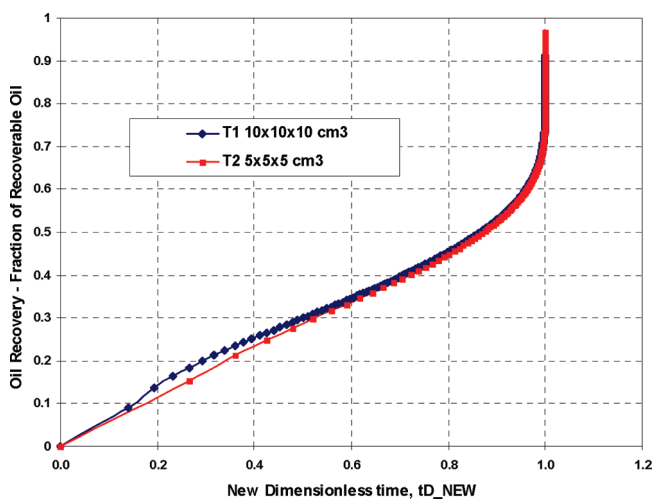


Figure 5. Oil recovery fraction of recoverable oil plotted vs new dimensionless time t_{D_NEW} .

point of the oil relative permeability curve, k_{rwe} is the end point of the oil relative permeability curve, n_w is the Corey exponent for the water relative permeability curve, and n_o is the Corey exponent for the oil relative permeability curve.

The relative permeability curves used in the simulation study are shown in Figure 2. Capillary pressure, P_C , versus S_w is modeled by the following expression described by Kashchiev and Firoozabadi:¹⁶

$$P_C = -B \ln \left[\frac{S_w - S_{wi}}{1 - S_{or} - S_{wi}} \right] \quad (23)$$

(16) Kashchiev, D.; Firoozabadi, A. *SPE paper*, 2002, No. 75166.

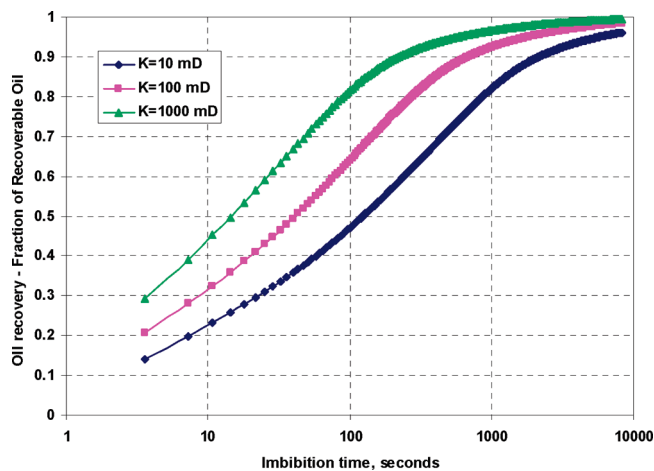


Figure 6. Oil recovery fraction of recoverable due to spontaneous imbibition of water into cubes with different permeabilities vs time. Cube dimensions for all three cases were 10 × 10 × 10 cm³.

where B is the constant characterizing the capillary forces ($= 0.4$ bar).

The value for the capillary pressure at the asymptotic point $S_{wi} = 0.24$ was put equal to 3.00 bar. The capillary pressure curve is depicted in Figure 3.

4. Results and Discussions

In order to validate the new scaling group, spontaneous imbibition data were generated using numerical simulation. The tests performed are listed in Table 1.

4.1. Scaling Rock Sample Size Differences Using L_c . The new scaling expression was first used to scale differences in cube dimensions described in tests 1 and 2. Test 1 has a cube length equal to 10 cm, whereas test 2 has a length of 5 cm. The simulated imbibition response is depicted in Figure 4, and the recovery curves are plotted versus t_{D_NEW} in Figure 5. The results show that the new scaling group is able to correctly account for differences in size by using the characteristic length term L_c . The simulated imbibition curves in Figure 4 both collapsed into one single curve when plotting the oil recovery fraction of recoverable oil versus the new dimensionless time t_{D_NEW} .

4.2. Scaling Rock Permeability Variations. The new scaling equation was further used to scale imbibition data for samples having differences in absolute permeability. Tests 3, 4, and 5 have permeabilities equal to 1000, 100, and 10 mD, respectively. Test data are specified in Table 1. The simulated imbibition responses are shown in Figure 6. The input capillary pressure in the simulated results for permeability equal to 1000 and 10 mD should also be scaled by the relationship:

$$P_{C2} = P_{C1} \sqrt{\frac{k_1}{k_2}} \quad (24)$$

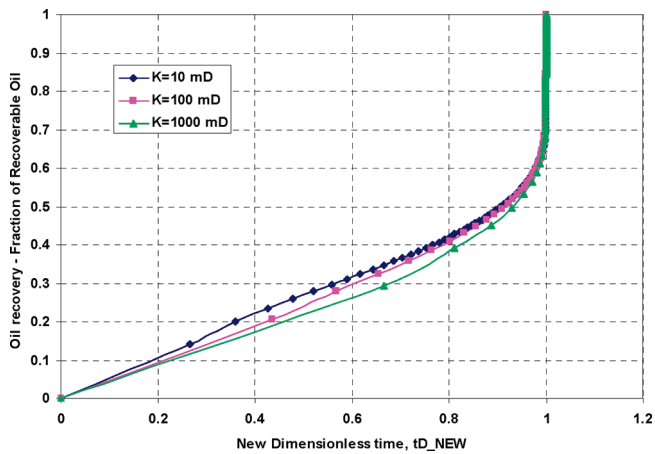


Figure 7. Oil recovery fraction of recoverable due to spontaneous imbibition of water into cubes with different permeabilities vs new dimensionless time t_{D_NEW} . Cube dimensions for all three cases were $10 \times 10 \times 10 \text{ cm}^3$.

to be able to correctly scale the results to one single curve when plotting oil recovery as a fraction of recoverable oil versus new dimensionless time, where P_{C1} is the capillary pressure for the rock sample with permeability k_1 and P_{C2} is the capillary pressure for the rock sample with permeability k_2 . Perfect scaling of the imbibition response was, however, then obtained and is depicted in Figure 7.

5. Conclusion

The following conclusions can be drawn from this work:

The Mattax and KYTE scaling group can be derived from the solution of the Washburn equation where gravity is neglected.

On the basis of the solution of the Washburn equation accounting for gravity, a new scaling group is introduced for spontaneous imbibition of water, which should be able to also account for gravity effects.

The new scaling equation reads $t_{D_NEW} = 1 + W(-e^{-1-t\sqrt{\frac{k}{2\phi}\frac{\sigma}{\mu_g L_c^2}}})$.

The new scaling group was tested using imbibition data generated using numerical simulations, and the scaling group was able to account properly for variations in characteristic length (size) and absolute permeability.

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Nomenclature

$a = [(2\sigma \cos \theta)/(\phi\mu_w)]k/R \text{ (L}^2/\text{T)}$
 $A_i = \text{area of the } i\text{th imbibition surface (L}^2\text{)}$
 $B = \text{constant characterizing the capillary forces (= 0.4 bar)}$

$b = \rho_w g k / \phi \mu_w \text{ (L/T)}$

$c = \text{the ratio between capillary and gravity forces}$

$e = \text{Euler's number (2.718282...)}$

$f(\theta) = \text{wettability function}$

$g = \text{acceleration due to gravity (L/T}^2\text{)}$

$h = \text{height (L)}$

$k = \text{absolute permeability (L}^2\text{)}$

$k_1 = \text{absolute permeability of medium 1 (L}^2\text{)}$

$k_2 = \text{absolute permeability of medium 2 (L}^2\text{)}$

$k_{roce} = \text{end point of the oil relative permeability curve}$

$k_{rwe} = \text{end point of the water relative permeability curve}$

$k_{re}^* = \text{relative permeability pseudofunction}$

$L = \text{length of the capillary tube (L)}$

$L_C = \text{characteristic length (L)}$

$L_H = \text{core sample height (L)}$

$l_i = \text{length from the } i\text{th imbibition surface to the no-flow boundary (L)}$

$n_w = \text{Corey exponent for the water relative permeability curve}$

$n_o = \text{Corey exponent for the oil relative permeability curve}$

$P_c = \text{capillary pressure (W/LT}^2\text{)}$

$P_c^* = \text{capillary pressure at } S_{wr} \text{ (W/LT}^2\text{)}$

$P_{C1} = \text{capillary pressure for rock sample with permeability } k_1 \text{ (W/LT}^2\text{)}$

$P_{C2} = \text{capillary pressure for rock sample with permeability } k_2 \text{ (W/LT}^2\text{)}$

$R = \text{tube radius (L)}$

$R(t) = \text{oil recovery at time } t \text{ (L}^3\text{)}$

$S = \text{fluid saturation of imbibing fluid}$

$S_{wi} = \text{initial water saturation}$

$S_w = \text{water saturation}$

$S_{wf} = \text{water saturation behind the imbibition front}$

$S_{or} = \text{residual oil saturation}$

$t = \text{imbibition time (T)}$

$t_D = \text{dimensionless time}$

$t_{D1} = \text{dimensionless time 1}$

$t_{DMK} = \text{dimensionless time from Mattax and KYTE}$

$t_{DXM} = \text{dimensionless time from Xie and Morrow}$

$t_{DLH} = \text{dimensionless time from Li and Horne}$

$t_{D_NEW} = \text{dimensionless time introduced in this paper}$

$V_b = \text{bulk volume of rock sample (L}^3\text{)}$

$W(x) = \text{Lambert's W function}$

$x = \text{function argument (L)}$

$\sigma = \text{oil-water interfacial tension (M/T}^2\text{)}$

$\phi = \text{fractional porosity}$

$\mu = \text{fluid viscosity (M/LT)}$

$\mu_e = \text{effective viscosity (M/LT)}$

$\mu_g = \text{geometrical mean of the fluid viscosities (M/LT)}$

$\mu_w = \text{water viscosity (M/LT)}$

$\mu_o = \text{oil viscosity (M/LT)}$

$\theta = \text{contact angle}$

$\rho_w = \text{water density (M/L}^3\text{)}$

$\Delta\rho = \text{difference in fluid densities (M/L}^3\text{)}$