

Calculation of Viscosity Scaling Groups for Spontaneous Imbibition of Water Using Average Diffusivity Coefficients

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Spontaneous imbibition is an important driving mechanism for oil recovery in fractured water-wet reservoirs. Reliable scaling relationships have to be used to scale laboratory test results to the field. This work addresses the scaling law described by Ma et al. (*J. Pet. Sci. Eng.* 1997, 18, 165.), with emphasis on the fluid viscosity term. Oil recovery curves versus time are generated using a mathematical expression, which has been shown to reproduce laboratory results very well for spontaneous imbibition of water into chalk cores (Standnes, D. C. *J. Pet. Sci. Eng.* 2006, 50, 151.). The expression includes an average constant diffusivity coefficient, which is calculated as an average value over the water saturation range. Calculated oil recovery rates versus time are then scaled using the Ma et al. scaling law describing mass transfer between matrix blocks and fractures. This law includes a geometrical mean term of the fluid viscosities found empirically to account for variation in oil and water viscosities. Excellent scaling is obtained for differences in rock sample size, oil viscosity (keeping water viscosity equal to 1.0 mPa s), and matched fluid viscosities. These results are in line with reported results in the literature. Oil recovery curves versus time are then calculated for different constant values of the oil viscosity (2, 4, 10, 22, and 43 mPa s) varying water viscosity from 1.0 mPa s to 500 times the oil viscosity for each oil viscosity value. Scaled oil recovery curves versus time show that the viscosity scaling group should be changed from the geometrical mean of the fluid viscosities to $\mu_w^{0.82}\mu_o^{0.18}$ [for viscosity ratios (ratio of water to oil viscosity) above unity] to obtain reasonable good scaling in line with experimental results reported by Fischer and Morrow (paper presented at the Eighth International Symposium on Reservoir Wettability and Its Effect on Oil Recovery, Houston TX, May 16–18, 2004). Oil recovery curves versus time are then calculated for different constant values of the water viscosity (2, 4, 10, 22, and 43 mPa s). The oil viscosity is varying from matched to 500 times the water viscosity for each water viscosity value. The results show that the viscosity scaling group should change from the geometrical mean of the fluid viscosities to approximately $\mu_w^{0.58}\mu_o^{0.42}$ (viscosity ratios below unity) to obtain reasonable good scaling. The same procedure is then applied to calculate the viscosity scaling group to be used to obtain good scaling for imbibition of water into gas-saturated rock samples. The results show that the viscosity scaling group in this case should be equal to $\mu_w^{0.95}\mu_{\text{gas}}^{0.05}$ to scale the calculated gas recovery versus time curves properly. Many of the results presented here are in accordance with experimental results, but many also need further comparison to experimental data to assess their correctness.

Introduction

Spontaneous imbibition (SI) of water into the matrix blocks is an important driving mechanism for oil recovery from fractured water-wet reservoirs. Water is sucked into the porous medium because of surface forces and expels the oil into the fracture system, where it can flow to toward the production well. Understanding and modeling of this process for the flow of oil and water in laboratory cores is therefore of outermost importance when predicting field oil recovery rates because of imbibition. It has recently been demonstrated that SI of water into cylindrical chalk cores having the top and bottom faces closed for fluid exchange can be described by the following expression:¹

$$S_{wAV}(t) = 1 - 4 \sum_{n=1}^{\infty} \frac{1}{Z_n^2} e^{-\frac{D_{AV}}{\phi} \left(\frac{Z_n}{a}\right)^2 t} \quad (1)$$

where S_{wAV} is the average water saturation in the rock sample at time t , Z_n is the zeros of the Bessel function J_0 , $n = 1, 2,$

3,..., a is the radius of the core (m), ϕ is the fractional porosity, D_{AV} is the average diffusivity coefficient (m^2/s), and t is the imbibition time (s).

Equation 1 is an analytical solution to the two-phase flow equation, with the assumption that the diffusivity coefficient is constant and independent of water saturation (some kind of average diffusivity coefficient). The oil production curves calculated from eq 1, however, fitted the experimental curves for SI of water into chalk rock samples extremely well over the whole water saturation range.¹ The first purpose of this paper is to provide further analysis of oil recovery rates calculated from eq 1 varying the fluid viscosities. Furthermore, to account for changes in fluid viscosities in eq 1; the constant diffusivity coefficient in the exponent in eq 1 is first calculated as a simple average value over the whole water saturation range from the following expression:²

$$D(S_w) = -\frac{kk_{ro}}{\mu_o} \frac{1}{1 + \frac{k_{ro}\mu_w}{k_{rw}\mu_o}} \frac{dP_c}{dS_w} \quad (2)$$

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(1) Standnes, D. C. *J. Pet. Sci. Eng.* 2006, 50, 151–160.

where k is the absolute permeability (m^2), k_{ro} is the relative permeability to oil, k_{rw} is the relative permeability to water, μ_o is the oil viscosity (Pa s), μ_w is the water viscosity (Pa s), S_w is the water saturation, and P_C is the capillary pressure (Pa).

The calculated oil recovery rates from eq 1 will be compared to predictions calculated from the well-established scaling law presented by Ma et al.³ reading

$$t_D = t \sqrt{\frac{k}{\phi \mu_w \mu_o}} \frac{1}{L_C^2} \quad (3)$$

where t_D is the dimensionless time, σ is the oil–water interfacial tension (N/m), and L_C is the characteristic length (m).

Equation 3 has been shown to scale oil recovery versus dimensionless time correctly for wide ranges of rock permeabilities,³ boundary conditions,⁴ and rock sample shapes and sizes.⁵ The idea is first to compare results using eqs 1 and 3 for variations in parameters, where eq 3 has been shown to account correctly for the imposed changes. Equation 1 should therefore give essentially the same results for oil recovery versus time if the parameter change is accounted for in a right manner. Equation 1 can in this way to some degree be validated, and oil recovery versus time curves can then be generated varying any parameter in eq 2. Changes in fluid viscosities are addressed in this work. It should be noticed that the literature covering this issue is extensive, and some recent publications are Li and Horne,⁶ Behbahani et al.,⁷ and Hatiboglu et al.⁸

The geometrical mean viscosity term in eq 3 is empirically based and several different expressions have been suggested to scale oil and gas recovery versus dimensionless time.⁹ The second purpose of this paper is to provide scaling relationships for SI of water into both oil- and gas-saturated cores, accounting for wide variations in both imbibing and displaced fluid viscosities. The procedure is as follows: The diffusivity coefficient in eq 2 is plotted as a function of water saturation for fixed input parameters. An simple average diffusivity coefficient is calculated varying the fluid viscosities in eq 3 while systematically keeping all other parameters fixed. SI curves are then calculated by eq 1 using the precalculated average diffusivity coefficient. The calculated oil recovery versus time curves are then analyzed using the scaling equation (eq 3). Deviations between oil recovery rates calculated from eq 1 and the results predicted from eq 3 can then be used to modify the empirical viscosity scaling group in eq 3 under the assumption that the oil recovery rates calculated from eq 1 are more or less correct.

Modeling Oil Recovery versus Time

Description of Flow Functions. Water and oil relative permeability are assumed to be properly represented by modified Corey functions¹⁰

$$k_{\text{rw}} = k_{\text{rwe}} \left[\frac{S_w - S_{\text{wi}}}{1 - S_{\text{or}} - S_{\text{wi}}} \right]^{n_w} \quad (4)$$

$$k_{\text{ro}} = k_{\text{roe}} \left[\frac{1 - S_{\text{or}} - S_w}{1 - S_{\text{or}} - S_{\text{wi}}} \right]^{n_o} \quad (5)$$

where S_{wi} is the initial water saturation, S_w is the water saturation, S_{or} is the residual oil saturation, k_{rwe} is the end point of the water relative permeability curve, k_{roe} is the end point of the oil relative permeability curve, n_w is the Corey exponent for the water relative permeability curve, and n_o is the Corey exponent for the oil relative permeability curve.

Capillary pressure, P_C , versus S_w is modeled by the following expression:²

$$P_C = -B \ln \left[\frac{S_w - S_{\text{wi}}}{1 - S_{\text{or}} - S_{\text{wi}}} \right] \quad (6)$$

where B is the constant characterizing the capillary forces (Pa).

This expression accounting only for variations in capillarity and changes in water saturation applies here because permeability, porosity, and oil–water interfacial tension are assumed constant for all cases investigated. Substitution of eqs 4–6 into eq 2 yields $D(S_w)$ as a function of water saturation. This function is plotted versus water saturation for the base case in Figure 1. The input parameters for the base case are given in Table 1. $D(S_w)$ is a bell-shaped function of water saturation as demonstrated by Kashchiev and Firoozabadi.²

The average diffusivity coefficient for the base case was $4.00 \times 10^{-8} \text{ m}^2/\text{s}$.

Calculating the Average Diffusivity Coefficient. To predict oil recovery rates using eq 1, it is first necessary to provide an average diffusivity coefficient. The following expression was applied when calculating the average diffusivity coefficient from the different $D(S_w)$ functions appearing when altering the fluid viscosities in eq 2:¹¹

$$D_{\text{AV}} = \frac{\int_{S_{\text{wi}}}^{S_{\text{wr}}} D(S_w) dS_w}{S_{\text{wr}} - S_{\text{wi}}} \quad (7)$$

where S_{wr} is the final water saturation when SI has ceased.

The integral was evaluated numerically using the trapezoidal approximation¹² for water saturation steps of 0.01 units. The average diffusivity coefficient was in general in the range from 4.00×10^{-8} to $10^{-11} \text{ m}^2/\text{s}$ for the base case and the cases with highest fluid viscosities, respectively. It should be mentioned that different more sophisticated weight average methods different from the simple average chosen here have been evaluated previously.¹³ Work has also been performed more directly on the nonlinear diffusion equation without any linearization.^{14,15}

Rock Sample and Fluid Properties. The dimensions of the rock samples (chalk) used in the calculations of the base case were 6.00 cm in diameter chalk samples and 3.20 cm in height. Permeability and porosity were equal to 2 mD and 0.424, respectively, and the porous medium was assumed to be strongly water-wet. The viscosity and density of the imbibing water phase were 1.0 mPa s and 998.2 kg/m^3 , whereas corresponding values for the oil phase were 1.0 mPa s and 731 kg/m^3 , respectively. The oil–water interfacial tension (IFT) was equal to 46 mN/m. All of these values are in agreement with data used when scaling experimental SI data into chalk samples.⁵ A viscosity of 0.0182 mPa s (μ_A) was applied when calculating water imbibition rates into air-saturated samples. Calculations of water imbibition rates into gas-saturated samples

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(14) Tavassoli, Z.; Zimmerman, R. W.; Blunt, M. J. *Transp. Porous Media* **2005**, 58, 173–189.

(15) Zimmerman, R. W.; Bodvarsson, G. S.; Kwicks, E. M. *Water Resour. Res.* **1990**, 26, 2797–2806.

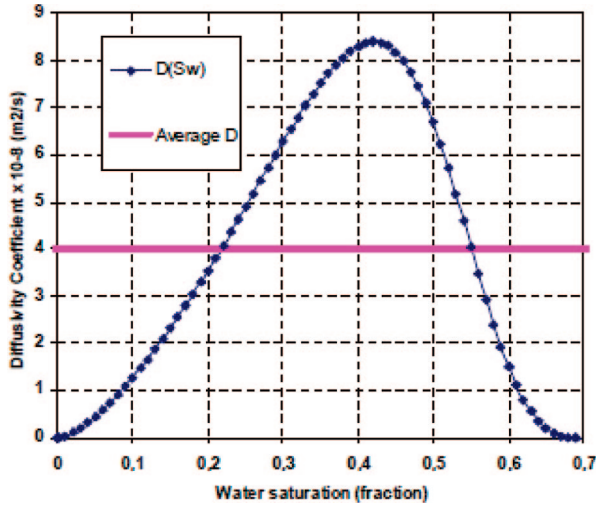


Figure 1. $D(S_w)$ plotted for the base case values given in Table 1.

Table 1. Parameters Used for the Base Case To Calculate the Average Diffusivity Coefficient

parameter	value	parameter	value	parameter	value
diameter	6.00 cm	μ_o	1.0 mPa s	k_{rwe}	0.1
height	3.20 cm	n_o	2.5	k_{roo}	1.0
porosity	0.424	n_w	2.5	B	804.8 kPa
permeability	0.001974 μm^2	S_{wi}	0		
μ_w	1.0 mPa s	S_{wr}	0.6899		

having viscosities 10 and 3 times lower and higher viscosity than air were also performed (referred to as μ_{vH} , μ_H , μ_L , and μ_{vL}). The gas–water IFT is, for simplicity, assumed to be constant and equal to the oil–water IFT.

Characteristic Length. The characteristic length term described by Ma et al.³ will be used here. It is given by the following expression:

$$L_C^2 = \frac{V_b}{\sum_{i=1}^n \frac{A_i}{l_i}} \quad (8)$$

where V_b is the bulk volume of the rock sample (m^3), A_i is the area of the i th imbibition surface (m^2), and l_i is the length from the i th imbibition surface to the no flow boundary (m).

For the rock sample with top and bottom faces closed for fluid exchange, the following expression applies:

$$L_C^2 = \frac{d^2}{8} \quad (9)$$

The oil recovery rate is independent of the rock sample height and scales with the diameter (d) of the rock sample squared.

Results and Discussion

Scaling Expressions When Varying the Characteristic Length and the Viscosity Term. When the characteristic length term is the only varying parameter, eq 3 takes the form

$$t_D = t \sqrt{\frac{k}{\phi} \frac{\sigma}{\mu_w \mu_o} \frac{1}{L_C^2}} = t \sqrt{\frac{2.9.87 \times 10^{-16}}{0.424} \frac{0.046}{\sqrt{10^{-3} 10^{-3} d^2}} \frac{8}{2.511 \times 10^{-5} d^2}} \quad (10)$$

The diameter should be given in meters, and t should be given in seconds. When the viscosity scaling group referred to as VSG is the only varying parameter, eq 3 transforms into

$$t_{DM} = t \sqrt{\frac{2.9.87 \times 10^{-16}}{0.424} \frac{0.046}{\text{VSG}} \frac{8}{0.0036}}$$

$$t_{DM} = \frac{0.006975}{\text{VSG}} t \quad (11)$$

where the time is expressed in seconds and the fluid viscosities in the VSG is expressed in mPa s. t_{DM} is the modified dimensionless time because of changes in the viscosity term. The VSG may in general be expressed as

$$\text{VSG} = \mu_w^{\text{VE}} \mu_o^{1-\text{VE}} \quad (12)$$

where the viscosity exponent, VE, can vary in the range $0 \leq \text{VE} \leq 1$. VE different from 0.5 will give modified viscosity scaling groups (see eq 3). These modified VSGs will imply new dimensionless time expressions, which will be referred to as t_{DM1} , t_{DM2} , and t_{DMG} . The two first modified dimensionless time expressions describe water imbibition into oil-saturated rock samples varying the water and the oil viscosity, respectively, whereas the latter refers to imbibition of water into gas-saturated rock.

Scaling Oil Recovery versus Dimensionless Time, t_D , Varying Rock Sample Size and Oil Viscosity. The approach of using the average diffusivity coefficient to predict oil recovery rates was first evaluated by varying the size of the rock sample and the viscosity of the displaced phase (keeping water viscosity equal to 1.0 mPa s). These examples were chosen because they are considered well-documented experimentally.⁶ The diameters of the rock samples were varied between 0.01 and 3.00 m. The calculated curves showing oil recovery versus time are depicted in Figure 2. The production curves cover 5 orders of magnitude in time. The ability of eq 1 to predict oil recovery versus time correctly was checked using the scaling law given in eq 10. The results show that perfect scaling was obtained when varying the diameter of the rock sample between 0.01 and 3.00 m (Figure 3).

Zhang et al.⁴ have shown that oil recovery versus time scales with the square root of the fluid viscosities. Their results were obtained using an aqueous phase of viscosity 1.0 mPa s and varying the oil viscosity from 1.0 to 156 mPa s. The maximum ratio between the displaced and imbibing fluid viscosity in this work is 500. Using the geometrical mean between the fluid viscosities in eq 3 gives perfect scaling of oil recovery versus dimensionless time t_D . The calculated oil recovery versus time and the scaled results using t_D are given in Figures 4 and 5, respectively.

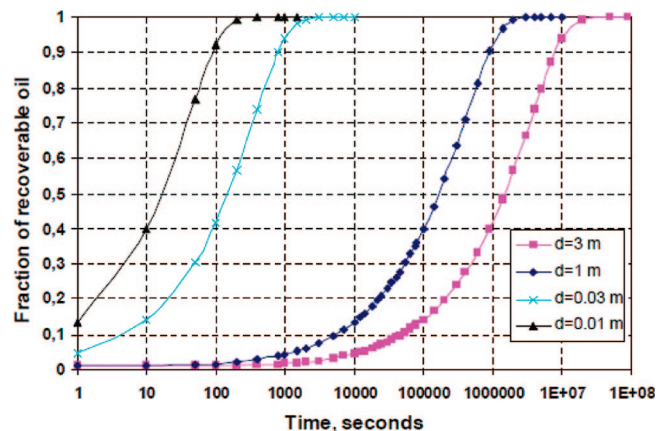


Figure 2. Calculated oil recovery versus time for different rock sample sizes (input parameters in eq 2 equal to the base case).

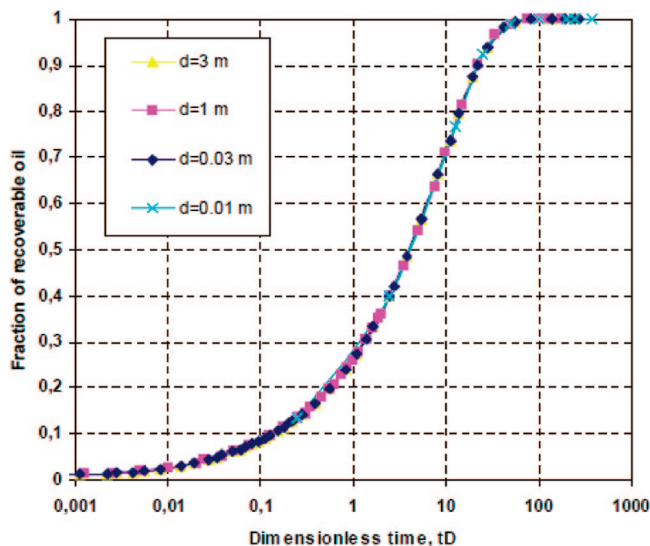


Figure 3. Oil recovery rates from Figure 2 scaled with t_D for different rock sample sizes.

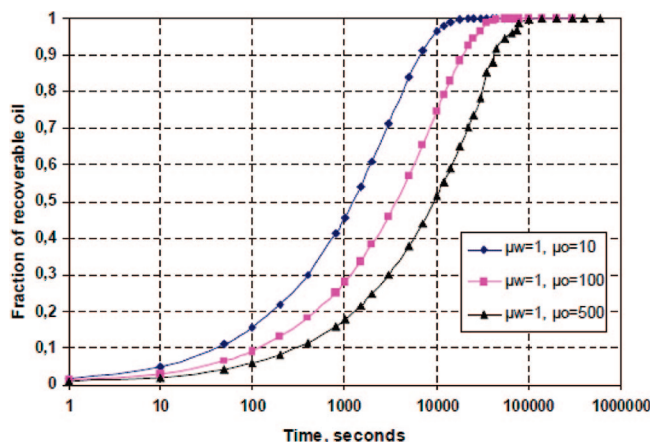


Figure 4. Calculated oil recovery versus time varying the oil viscosity between 10 and 500 mPa s while keeping the imbibing water viscosity constant at 1.0 mPa s.

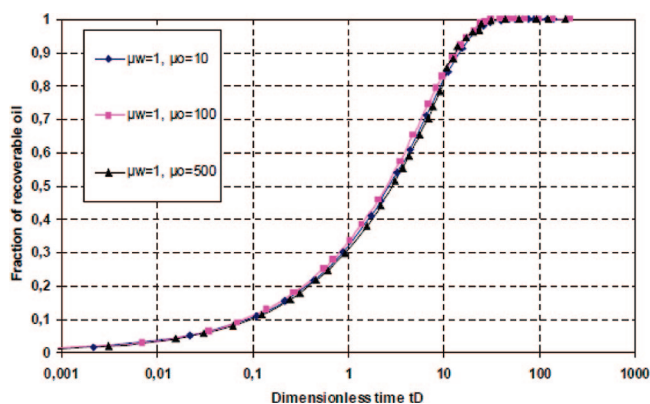


Figure 5. Oil recovery versus t_D for water viscosities of 1.0 mPa s while varying the oil viscosities.

The imbibing fluid viscosity was kept equal to 1.00 mPa s as in the experimental results reported by Zhang et al.⁴ The results presented in this section indicate that changing the input parameters in eq 1 (the applied average diffusivity coefficient change because of changes in input viscosities in eq 2) and the scaling law (eq 3) gives the same response regarding oil recovery rate versus time. Hence, calculated average diffusivity coefficients are able to predict recovery of the non-wetting phase versus dimensionless time reasonably well.

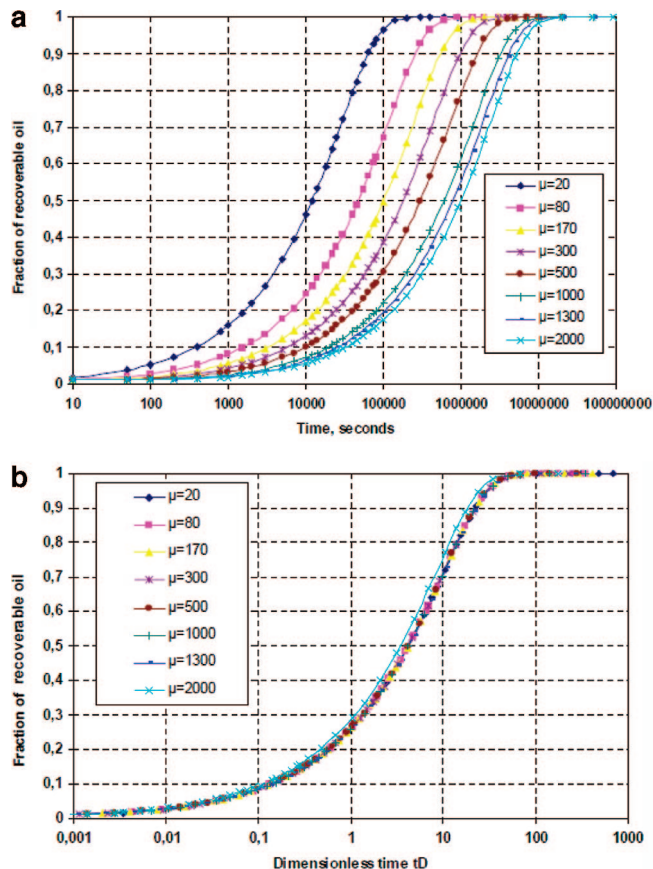


Figure 6. (a) Calculated oil recovery versus time while varying oil and water viscosity (matched). (b) Scaled oil recovery versus t_D for matched fluid viscosities.

Scaling Oil Recovery versus Dimensionless Time (t_D) for Matched Viscosities. Fischer and Morrow¹⁶ have reported large data sets containing SI of water into oil-saturated rock samples while varying both the water and the oil viscosity over 2.5 orders of magnitude. They scaled the imbibition data using only the water viscosity for the case of matched oil and water viscosities and obtained reasonably good results. Figure 6a shows calculated oil recovery versus time for matched fluid viscosities in the range from 20 to 2 000 mPa s (all other parameters in Table 1 are held constant).

The corresponding curves plotting oil recovery versus dimensionless time, t_D , are depicted in Figure 6b. The results show that perfect scaling was obtained using the square root of time expression for the fluid viscosities. The results obtained by the numerical scheme presented in this paper are therefore thus far in excellent agreement with experimental results.

Scaling Oil Recovery versus t_D and t_{DMI} for $\mu_o = 2, 4, 10, 22$, and 43 mPa s while Varying the Water Viscosity. Oil recovery versus time for varying water viscosities between 1 mPa s to 500 times the oil viscosity were then calculated (all other parameters were equal to the base case calculation) while keeping the oil viscosity constant to 2, 4, 10, 22, and 43 mPa s. Figure 7 shows the recovery rates for the case of constant $\mu_o = 22$ mPa s. The imbibing water viscosity varied from 1 mPa s to 500 times the oil viscosity [viscosity ratio (VR) defined as the ratio of water viscosity to oil viscosity = 500] for each specific oil viscosity. The corresponding curves for oil recovery

(16) Fischer, H.; Morrow, N. R. Paper presented at the 8th International Symposium on Reservoir Wettability and Its Effect on Oil Recovery, Houston TX, May 16–18, 2004.

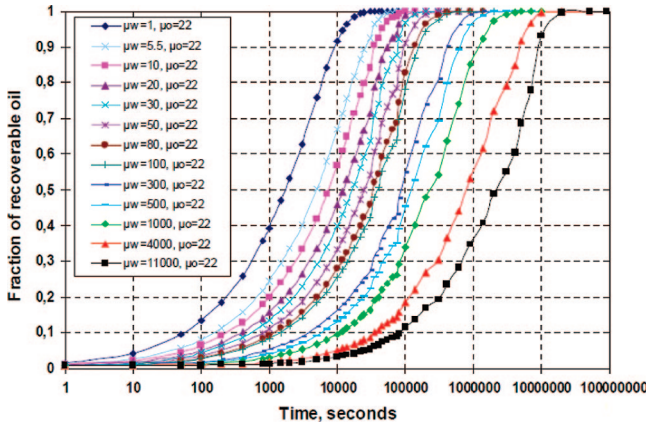


Figure 7. Calculated SI curves for constant oil viscosity of 22 mPa s while varying water viscosity between 1 and 11 000 mPa s.

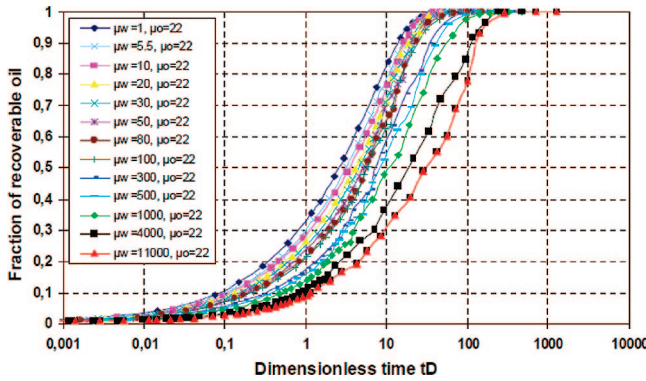


Figure 8. Oil recovery versus t_D curves for constant oil viscosity of 22 mPa s while varying water viscosity between 1 and 11 000 mPa s.

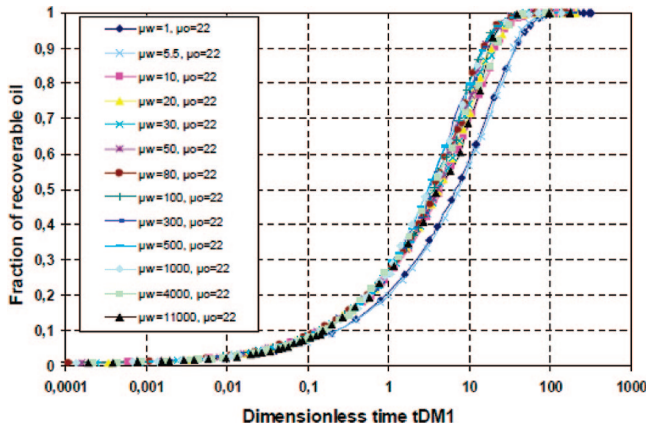


Figure 9. Oil recovery versus t_{DM1} curves for constant oil viscosity of 22 mPa s while varying water viscosity between 1 and 11 000 mPa s. The optimal scaling shown was obtained with a VE = 0.82.

versus dimensionless time (t_D) using eq 3 to account for the viscosity term is shown in Figure 8 for the case $\mu_o = 22$ mPa s.

The results clearly show that using the square root of the oil and the water viscosity fails to account for the variation in aqueous phase viscosity. A closer inspection of the data, however, shows that significant deviation from the scaling starts when the VR exceeds 5. It should be noticed that this result is in perfect agreement with the data observed by Fischer and Morrow.¹⁶ They observed a clear tendency for the dimensionless time to increase with increasing VR, and significant deviations from the general trend appeared for VRs > 4. Figure 9 shows oil recovery versus modified dimensionless time (t_{DM1}) again for the case $\mu_o = 22$ mPa s. The exponent for the water and the

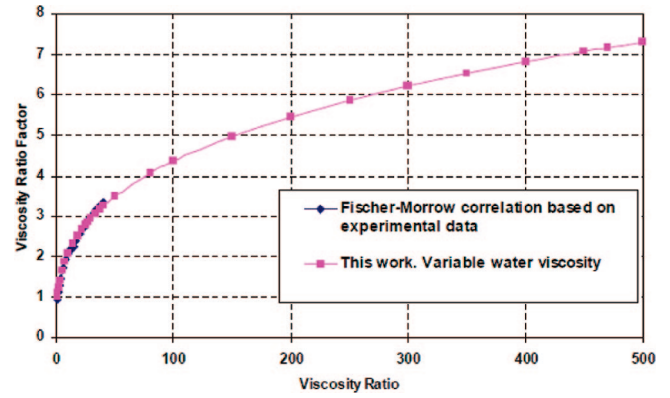


Figure 10. VRF as a function of VR for imbibition of water into oil-saturated rock samples with constant oil viscosities.

oil viscosity was altered according to eq 12. Optimal scaling was estimated by visual inspection while varying the exponents in steps of 0.01.

This procedure gave a VE equal to 0.82 for all of the oil viscosity values (2, 4, and 22 mPa s), except for the highest oil viscosity (43 mPa s), which gave an exponent of 0.84. It was not possible to obtain an overall good scaling for the cases where the water viscosity falls significantly below the actual oil viscosity ($\mu_w = 1$ and 5.5 mPa s). These cases were therefore not included when determining the VE giving the optimal scaling. The reason why these cases failed to scale will become evident in the next section.

Hence, the results obtained in this section are valid for cases where the water viscosity is equal to or higher than the oil viscosity (the oil viscosity should also be equal to or larger than 2.0 mPa s). Fischer and Morrow¹⁶ also observed an increase in dimensionless time with an increasing VR, as mentioned above. They therefore introduced a viscosity ratio factor (VRF) to obtain reasonably good scaling results. The VRF was expressed as a function of VR

$$f_1(\text{VR}) = -0.007(\text{VR})^2 + 0.203(\text{VR}) + 0.759 \quad (13)$$

$$\text{VR} \leq 14 \text{ and}$$

$$f_2(\text{VR}) = 1.122 \ln(\text{VR}) - 0.794 \quad (14)$$

$$14 < \text{VR} < 40$$

The VRF describes how much the modified dimensionless time, t_{DM1} , should change in addition to the square root dependency of the fluid viscosities given in eq 3 to obtain good scaling. This change can be quantified in the following way for the approach presented in this work:

$$f_w(\text{VR}) = \frac{\text{VSG}}{\sqrt{\mu_w \mu_o}} = \frac{\mu_w^{0.82} \mu_o^{0.18}}{\mu_w^{0.5} \mu_o^{0.5}} = (\text{VR})^{0.32} \quad (15)$$

$$1 < \text{VR} < 500$$

The functions given in eqs 13–15 are plotted in Figure 10. It can be seen that the $f_w(\text{VR})$ expression coincides very well with the correlation given by Fischer and Morrow.¹⁶ In addition, $f_w(\text{VR})$ also predicts the behavior of the VRF for VRs up to 500.

It should be mentioned that Fischer and Morrow's SI data were obtained on cores having all faces open to fluid exchange. Reasonable agreement between the all faces open case and the radial case considered here is, however, expected because

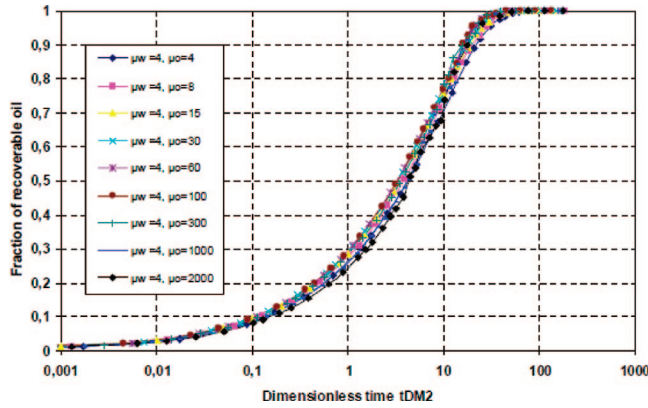


Figure 11. Oil recovery curves versus t_{DM2} for constant water viscosity of 4 mPa s. Oil viscosity varied from 4 to 2000 mPa s. The optimal scaling shown was obtained with a $VE = 0.58$.

the core lengths in their experiments were much larger than the diameter ($2H/D \sim 3.3$).¹⁷

Scaling Oil Recovery versus t_{DM2} for $\mu_w = 2, 4, 10, 22$, and 43 mPa s while Varying the Oil Viscosity. Figure 11 shows oil recovery versus modified dimensionless time (t_{DM2}) for varying oil viscosities while keeping the water viscosity constant at 4 (all other parameters were equal to the base case calculation). The square root expression for the fluid viscosities was again unable to account for all of the VRs (not shown). Severe deviations from the general scaling trend appeared approximately for VR less than 0.2. The VR varied in the range from $1/500$ to 1 for each specific water viscosity. The results were again scaled while varying the VE until a reasonable good scaling was obtained. Optimal scaling was obtained for VE values of 0.56, 0.58, 0.59, 0.60, and 0.60 corresponding to water viscosities of 2, 4, 10, 22, and 43 mPa s, respectively. The VRF as a function of VR for different constant water viscosities can then be expressed as

$$f_{02}(VR) = \left(\frac{\mu_w}{\mu_o}\right)^{0.06} = VR^{0.06} \quad (16)$$

$$f_{04}(VR) = \left(\frac{\mu_w}{\mu_o}\right)^{0.08} = VR^{0.08} \quad (17)$$

$$f_{010}(VR) = \left(\frac{\mu_w}{\mu_o}\right)^{0.09} = VR^{0.09} \quad (18)$$

$$f_{022}(VR) = f_{043}(VR) = \left(\frac{\mu_w}{\mu_o}\right)^{0.1} = VR^{0.10} \quad (19)$$

where $0.002 \leq VR \leq 1$ in all cases. These functions are plotted in Figure 12. What is most important about these expressions is that the VRF seems to be much less pronounced for the constant water viscosity and the varying oil viscosity case than for the opposite case. These predictions need, however, more experimental data to be validated.

The results obtained in this section explain the scaling failure for the cases in the previous section where the water viscosity was significantly lower than the oil viscosity (water viscosity of 1.0 and 5.5 mPa s for oil viscosity of 22 mPa s for example). These two cases should according to eqs 16 and 17 scale approximately with VSGs equal to $\mu_w^{0.56}\mu_o^{0.44}$ and $\mu_w^{0.58}\mu_o^{0.42}$ for the 1.0 and 5.5 mPa s case, respectively. If these exponents are included in Figure 9 rather than $\mu_w^{0.82}\mu_o^{0.18}$, reasonably good

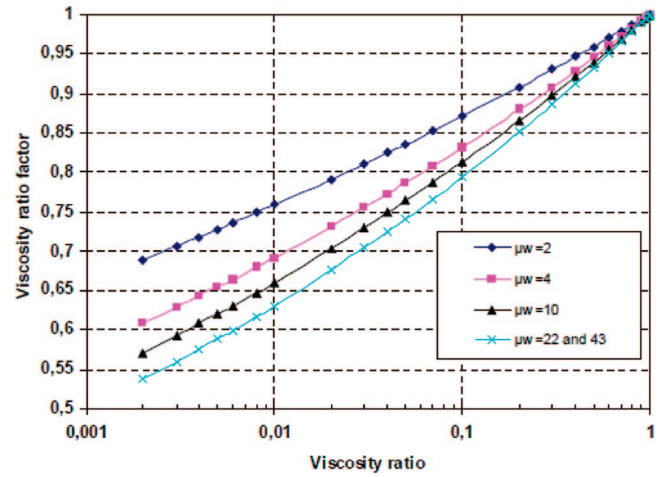


Figure 12. VRF as a function of VR for imbibition of water with constant viscosity into oil-saturated rock samples with varying oil viscosities.

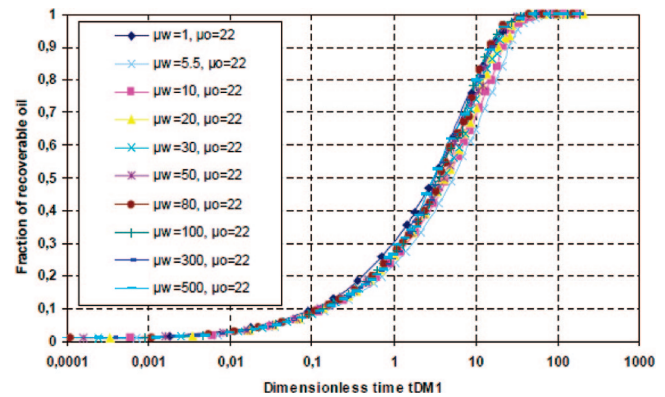


Figure 13. Figure 9 with the $\mu_w = 1, \mu_o = 22$ and the $\mu_w = 5.5, \mu_o = 22$ cases scaled using $\mu_w^{0.56}\mu_o^{0.44}$ and $\mu_w^{0.58}\mu_o^{0.42}$ instead of $\mu_w^{0.82}\mu_o^{0.18}$.

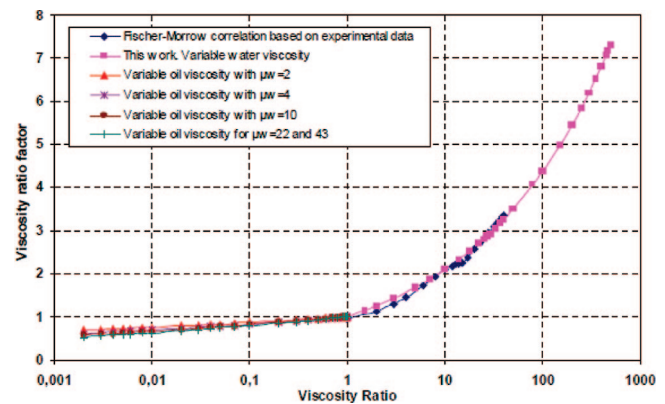


Figure 14. VRF as a function of VR in the range of 0.002–500.

scaling is obtained for all of the data (Figure 13). The results in the last two sections also allow for the drawing of curves for the VRF covering the entire range from 0.002 to 500 (Figure 14). It is important to recall that these curves only are valid for cases where both fluid viscosities exceed 1.0 mPa s.

Scaling Water Imbibed versus t_{DMG} for Rock Samples Saturated with Five Different Gas Qualities while Varying the Water Viscosity. It has been observed experimentally that the square root expression for the viscosity term in eq 3 is unable to scale imbibition of water into rock samples saturated with air.⁹

Figure 15 shows calculated SI curves for water into rock samples saturated with air together with four other cases where

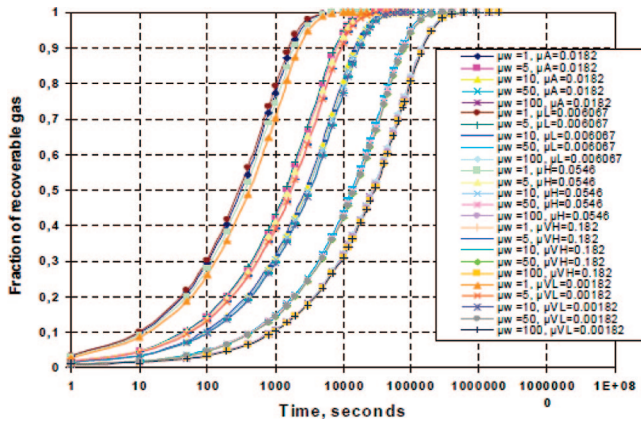


Figure 15. Gas recovery versus time for imbibition of water into rock samples saturated with five different gas qualities. The gas viscosity varied from 0.001 82 (VL) to 0.182 (VH) mPa s, whereas the water viscosity varied in the range from 1.0 to 100 mPa s.

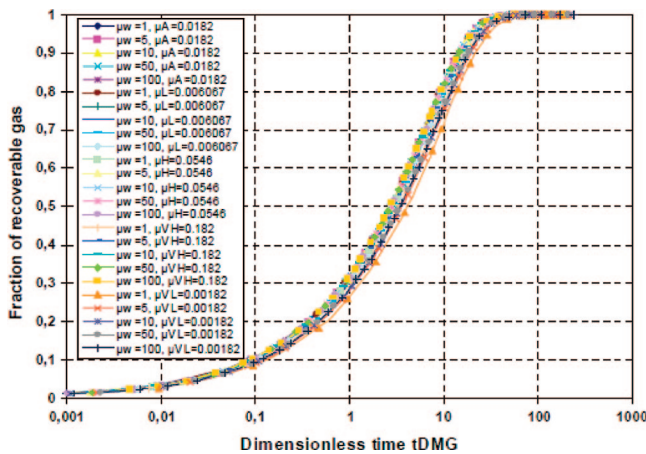


Figure 16. Gas recovery versus t_{DMG} for imbibition of water into rock samples saturated with five different gas qualities. The optimal scaling shown was obtained with a VE = 0.95.

the gas viscosity was equal to 10 and 3 times lower and higher than the air viscosity, respectively (all other parameters were equal to the base case calculation).

The data therefore cover a broad range of gas viscosities ranging from 0.00182 to 0.182 mPa s (water viscosity varied at the same time in the range from 1.0 to 100 mPa s for each gas viscosity).

Figure 16 shows gas recovery versus modified dimensionless time, t_{DMG} , using a VE of 0.95 for all five cases tested. Very good scaling is obtained, and the VRF can therefore in all cases be expressed as

$$f_g(VR) = \left(\frac{\mu_w}{\mu_{gas}} \right)^{0.45} = VR^{0.45} \quad (20)$$

$$\text{for } 5.5 \leq VR \leq 55000$$

This function is plotted in Figure 17 for VRs ranging from 5.5 to approximately 55 000. The results indicate that the scaling of gas/water deviates significantly from the oil/water case. The VRF is apparently a very important factor when scaling gas/water imbibition results. The results obtained in this section are calculated using the oil–water IFT instead of gas–water IFT. The gas recovery curves versus time will therefore be slightly delayed in comparison to corresponding curves if the somewhat higher gas–water IFT had been applied. The scaling results

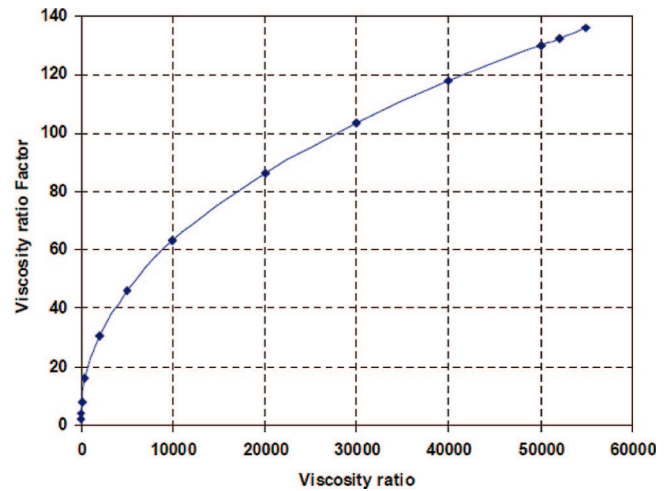


Figure 17. VRF as a function of VR ($=\mu_w/\mu_{gas}$) for imbibition of water with constant viscosity into gas-saturated rock samples with varying viscosity.

presented should, however, be valid because the difference in IFT is compensated for in the scaling expression.

It is important to remember that all results in this section need to be more investigated and tested experimentally before drawing any conclusion about their validity. It is interesting to notice that Morrow and Mason⁹ reported that gas recovery scales weakly with the VSG $\mu_o^{0.8}\mu_{gas}^{0.2}$ when oil was used as the imbibing fluid. The correlation showed in general much more spread than for the corresponding correlations developed for oil/water systems. The VE found experimentally (0.8) is, however, not very different from the best VE estimated in this work (0.95).

Sensitivity Tests while Varying Capillary Pressure, n_w , and k_{rwe} . The validity of the results obtained by the numerical scheme was also tested by performing sensitivity runs while varying successively the capillary pressure ($B = 402.4$ kPa), the curvature of the relative permeability to water ($n_w = 4.5$), and the end point value on the relative permeability curve to water ($k_{rwe} = 0.25$).

The tests were performed using the data set for the matched viscosities. Calculated oil recovery versus time curves were generated using eq 1, with the average diffusivity coefficient calculated using the above-mentioned values for matched fluid viscosities in the range of 20–2000 mPa s (all other parameters were equal to the base case). The oil production curves were then scaled according to eq 3, which is valid for matched fluid viscosities (Figure 18).

All of the scaled curves collapsed into one single curve for each of the parameters, confirming the ability of eq 1 to predict oil recovery rates according to the scaling law, independent of the magnitude of P_C , the curvature, and the value of the end point on the water relative permeability curve.

Conclusions

It is important to notice that all of the modified scaling relationships presented are only valid under the assumption that the SI model (eq 1) accounts correctly for the variation in fluid viscosities when calculating oil recovery versus time. The following conclusions can then be drawn from this work: (1) There is strong evidence for the applicability of using average diffusivity coefficients in eq 1 to calculate oil recovery versus time by SI. Calculated oil recovery versus time was successfully scaled with the well-established scaling law presented by Ma

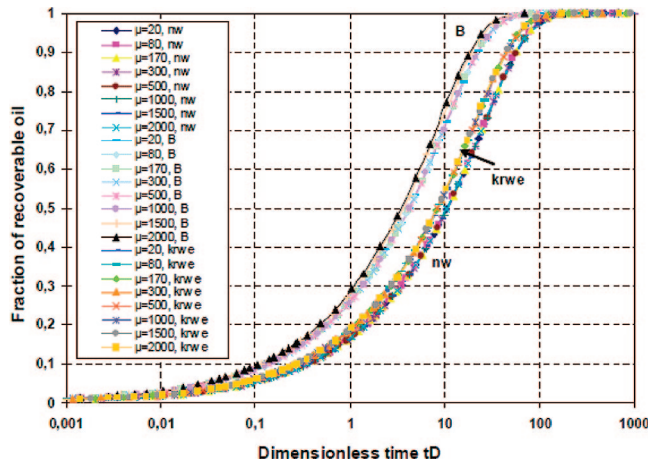


Figure 18. Oil recovery versus t_D for matched fluid viscosities and k_{rwe} changed from 0.1 to 0.25, B changed from 804.8 to 402.4 kPa, and n_w changed from 2.5 to 4.5.

et al.³ while varying rock sample size, oil viscosity, and matched oil and water viscosities. (2) The geometrical mean of the fluid viscosities was not able to scale oil recovery versus time for constant oil viscosities equal to 2, 4, 10, 22, and 43 mPa s while varying water viscosity from 1 mPa s to 500 times the oil viscosity. The VSG should be equal to $\mu_w^{0.82}\mu_o^{0.18}$ to obtain good scaling. (3) The geometrical mean of the fluid viscosities was not able to scale oil recovery versus time for constant water viscosities equal to 2, 4, 10, 22, and 43 mPa s while varying oil viscosity from matched to 500 times the water viscosity. The viscosity exponent should be equal to 0.56, 0.58, 0.59, 0.60, and 0.60 corresponding to water viscosities of 2, 4, 10, 22 and 43 mPa s, respectively, to obtain good scaling. (4) VRFs are established for VR in the range from 0.002 to 500. The calculated VRF for VR ranging from 4 to 40 is in good agreement with a VRF correlation established by Fischer and Morrow.¹⁶ (5) The geometrical mean of the fluid viscosities was not able to scale gas recovery versus time in line with experimental results. The VSG should be equal to $\mu_w^{0.95}\mu_{gas}^{0.05}$ to obtain good scaling for imbibition of water into rock samples saturated with gas. (6) VRF as a function of VR is established for imbibition into gas-saturated rock samples. The VRF curve indicates much stronger dependency upon the VR than for corresponding VRF curves versus VR for oil/water systems. (7) Many of the results generated by the numerical scheme using average diffusivity coefficients are in line with well-established experimental data. Other results need, however, more empirical testing to be validated.

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Nomenclature

A_i = area of the i th imbibition surface (m^2)
 a = radius of the core (m)

B = constant characterizing the capillary forces (Pa)
 D_{AV} = average diffusivity coefficient (m^2/s)
 d = diameter (m)
 f_1 = function describing the VRF for $VR \leq 14$
 f_2 = function describing the VRF for $14 \leq VR \leq 40$
 f_g = function describing the VRF for imbibition into gas-saturated rock samples
 $f_{o\#}$ = function describing the VRF for variable oil viscosities keeping water viscosity equal to #
 f_w = function describing the VRF for variable water viscosities
 H = height of the cylindrical rock sample (m)
 IFT = interfacial tension (N/m)
 k = absolute permeability (m^2)
 k_{ro} = relative permeability to oil
 k_{rw} = relative permeability to water
 $k_{ro\#}$ = end point of the oil relative permeability curve
 k_{rwe} = end point of the water relative permeability curve
 L_C = characteristic length (m)
 l_i = length from the i th imbibition surface to the no flow boundary (m)
 n_w = Corey exponent for the water relative permeability curve
 n_o = Corey exponent for the oil relative permeability curve
 P_c = capillary pressure (Pa)
 SI = spontaneous imbibition
 S_{or} = residual oil saturation (fraction)
 S_w = water saturation (fraction)
 S_{wi} = initial water saturation (fraction)
 S_{wAV} = average water saturation in the rock sample at time t (fraction)
 t = imbibition time (s)
 t_D = dimensionless time
 t_{DM1} = modified dimensionless time for imbibition of aqueous phases with different viscosities while keeping the oil viscosity constant
 t_{DM2} = modified dimensionless time for imbibition of oleic phases with different viscosities while keeping the water viscosity constant
 t_{DMG} = modified dimensionless time for imbibition of aqueous phases with different viscosities while keeping the gas viscosity constant
 VE = viscosity exponent
 VR = viscosity ratio
 VRF = viscosity ratio factor
 VSG = viscosity scaling group
 V_b = bulk volume of the rock sample (m^3)
 Z_n = zeros of the Bessel function J_0 , $n = 1, 2, 3, \dots$
 σ = oil–water interfacial tension (N/m)
 ϕ = fractional porosity
 μ_A = air viscosity (Pa s)
 μ_{gas} = gas viscosity (Pa s)
 μ_H = viscosity of air with $3\mu_A$ (Pa s)
 μ_L = viscosity of air with $\mu_A/3$ (Pa s)
 μ_o = oil viscosity (Pa s)
 μ_{VH} = viscosity of air with $10\mu_A$ (Pa s)
 μ_{VL} = viscosity of air with $\mu_A/10$ (Pa s)
 μ_w = water viscosity (Pa s)

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