MSc Lecture: Enumerative Geometry

Lecture by Helge Ruddat; Exercises by Tim Gräfnitz; Tu & We 16:15 – 17:45 on Zoom

Description

Hermann Schubert developed an effective method for solving counting problems in geometry in his book "Kalkül der abzählenden Geometrie" that appeared in 1879. A basic example is the question “how many lines in R^3 meet four given lines in general position?”. Do you know the answer? The calculus is now known as “Schubert calculus”, see (1),(2),(3).
Schubert calculus is immensely powerful, e.g. you can use it to find that the number of twisted cubic curves in 3-space that are tangential to 12 quadric surfaces is 5.819.539.783.680.
Even though it clearly worked, for a long time it has been unknown what the theoretical basis for the calculus truly is, i.e. nobody knew **why** it worked. It became the famous Hilbert problem 15 to find a theoretical basis for Schubert calculus.

This was looong time ago. Today we know that intersection theory, the spaces of stable maps and Gromov-Witten theory form the right theoretical framework that underlies Schubert calculus - though there are also competing alternative approaches like Donaldson-Thomas theory. The purpose of this lecture is to introduce the basics of Gromov-Witten theory with a particular view to mirror symmetry and thus the relationship with tropical curve counting.

Prerequisites

When taking this class, you should have some basic understanding of algebraic geometry or complex geometry or complex analysis, that is you should either know what an algebraic variety is or know what a complex manifold or Riemann surface is. These two notions converge nicely into the notion of compact Riemann surface alias projective algebraic curve which is the one-dimensional version of each and these are equivalent. We will talk a lot about Riemann surfaces alias curves. This class is new in Hamburg and will be tailored in real time to suit whoever is attending this lecture.

Coordinates

The lecture will be held on Zoom, every Tuesday 8:30-10:00 and Wednesday 16:15-17:45 starting November 3rd. More information can be found on Stine.

There will also be a weekly exercise class led by Tim Gräfnitz whose time isn’t set up yet. Attendees can collect points on exercises they solve. Achieving the minimum number of points will grant permission to take the oral exam at the end.

Literature

(1) https://en.wikipedia.org/wiki/Schubert_calculus

(2) “Kalkül der abzählenden Geometrie" von Hermann Schubert, Springer Berlin Heidelberg, 384S.

(3) "Problem 15. Rigorous foundation of Schubert's enumerative calculus", by Steven Kleiman, in F.Browder Hilberts problems, Proceedings of the Symposium in Pure Mathematics of the American Mathematical Society, Held at Northern Illinois University 1974, S. 445-482.

(4) “Moduli of Curves” by Joe Harris and Ian Morrison

(5) “Notes on stable maps and quantum cohomology”, Rahul Pandharipande and William Fulton, 1995

(6) “Mirror Symmetry and Algebraic Geometry” by David Cox and Sheldon Katz, 1998