FLERE OPPGAVETEKSTER
FRA KAP. 6

6.10 The simple harmonic oscillator in one dimension can also be solved by the method of ladder operators. This solution is simpler and more elegant than the one in Chapter 4.

(a) For a particle of mass m in a one-dimensional simple harmonic oscillator potential $V(x) = \frac{1}{2}Kx^2$, the Hamiltonian operator is

$$H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{1}{2}Kx^2$$

Define the ladder operators a_- and a_+ to be given by

$$a_+ = \sqrt{\frac{K}{2}}x - \frac{\hbar}{\sqrt{2m}}\frac{\partial}{\partial x}$$

and

$$a_- = \sqrt{\frac{K}{2}}x + \frac{\hbar}{\sqrt{2m}}\frac{\partial}{\partial x}$$

Show that

$$[H, a_+] = \hbar\omega a_+$$

and

$$[H, a_-] = -\hbar\omega a_-$$

where $\omega = \sqrt{K/m}$.

(b) Suppose that $\psi(x)$ is a solution of the time-independent Schrödinger equation for the harmonic oscillator with energy E. Show that $a_+\psi(x)$ is also a solution of the time-independent Schrödinger equation for the harmonic oscillator with energy $E + \hbar\omega$. Show that $a_-\psi(x)$ is a solution with energy $E - \hbar\omega$.

(c) Show that $a_+a_- = H - \hbar\omega/2$.

(d) There is no upper bound on the possible values for E, but there is a lower bound; the energy cannot be negative. This means that if $\psi_0(x)$ is the ground state wave function, then $a_-\psi_0(x) = 0$. Using the relation derived in part (c), show that the ground state wave function has energy $\hbar\omega/2$.

(e) Write out the equation $a_-\psi_0(x) = 0$ as a differential equation and solve it to find the ground-state wave function.

6.14 The "radius of the hydrogen atom" is often taken to be on the order of about 10^{-10} m. If a measurement is made to determine the location of the electron for hydrogen in its ground state, what is the probability of finding the electron within 10^{-10} m of the nucleus?

6.15 **(a)** The electron in a hydrogen atom is in the $l = 1$ state having the lowest possible energy and the highest possible value for m_l. What are the n, l, and m_l quantum numbers?

(b) A particle is moving in an unknown central potential. The wave function of the particle is spherically symmetric. What are the values of l and m_l?