OPPGAVETEKSTER
KAP. 5

5.4 **(a)** The operators A, B, and C are all Hermitian with $[A, B] = C$. Show that $C = 0$.

(b) The operators A and B are both Hermitian with $[A, B] = i\hbar$. Determine whether or not AB is a Hermitian operator.

5.6 **(a)** Let Q be an operator which is not a function of time, and let H be the Hamiltonian operator. Show that

$$i\hbar \frac{\partial}{\partial t}\langle q \rangle = \langle [Q, H] \rangle$$

Here $\langle q \rangle$ is the expectation value of Q for an arbitrary time-dependent wave function Ψ, which is not necessarily an eigenfunction of H, and $\langle [Q, H] \rangle$ is the expectation value of the commutator of Q and H for the same wave function. This result is known as *Ehrenfest's theorem*.

(b) Use this result to show that

$$\frac{\partial}{\partial t}\langle p \rangle = \left\langle -\frac{\partial V}{\partial x} \right\rangle$$

What is the classical analog of this equation?

5.8 Suppose that the operator T is defined by $T = \alpha Q^\dagger Q$, where α is a real number, and Q is an operator (not necessarily Hermitian). Show that T is Hermitian.

5.10 Suppose that two operators P and Q satisfy the commutation relation

$$[P, Q] = Q$$

Suppose that ψ is an eigenfunction of the operator P with eigenvalue p. Show that $Q\psi$ is also an eigenfunction of P, and find its eigenvalue.