FLERE OPPGAVETEKSTER
KAP. 4

4.11 Consider the semi-infinite square well given by $V(x) = -V_0 < 0$ for $0 \le x \le a$ and $V(x) = 0$ for $x > a$. There is an infinite barrier at $x = 0$ (hence the name "semi-infinite"). A particle with mass m is in a bound state in this potential with energy $E \le 0$.

(a) Solve the Schrödinger equation to derive $\psi(x)$ for $x \ge 0$. Use the appropriate boundary conditions and normalize the wave function so that the final answer does not contain any arbitrary constants.

(b) Show that the allowed energy levels E must satisfy the equation

$$\tan\left(\frac{\sqrt{2m(E+V_0)}}{\hbar}a\right) + \sqrt{\frac{-(E+V_0)}{E}} = 0$$

(c) The equation in part (b) cannot be solved analytically to give the allowed energy levels, but simple solutions exist in certain special cases. Determine the conditions on V_0 and a so that a bound state exists with $E = 0$.

4.12 A particle of mass m moves in a harmonic oscillator potential. The particle is in the first excited state.

(a) Calculate $\langle x \rangle$ for this particle.

(b) Calculate $\langle p \rangle$ for this particle.

(c) Calculate $\langle p^2 \rangle$ for this particle.

(d) At what positions are you most likely to find the particle? At what positions are you least likely to find it?

4.13 The oscillation frequencies of a diatomic molecule are typically 10^{12} Hz-10^{14} Hz. Derive an order of magnitude estimate for the harmonic oscillator constant K for such molecules.

4.14 A particle of mass m is bound in a one-dimensional power law potential $V(x) = Kx^\beta$, where β is an even positive integer. Show that the allowed energy levels are proportional to $m^{-\beta/(2+\beta)}$.