NOEN OPPGAVETEKSTER
KAP. 2 & 3

2.8 Consider the identity operator I, defined by $I[f(x)] = f(x)$.

(a) Show that I is a linear operator.

(b) Find the eigenfunctions and corresponding eigenvalues of I.

2.12 Consider the following operator L:

$$L[f(x)] = \int_0^x f(s)\,ds$$

(a) Show that L is a linear operator.

(b) Find the eigenfunctions of L, or show that L has no eigenfunctions.

3.1 A particle of mass m is moving in one dimension in a potential $V(x, t)$. The wave function for the particle is

$$\Psi(x, t) = Axe^{-(\sqrt{km}/2\hbar)x^2} e^{-i\sqrt{k/m}(3/2)t}$$

for $-\infty < x < +\infty$, where k and A are constants.

(a) Show that V is independent of t, and determine $V(x)$.

(b) Normalize this wave function.

(c) Using the normalized wave function, calculate $\langle x \rangle$, $\langle x^2 \rangle$, $\langle p \rangle$, and $\langle p^2 \rangle$.

3.3 The wave function for a particle is $\Psi(x, t) = \sin(kx)[i\cos(\omega t/2) + \sin(\omega t/2)]$, where k and ω are constants.

(a) Is this particle in a state of definite momentum? If so, determine the momentum.

(b) Is this particle in a state of definite energy? If so, determine the energy.

3.7 Suppose that a wave function $\Psi(\mathbf{r}, t)$ is normalized. Show that the wave function $e^{i\theta}\Psi(\mathbf{r}, t)$, where θ is an arbitrary real number, is also normalized.

3.8 Suppose that ψ_1 and ψ_2 are two different solutions of the time-independent Schrödinger equation with the same energy E.

(a) Show that $\psi_1 + \psi_2$ is also a solution with energy E.

(b) Show that $c\psi_1$ is also a solution of the Schrödinger equation with energy E.