Sample Problem 7.4 Derive an expression for the flow rate q over the spillway shown in the accompanying figure per foot of spillway perpendicular to the sketch. Assume that the sheet of water is relatively thick, so that surface-tension effects may be neglected. Assume also that gravity effects predominate so strongly over viscosity that viscosity may be neglected.

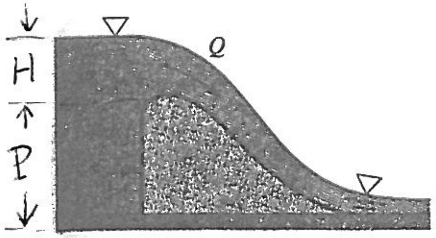

Figure S7.4

Solution

Under the assumed conditions the variables that affect q would be the head H, the acceleration of gravity g, and possibly the spillway height P. Thus

$$q = f(H, g, P)$$

or

$$f_1(q, H, g, P) = 0$$

In this case there are $n = 4$ variables and $m = 2$ dimensions. Two variables can easily be found that cannot be formed into a dimensionless group; therefore $k = m = 2$, and so there are $n - k = 2$ pi groups and

$$\phi(\Pi_1, \Pi_2) = 0$$

Using q and H as the primary variables,

$$\Pi_1 = q^{a_1} H^{b_1} g$$

$$\Pi_2 = q^{a_2} H^{b_2} P$$

Working with Π_1,

$$L^0 T^0 = \left(\frac{L^3}{TL}\right)^{a_1} L^{b_1} \left(\frac{L}{T^2}\right)$$

L: $\quad 0 = 2a_1 + b_1 + 1$

T: $\quad 0 = -a_1 - 2$

Hence

$$a_1 = -2, \quad b_1 = 3$$

$$\Pi_1 = q^{-2} H^3 g = \frac{gH^3}{q^2}$$

Working with Π_2,

$$L^0 T^0 = \left(\frac{L^3}{TL}\right)^{a_2} L^{b_2} L$$

L: $\quad 0 = 2a_2 + b_2 + 1$

T: $\quad 0 = -a_2$

Hence

$$a_2 = 0, \quad b_2 = -1$$

$$\Pi_2 = q^0 H^{-1} P = \frac{P}{H}$$

Finally, $\phi(\Pi_1, \Pi_2) = 0$ can be written as

$$\Pi_1^{-1/2} = \phi_1(\Pi_2^{-1})$$

i.e.,

$$\frac{q}{\sqrt{g} H^{3/2}} = \phi_1\left(\frac{H}{P}\right)$$

or

$$q = \phi_1\left(\frac{H}{P}\right)\sqrt{g} H^{3/2}$$

Thus dimensional analysis indicates that the flow rate per unit length of spillway is proportional to $\sqrt{g}$ and to $H^{3/2}$. The flow rate also is affected by the H/P ratio. This relationship is discussed in Sec. 11.13.

If viscosity were included as one of the variables, another dimensionless group would have resulted. This dimensionless group would have had the form of a Reynolds number. With surface tension included as a variable, the resulting dimensionless group would have been a Weber number.