Sample Problem 6.2 Determine the magnitude and direction of the resultant force exerted on this double nozzle by water flowing through it as shown in Fig. S6.2. Both nozzle jets have a velocity of 12 m/s. The axes of the pipe and both nozzles lie in a horizontal plane. $\gamma = 9.81$ kN/m^3. Neglect friction

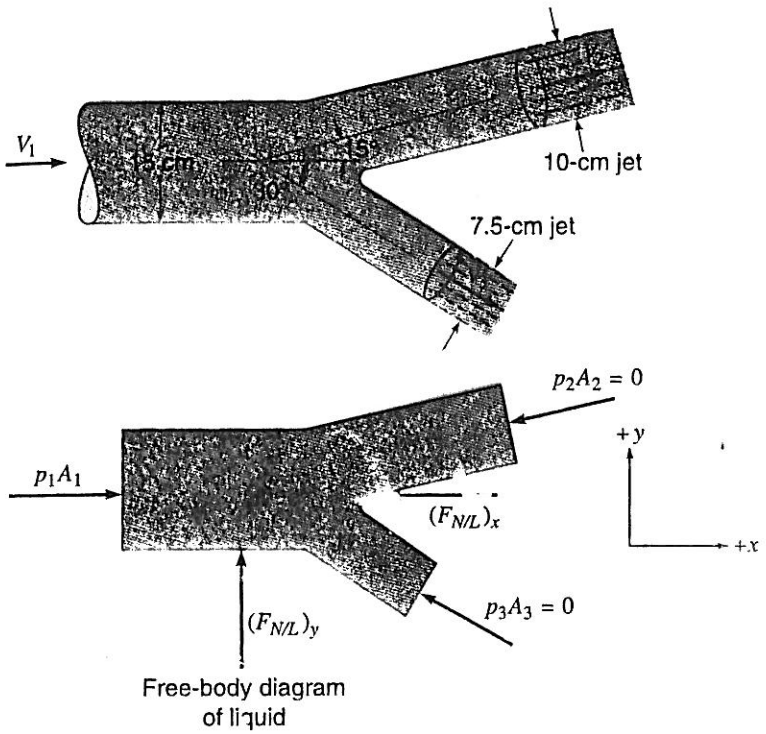

Figure S6.2

Solution

Continuity: $$A_1V_1 = A_2V_2 + A_3V_3$$

$$15^2V_1 = 10^2(12) + 7.5^2(12), \quad V_1 = 8.33 \text{ m/s}$$

$$Q_1 = \frac{\pi}{4}(0.15)^2 8.33 = 0.1473 \text{ m}^3/\text{s}, \quad Q_2 = 0.0942 \text{ m}^3/\text{s}, \quad Q_3 = 0.0530 \text{ m}^3/\text{s}$$

Jets 2 and 3 are "free," i.e., in the atmosphere, so $p_2 = p_3 = 0$.

Energy equation: $$\frac{p_1}{\gamma} + \frac{8.33^2}{2(9.81)} = 0 + \frac{12^2}{2(9.81)}$$

$$\frac{p_1}{\gamma} = 3.80 \text{ m}, \quad p_1 = 37.3 \text{ kN/m}^2, \quad p_1A_1 = 0.659 \text{ kN}$$

$$\sum F_x = p_1A_1 - (F_{N/L})_x = (\rho Q_2V_{2x} + \rho Q_3V_{3x}) - \rho Q_1V_{1x}$$

$$\rho = \frac{\gamma}{g} = \frac{9.81 \text{ kN/m}^3}{9.81 \text{ m/s}^2} = 1.0 \frac{\text{kN}\cdot\text{s}^2}{\text{m}^4} = 10^3 \frac{\text{kg}}{\text{m}^3}$$

$$V_{2x} = V_2 \cos 15° = 12(0.966) = 11.6 \text{ m/s}$$

$$V_{3x} = V_3 \cos 30° = 12(0.866) = 10.4 \text{ m/s}, \quad V_{1x} = V_1 = 8.33 \text{ m/s}$$

$$0.659 - (F_{N/L})_x = 10^3(0.0942)(11.6) + 10^3(0.0530)(10.4) - 10^3(0.1473)8.33$$

$$= 0.417 \text{ kN}$$

$$(F_{N/L})_x = 0.659 - 0.417 = 0.242 \text{ kN} \leftarrow$$

$$\sum F_y = (F_{N/L})_y = (\rho Q_2V_{2y} + \rho Q_3V_{3y}) - \rho Q_1V_{1y}$$

$$V_{2y} = V_2 \sin 15° = 12(0.259) = 3.11 \text{ m/s}$$

$$V_{3y} = -V_3 \sin 30° = -12(0.50) = -6.00 \text{ m/s}, \quad V_{1y} = 0$$

$$(F_{N/L})_y = 10^3(0.0942)(3.11) + 10^3(0.0530)(-6.00) - 10^3(0.1473)(0)$$

$$= 0.291 - 0.318 - 0 = -0.027 \text{ kN} \uparrow = 0.027 \text{ kN} \downarrow$$

The minus sign indicates that the assumed direction of $(F_{N/L})_y$ was wrong. Therefore $(F_{N/L})_y$ acts in the negative y direction. $F_{L/N}$ is equal and opposite to $F_{N/L}$.

$$(F_{L/N})_x = 0.247 \text{ kN} \rightarrow \quad \text{(in the positive } x \text{ direction)}$$

$$(F_{L/N})_y = 0.027 \text{ kN} \uparrow \quad \text{(in the positive } y \text{ direction)}$$

$$F_{L/N} = 0.243 \text{ kN at } 5.90° \measuredangle \quad \text{ANS}$$