Sample Problem 3.5 Figure S3.5 shows a gate, 2 ft wide perpendicular to the sketch. It is pivoted at hinge H. The gate weighs 500 lb. Its center of gravity is 1.2 ft to the right of and 0.9 ft above H. For what values of water depth x above H will the gate remain closed? Neglect friction at the pivot and neglect thickness of the gate.

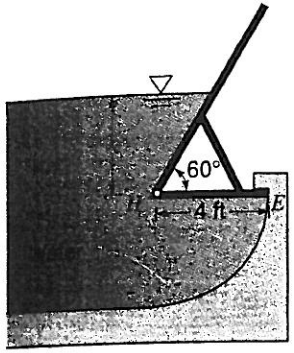


Figure S3.5

Solution

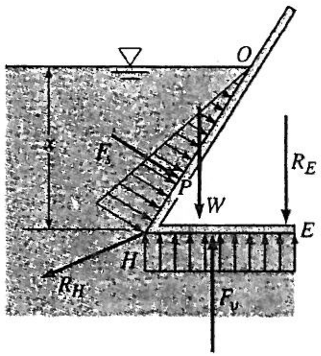
In addition to the reactive forces R_H at the hinge and R_E at end E, there are three forces acting on the gate: its weight W, the vertical hydrostatic force F_v upward on the rectangular bottom of the gate, and the slanting hydrostatic force F_s acting at right angles to the sloping rectangular portion of the gate. The magnitudes of the latter three forces are as follows:

Given: $$W = 500 \text{ lb}$$

Eq. (3.16): $$F_v = \gamma h_c A = \gamma(x)(4\times 2) = 8\gamma x$$

Eq. (3.16): $$F_s = \gamma h_c A = \gamma(x/2)\left(\frac{x}{\cos 30^\circ}\times 2\right) = 1.155\gamma x^2$$

A diagram showing these three forces is as follows:



The moment arms of W and F_v with respect to H are 1.2 ft and 2.0 ft respectively. The moment arm of F_s gets larger as the water depth increases because the location of the center of pressure changes. The location of the center of pressure of F_s may be found from Eq. (3.18):

$$y_p = y_c + \frac{I_c}{y_c A}, \quad \text{where} \quad I_c = \frac{bh^3}{12}$$

with $h = x/\cos 30^\circ$ and $y_c = 0.5h$. So

$$y_p = \frac{0.5x}{\cos 30^\circ} + \frac{(1/12)2(x/\cos 30^\circ)^3}{(0.5x/\cos 30^\circ)[2(x/\cos 30^\circ)]}$$

i.e., for F_s: $$OP = y_p = 0.577x + \frac{2x}{12\cos 30^\circ} = 0.770x$$

Hence the moment arm of F_s with respect to H is $PH = x/\cos 30^\circ - 0.770x = 0.385x$. [*Note:* In this case Eq. (3.18) need not be used to find the lever arm of F_s because the line of action of F_s for the triangular distributed load on the rectangular area is known to be at the third point between H and O, i.e., $HP = (1/3)(x/\cos 30^\circ) = 0.385x$.]

When the gate is about to open (incipient rotation), $R_E = 0$ and the sum of the moments of all forces about H is zero, viz

$$\sum M = F_s(0.385x) + W(1.2) - F_v(2.0) = 0$$

i.e., $$1.155\gamma x^2(0.385x) + 500(1.2) - 8\gamma x(2) = 0$$

Substituting $\gamma = 62.4 \text{ lb/ft}^3$ gives

$$27.73x^3 + 600 - 998.4x = 0$$

Without an equation solver, we can solve this cubic equation by trials, seeking x values that make the left-hand side of the equation equal to zero. After two trial values, use linear interpolation to estimate the next trial value, as follows:

Trial x	*Left-hand side*	*Trial x*	*Left-hand side*	*Trial x*	*Left-hand side*
0.1	500.2	10.0	18,349	−10.0	−17,149
0.5	104.3	5.0	−925.3	−5.0	2125
0.6	6.95	6.0	600.0	−5.55	1400
0.61	−2.73	5.60	−120.6	−6.61	−810
		5.67	−5.58	−6.22	−136.3
				−6.28	1.15

Thus $x = 0.61$ ft or 5.67 ft or a negative (meaningless) root. Therefore, from inspection of the moment equation, the gate will remain closed when $0.61 \text{ ft} < x < 5.67$ ft. ***ANS***