Chapter 1

Introduction

We start by summarizing some basic equations and derived relations which will be assumed known from previous courses. After that, a justification of the contents and objective of this course will follow.

1.1 Recapitulation

1.1.1 Notation and vector relations

In this course we will treat fluids which will be assumed to be

- *one-phase*,
- *incompressible* (unless otherwise indicated explicitly), as well as
- *Newtonian* (viscosity function independent of the angular deformation velocity in the fluid [Irgens 1983]);

accordingly the density ρ as well as the dynamical and kinematical viscosities μ and $\nu\,(=\mu/\rho)$ will be treated as constants.

The SI system is being used in the compendium. In cases where the notation varies between common textbooks, the notation of [Tritton 1988] is mostly used.

Fluid dynamics is based on the assumption that a description in terms of *fields* is valid for the local velocity $\boldsymbol{u}$ and pressure p, that is, $\boldsymbol{u} = \boldsymbol{u}(x, y, z, t)$ and $p = p(x, y, z, t)$ in cartesian coordinates. A more compact notation which does not presuppose anything about the type of coordinate system, is $\boldsymbol{u} = \boldsymbol{u}(\boldsymbol{r}, t)$, where $\boldsymbol{r}$ is the position vector. The alternative cartesian notations

$$\boldsymbol{r} = (x, y, z) = (x_1, x_2, x_3) \tag{1.1}$$

$$\boldsymbol{u} = (u, v, w) = (u_1, u_2, u_3) \tag{1.2}$$

will be used, as well as a compact notation for the partial derivation operator:

$$\partial_i \equiv \frac{\partial}{\partial x_i} \quad (i = 1, 2, 3)\ , \qquad \partial_t \equiv \frac{\partial}{\partial t} \tag{1.3}$$

The vector operator ∇ (*del* or *nabla*) is defined in cartesian coordinates as

$$\begin{aligned}\nabla &= \delta_{ij}\hat{\boldsymbol{e}}_i\partial_j \\ &= \hat{\boldsymbol{e}}_i\partial_i \\ &= \hat{\boldsymbol{e}}_1\partial_1 + \hat{\boldsymbol{e}}_2\partial_2 + \hat{\boldsymbol{e}}_3\partial_3 \qquad (1.4)\end{aligned}$$

where $\hat{\boldsymbol{e}}_i$ $(i = 1, 2, 3)$ are orthogonal unit vectors, and the *Kroneckerdelta* δ_{ij} is defined by

$$\delta_{ij} = \begin{cases} 1 & \text{for } i = j \\ 0 & \text{for } i \neq j \end{cases} \qquad (1.5)$$

Equation 1.4 is an example of a convention we will be using consistently in this course, to avoid writing the summation symbol:

- *Sum convention*: For pairs of identical latin indices in products of cartesian coordinates, a sum over the index values from 1 to 3 is implied

The operator appears in the expressions

$$\begin{aligned}\operatorname{grad}\Phi &\equiv \nabla\Phi\,, \qquad (\operatorname{grad}\Phi)_i = \partial_i\Phi \quad (i = 1, 2, 3) \qquad (1.6)\\ \operatorname{div}\boldsymbol{u} &\equiv \nabla\cdot\boldsymbol{u} = \partial_i u_i \qquad (1.7)\\ \operatorname{curl}\boldsymbol{u} &\equiv \nabla\times\boldsymbol{u}\,, \qquad (\operatorname{curl}\boldsymbol{u})_i = \epsilon_{ijk}\partial_j u_k \qquad (1.8)\end{aligned}$$

where Φ is a scalar field, $\boldsymbol{u}$ a vector field, and ϵ_{ijk} is the *Levi-Civitá tensor* which allows us to write components of vector products in a compact way. It is defined by

$$\epsilon_{ijk} = \begin{cases} 1 & \text{for } (ijk) \text{ cyclic permutation of } (123) \\ -1 & \text{for } (ijk) \text{ anticyclic permutation of } (123) \\ 0 & \text{otherwise} \end{cases} \qquad (1.9)$$

We will need the general relations

$$\begin{aligned}\nabla\times(\nabla\Phi) &\equiv 0 \qquad (1.10)\\ \nabla\cdot(\nabla\times\boldsymbol{u}) &\equiv 0 \qquad (1.11)\\ \nabla\times(\nabla\times\boldsymbol{u}) &= \nabla(\nabla\cdot\boldsymbol{u}) - \nabla^2\boldsymbol{u} \qquad (1.12)\\ (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} &= \tfrac{1}{2}\nabla(\boldsymbol{u}\cdot\boldsymbol{u}) - \boldsymbol{u}\times(\nabla\times\boldsymbol{u}) \qquad (1.13)\\ \nabla\times((\boldsymbol{u}\cdot\nabla)\boldsymbol{u}) &= (\boldsymbol{u}\cdot\nabla)(\nabla\times\boldsymbol{u}) + (\nabla\cdot\boldsymbol{u})(\nabla\times\boldsymbol{u}) - ((\nabla\times\boldsymbol{u})\cdot\nabla)\boldsymbol{u} \qquad (1.14)\end{aligned}$$

which are derived in Appendix A by methods which are also valid for more complicated products. The scalar multiplication symbol and the parentheses are customarily deleted in cases where the meaning is obvious.

The operations grad, div, curl og ∇^2 (the *(Laplace operator)* can of course also be expressed in curvilinear coordinates. We will not present the derivations of any such expressions, many of which are complicated. In Appendix A, however, some particular expressions in cylindrical and spherical polar coordinates are listed; we will need them in later chapters. A more complete presentation can be found in formula collections like [Rottmann 1995], and in the textbooks in fluid dynamics from the series in theoretical physics mentioned in the list of references.