Frequent Itemsets and Association Rule Mining

Vinay Setty
vinay.j.setty@uis.no

Slides credit: http://www.mmds.org/

Association Rule Discovery

Supermarket shelf management – Market-basket model:

- **Goal:** Identify items that are bought together by sufficiently many customers
- **Approach:** Process the sales data collected with barcode scanners to find dependencies among items
- **A classic rule:**
  - If someone buys diaper and milk, then he/she is likely to buy beer
  - Don’t be surprised if you find six-packs next to diapers!

The Market-Basket Model

- A large set of **items**
  - e.g., things sold in a supermarket
- A **large set** of **baskets**
- Each basket is a **small subset of items**
  - e.g., the things one customer buys on one day
- Want to discover **association rules**
  - People who bought {x,y,z} tend to buy {v,w}
    - Amazon!

Input:

TID	*Items*
1	**Bread, Coke, Milk**
2	**Beer, Bread**
3	**Beer, Coke, Diaper, Milk**
4	**Beer, Bread, Diaper, Milk**
5	**Coke, Diaper, Milk**

Output:

Rules Discovered:

{Milk} --> {Coke}

{Diaper, Milk} --> {Beer}

Applications – (1)

- **Items** = products; **Baskets** = sets of products someone bought in one trip to the store
- **Real market baskets:** Chain stores keep TBs of data about what customers buy together
  - Tells how typical customers navigate stores, lets them position tempting items
  - Suggests tie-in "tricks", e.g., run sale on diapers and raise the price of beer
  - Need the rule to occur frequently, or no $$'s
- ***Amazon's people who bought X also bought Y***

Applications – (2)

- **Baskets** = sentences; **Items** = documents containing those sentences
  - Items that appear together too often could represent plagiarism
  - Notice items do not have to be "in" baskets

- **Baskets** = patients; **Items** = drugs & side-effects
  - Has been used to detect combinations of drugs that result in particular side-effects
  - **But requires extension:** Absence of an item needs to be observed as well as presence

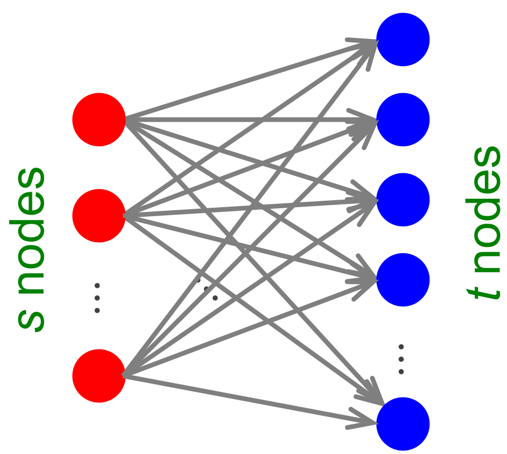
More generally

- **A general many-to-many mapping (association) between two kinds of things**
  - But we ask about connections among “items”, not “baskets”

- **For example:**
  - Finding communities in graphs (e.g., Twitter)

Example:

- **Finding communities in graphs (e.g., Twitter)**
- **Baskets** = nodes; **Items** = outgoing neighbors
  - Searching for complete bipartite subgraphs $K_{s,t}$ of a big graph



A dense 2-layer graph

- **How?**
  - View each node i as a basket B_i of nodes i it points to
  - $K_{s,t}$ = a set Y of size t that occurs in s buckets B_i
  - Looking for $K_{s,t}$ → set of support s and look at layer t – all frequent sets of size t

Outline

First: Define

- **Frequent itemsets**
- **Association rules:**
  - Confidence, Support, Interestingness

Then: Algorithms for finding frequent itemsets

- **Finding frequent pairs**
- **A-Priori algorithm**

Frequent Itemsets

- **Simplest question:** Find sets of items that appear together "frequently" in baskets
- ***Support*** for itemset I: Number of baskets containing all items in I
  - (Often expressed as a fraction of the total number of baskets)
- Given a ***support threshold*** s, then sets of items that appear in at least s baskets are called ***frequent itemsets***

TID	*Items*
1	**Bread, Coke, Milk**
2	**Beer, Bread**
3	**Beer, Coke, Diaper, Milk**
4	**Beer, Bread, Diaper, Milk**
5	**Coke, Diaper, Milk**

Support of {Beer, Bread} = 2

Example: Frequent Itemsets

- **Items** = {milk, coke, pepsi, beer, juice}
- **Support threshold** = 3 baskets

  B_1 = {m, c, b} **B_2** = {m, p, j}

  B_3 = {m, b} **B_4** = {c, j}

  B_5 = {m, p, b} **B_6** = {m, c, b, j}

  B_7 = {c, b, j} **B_8** = {b, c}

- **Frequent itemsets:** {m}, {c}, {b}, {j},

  {m,b}, {b,c}, {c,j}.

Association Rules

- **Association Rules:**
  If-then rules about the contents of baskets
- $\{i_1, i_2, \ldots, i_k\} \rightarrow j$ means: "if a basket contains all of $i_1, \ldots, i_k$ then it is ***likely*** to contain j"
- **In practice there are many rules, want to find significant/interesting ones!**
- ***Confidence*** of this association rule is the probability of j given $I = \{i_1, \ldots, i_k\}$

$$\text{conf}(I \rightarrow j) = \frac{\text{support}(I \cup j)}{\text{support}(I)}$$

Interesting Association Rules

- **Not all high-confidence rules are interesting**
  - The rule $X \rightarrow \textit{milk}$ may have high confidence for many itemsets X, because milk is just purchased very often (independent of X) and the confidence will be high
- **Interest** of an association rule $I \rightarrow j$:
difference between its confidence and the fraction of baskets that contain j

$$\text{Interest}(I \rightarrow j) = \text{conf}(I \rightarrow j) - \Pr[j]$$

  - Interesting rules are those with high positive or negative interest values (usually above 0.5)

Example: Confidence and Interest

B_1 = {m, c, b} B_2 = {m, p, j}

B_3 = {m, b} B_4= {c, j}

B_5 = {m, p, b} B_6 = {m, c, b, j}

B_7 = {c, b, j} B_8 = {b, c}

- **Association rule: {m, b} →c**
  - **Confidence** = 2/4 = 0.5
  - **Interest** = |0.5 – 5/8| = 1/8
    - Item **c** appears in 5/8 of the baskets
    - Rule is not very interesting!

Finding Association Rules

- **Problem: Find all association rules with support $\geq s$ and confidence $\geq c$**
  - **Note:** Support of an association rule is the support of the set of items on the left side
- **Hard part: Finding the frequent itemsets!**
  - If $\{i_1, i_2, \ldots, i_k\} \rightarrow j$ has high support and confidence, then both $\{i_1, i_2, \ldots, i_k\}$ and $\{i_1, i_2, \ldots, i_k, j\}$ will be "frequent"

$$\text{conf}(I \rightarrow j) = \frac{\text{support}(I \cup j)}{\text{support}(I)}$$

Mining Association Rules

- **Step 1:** Find all frequent itemsets I
  - (we will explain this next)
- **Step 2: Rule generation**
  - For every subset A of I, generate a rule $A \rightarrow I \setminus A$
    - Since I is frequent, A is also frequent
    - **Variant 1:** Single pass to compute the rule confidence
      - confidence(***A,B→C,D***) = support(**A,B,C,D**) / support(**A,B**)
    - **Variant 2:**
      - **Observation:** If **A,B,C→D** is below confidence, so is **A,B→C,D**
      - Can generate "bigger" rules from smaller ones!
  - **Output the rules above the confidence threshold**

Example

$B_1 = \{m, c, b\}$ $B_2 = \{m, p, j\}$

$B_3 = \{m, c, b, n\}$ $B_4 = \{c, j\}$

$B_5 = \{m, p, b\}$ $B_6 = \{m, c, b, j\}$

$B_7 = \{c, b, j\}$ $B_8 = \{b, c\}$

- **Support threshold** $s = 3$, **confidence** $c = 0.75$
- **1) Frequent itemsets:**
  - {b,m} {b,c} {c,m} {c,j} {m,c,b}
- **2) Generate rules:**
  - ~~**b**→**m**: c=4/6~~ **b**→**c**: c=5/6 ~~**b,c**→**m**: c=3/5~~
  - **m**→**b**: c=4/5 … **b,m**→**c**: c=3/4
  - ~~**b**→**c,m**: c=3/6~~

Compacting the Output

- **To reduce the number of rules we can post-process them and only output:**
  - **Maximal frequent itemsets:** No immediate superset is frequent
    - Gives more pruning

  or

  - **Closed itemsets:** No immediate superset has the same count (> 0)
    - Stores not only frequent information, but exact counts

Example: Maximal/Closed

	Support	Maximal(s=3)	Closed
A	4	No	No
B	5	No	Yes
C	3	No	No
AB	4	Yes	Yes
AC	2	No	No
BC	3	Yes	Yes
ABC	2	No	Yes

Frequent, but superset BC also frequent.

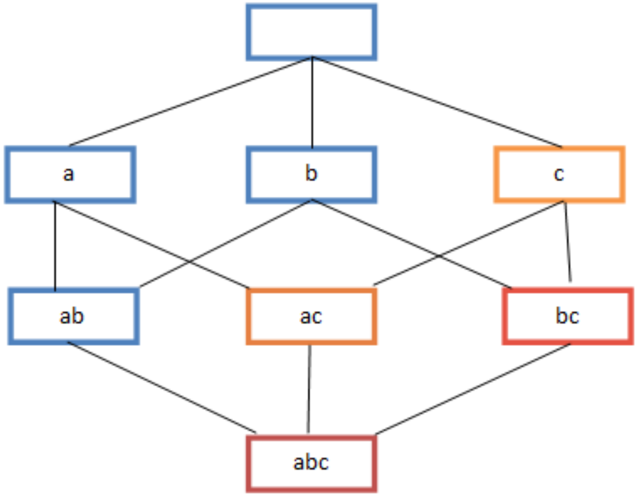
Frequent, and its only superset, ABC, not freq.

Superset BC has same count.

Its only super-set, ABC, has smaller count.

A-Priori Algorithm

A-Priori Algorithm – (1)

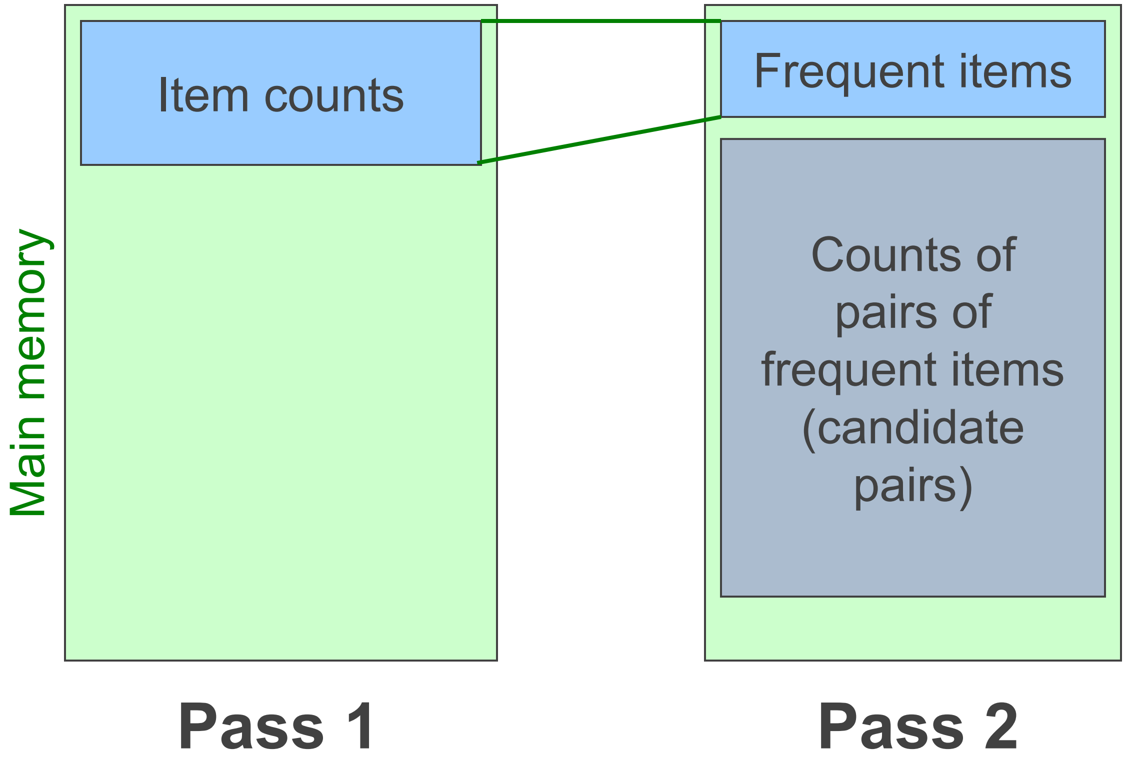


- A **two-pass** approach called ***A-Priori*** limits the need for main memory
- **Key idea: *monotonicity***
  - If a set of items I appears at least s times, so does every **subset** J of I
- **Contrapositive for pairs:**
  If item i does not appear in s baskets, then no pair including i can appear in s baskets
- **So, how does A-Priori find freq. pairs?**

A-Priori Algorithm – (2)

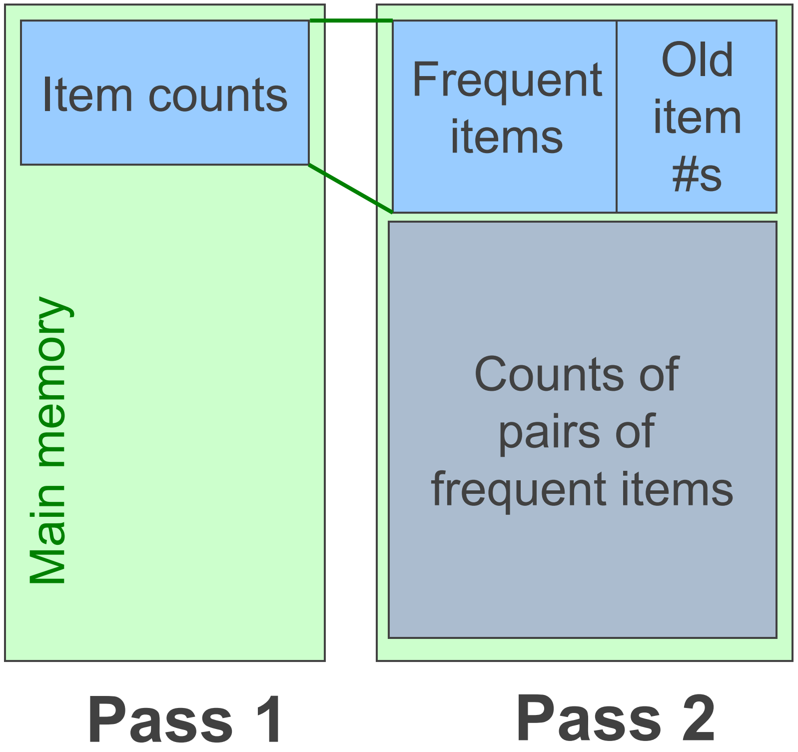
- **Pass 1:** Read baskets and count in main memory the occurrences of each **individual item**
  - Requires only memory proportional to #items
- **Items that appear $\geq s$ times are the frequent items**
- **Pass 2:** Read baskets again and count in main memory only those pairs where both elements are frequent (from Pass 1)
  - Requires memory proportional to square of **frequent** items only (for counts)
  - Plus a list of the frequent items (so you know what must be counted)

Main-Memory: Picture of A-Priori



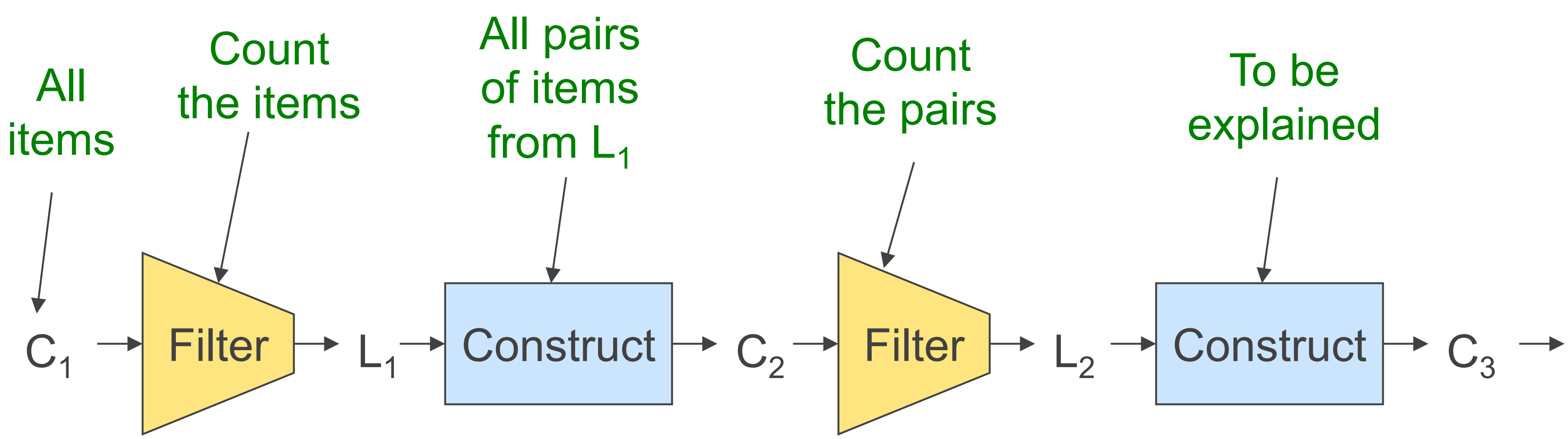
Detail for A-Priori

- You can use the triangular matrix method with ***n*** = number of frequent items
  - May save space compared with storing triples
- **Trick:** re-number frequent items 1,2,… and keep a table relating new numbers to original item numbers



Frequent Triples, Etc.

- **For each *k*, we construct two sets of *k-tuples*** (sets of size *k*):
  - C_k = ***candidate k-tuples*** = those that might be frequent sets (support $\geq$ **s**) based on information from the pass for ***k*–1**
  - L_k = the set of truly frequent ***k***-tuples



Example

** Note here we generate new candidates by generating C_k from L_{k-1} and L_1.
But that one can be more careful with candidate generation. For example, in C_3 we know {b,m,j} cannot be frequent since {m,j} is not frequent

- **Hypothetical steps of the A-Priori algorithm**
  - C_1 = { {b} {c} {j} {m} {n} {p} }
  - Count the support of itemsets in C_1
  - Prune non-frequent: L_1 = { b, c, j, m }
  - Generate C_2 = { {b,c} {b,j} {b,m} {c,j} {c,m} {j,m} }
  - Count the support of itemsets in C_2
  - Prune non-frequent: L_2 = { {b,m} {b,c} {c,m} {c,j} }
  - Generate C_3 = { {b,c,m} {b,c,j} {b,m,j} {c,m,j} }
  - Count the support of itemsets in C_3 **
  - Prune non-frequent: L_3 = { {b,c,m} }

Generating Candidates – Full Example

Database D

TID	Items
100	1 3 4
200	2 3 5
300	1 2 3 5
400	2 5

Scan D

minsup = 2

C_1

itemset	sup.
{1}	2
{2}	3
{3}	3
{4}	1
{5}	3

L_1

itemset	sup.
{1}	2
{2}	3
{3}	3
{5}	3

C_2

itemset
{1 2}
{1 3}
{1 5}
{2 3}
{2 5}
{3 5}

Scan D

C_2

itemset	sup
{1 2}	1
{1 3}	2
{1 5}	1
{2 3}	2
{2 5}	3
{3 5}	2

L_2

itemset	sup
{1 3}	2
{2 3}	2
{2 5}	3
{3 5}	2

C_3

itemset
{2 3 5}

Scan D

L_3

itemset	sup
{2 3 5}	2

Scan D

C_4 is empty

Pruning Step

L_2

itemset	sup
{1 3}	2
{2 3}	2
{2 5}	3
{3 5}	2

C_3

itemset
{2 3 5}

itemset
{1 3 5}

Check to see of all these itemsets are frequent

{2 3}	{1 3}
{3 5}	{3 5}
{2 5}	{1 5}

Not frequent!

- For an itemset of size k, check if all the itemsets of size k-1 are also frequent
- If any of the k-1 sized itemsets are not frequent prune the itemset of size k

A-Priori for All Frequent Itemsets

- One pass for each ***k*** (itemset size)
- Needs room in main memory to count each candidate ***k***–tuple
- For typical market-basket data and reasonable support (e.g., 1%), ***k* = 2** requires the most memory

- **Many possible extensions:**
  - Association rules with intervals:
    - For example: Men over 65 have 2 cars
  - Association rules when items are in a taxonomy
    - Bread, Butter $\longrightarrow$ FruitJam
    - BakedGoods, MilkProduct $\longrightarrow$ PreservedGoods
  - Lower the support ***s*** as itemset gets bigger

Frequent Itemsets in $\leq$ 2 Passes

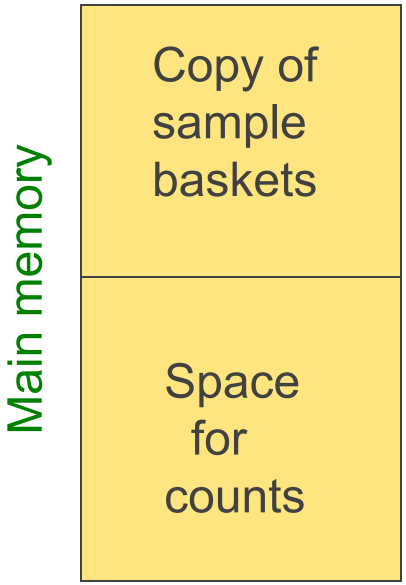
Frequent Itemsets in ≤ 2 Passes

- A-Priori takes k passes to find frequent itemsets of size k
- **Can we use fewer passes?**
- Use 2 or fewer passes for all sizes,
  but may miss some frequent itemsets
  - Random sampling
  - SON (Savasere, Omiecinski, and Navathe)
  - Toivonen (see textbook)

Random Sampling (1)

- Take a random sample of the market baskets

- Run a-priori or one of its improvements in main memory
  - So we don't pay for disk I/O each time we increase the size of itemsets
  - Reduce support threshold proportionally
    to match the sample size



Random Sampling (2)

- Optionally, verify that the candidate pairs are truly frequent in the entire data set by a second pass (avoid false positives)

- But you don't catch sets frequent in the whole but not in the sample
  - Smaller threshold, e.g., $s/125$, helps catch more truly frequent itemsets
    - But requires more space

SON Algorithm – (1)

- Repeatedly read small subsets of the baskets into main memory and run an in-memory algorithm to find all frequent itemsets
  - Note: we are not sampling, but processing the entire file in memory-sized chunks

- An itemset becomes a candidate if it is found to be frequent in *any* one or more subsets of the baskets.

SON Algorithm – (2)

- On a **second pass**, count all the candidate itemsets and determine which are frequent in the entire set

- **Key "monotonicity" idea:** an itemset cannot be frequent in the entire set of baskets unless it is frequent in at least one subset.

SON Summary

- Pass 1 – Batch Processing
  - Scan data on disk
  - Repeatedly fill memory with new batch of data
  - Run sampling algorithm on each batch
  - Generate candidate frequent itemsets
- Candidate Itemsets – if frequent in some batch
- Pass 2 – Validate candidate itemsets
- **Monotonicity Property**

Itemset X is frequent overall → frequent in at least one batch

SON – Distributed Version

- SON lends itself to distributed data mining
- Baskets distributed among many nodes
  - Compute frequent itemsets at each node
  - Distribute candidates to all nodes
  - Accumulate the counts of all candidates

SON: Map/Reduce

- **Phase 1:** Find candidate itemsets
  - Map?
  - Reduce?

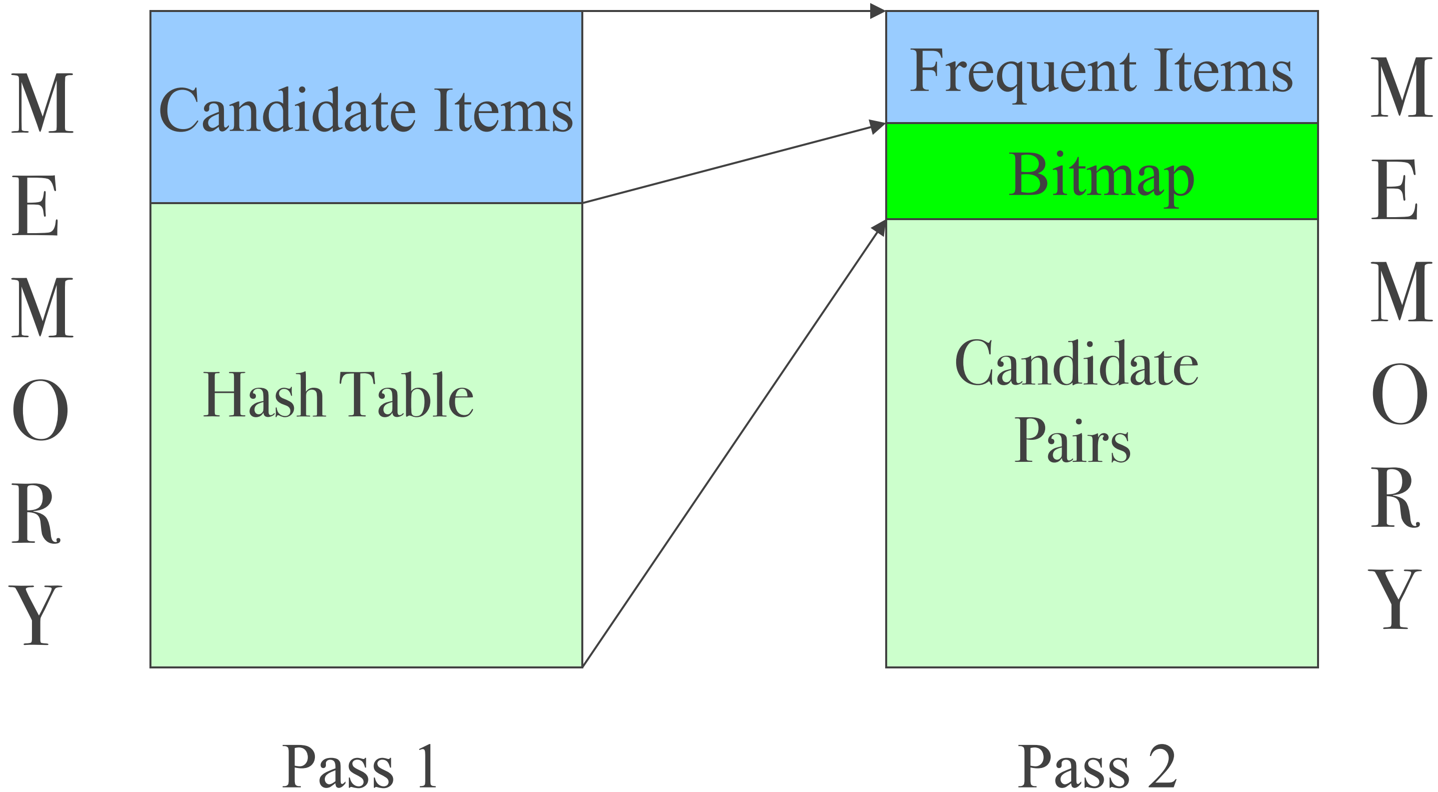
- **Phase 2:** Find true frequent itemsets
  - Map?
  - Reduce?

PCY (Park-Chen-Yu) Algorithm

(Park-Chen-Yu) PCY Idea

- Improvement upon A-Priori
- Observe – during Pass 1, memory mostly idle
- Idea
  - Use idle memory for hash-table H
  - Pass 1 – hash pairs from b into H
    - Increment counter at hash location
  - At end – bitmap of high-frequency hash locations
  - Pass 2 – bitmap extra condition for candidate pairs

Memory Usage PCY



PCY Algorithm

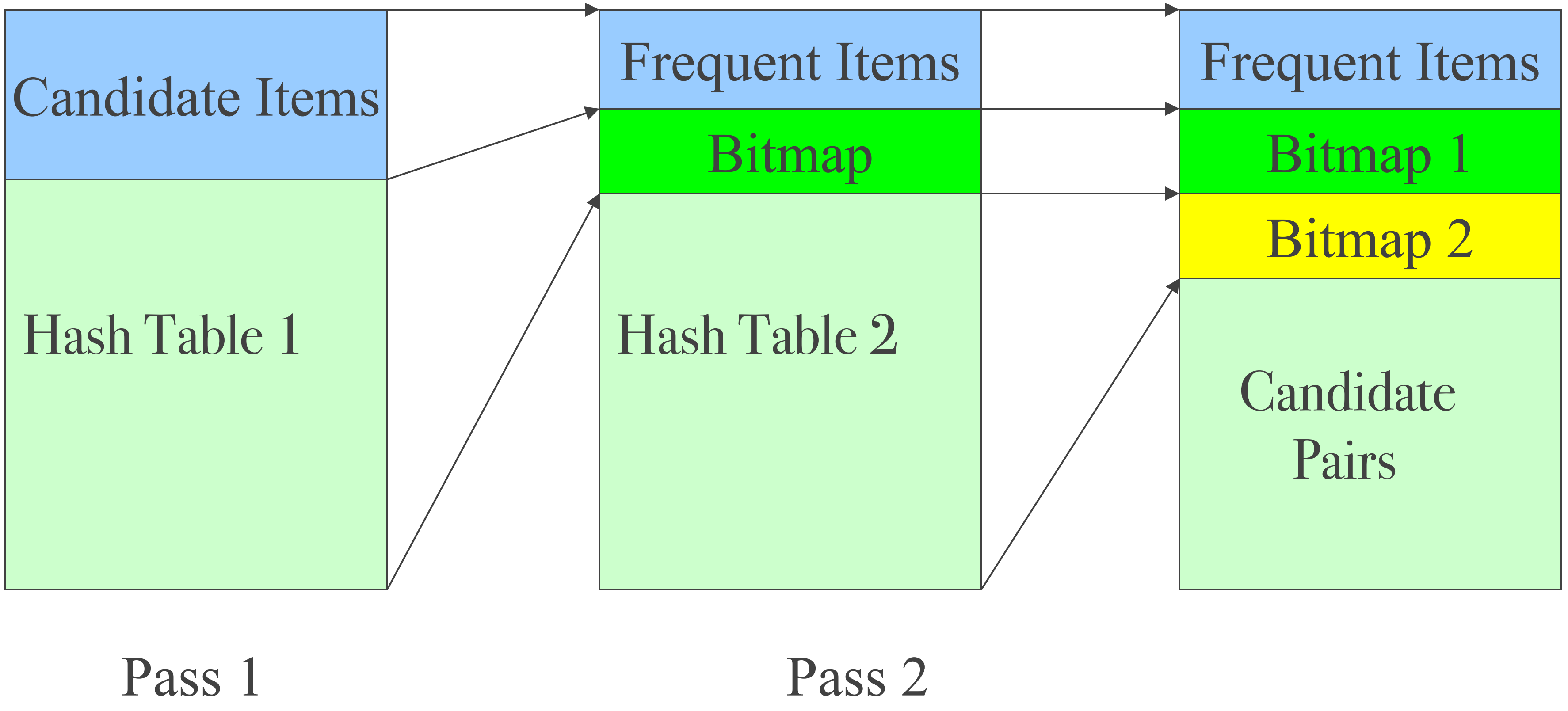
- **Pass 1**
  - m counters and hash-table T
  - Linear scan of baskets b
  - Increment counters for each item in b
  - Increment hash-table counter for each item-pair in b
- Mark as frequent, f items of count at least s
- Summarize T as bitmap (count > s → bit = 1)
- **Pass 2**
  - Counter only for F qualified pairs (X_i,X_j):
    - both are frequent
    - pair hashes to frequent bucket (bit=1)
  - Linear scan of baskets b
  - Increment counters for candidate qualified pairs of items in b

Multi-Stage PCY

- **Problem** – False positives from hashing
- **New Idea**
  - Multiple rounds of hashing
  - After Pass 1, get list of qualified pairs
  - In Pass 2, hash only qualified pairs
  - Fewer pairs hash to buckets → less false positives

    (buckets with count >s, yet no pair of count >s)
  - In Pass 3, less likely to qualify infrequent pairs
- Repetition – reduce memory, but more passes
- Failure – memory < O(f+F)

Multi-Stage PCY Memory



Literature

- **Mining of Massive Datasets Jure Leskovec, Anand Rajaraman, Jeff Ullman, Chapter 6**
- http://mmds.org
  http://infolab.stanford.edu/~ullman/mmds/ch6.pdf