Chapter 3

Fixed filter banks

As we saw in Chapter 2, several approaches to multi channel filtering for texture segmentation are proposed. There are different, and to some extent conflicting, objectives for the choice of a filter bank. The principal objective is good feature separation, i.e., good classification or segmentation accuracy. However, issues as feature extraction complexity, feature dimensionality, memory requirements, and how easy the feature extractor may be tuned may also be important.

Two new multi channel filter approaches are presented in this chapter, addressing some of these objectives. In Section 3.1, alternatives to the Gabor filter bank sub-band decomposition of Jain and Farrokhnia [50] are suggested. The idea for suggesting new decompositions, is that they may yield improved segmentation accuracies. In Section 3.2, filters applied successfully to image compression [46] are introduced for texture feature extraction. Furthermore, the use of critically sampled filter banks is proposed. These approaches are advantageous with respect to their computational properties. It is therefore interesting to see if they can compete with the more computationally complex filter banks.

3.1 Non-dyadic Gabor filter bank decomposition

It has been common to decompose the frequency band by the octave band (dyadic) decomposition [50, 87, 126, 129]. This choice is motivated by the similarities between the octave band decomposition and the early vision of mammals. Octave band decompositions imply finer frequency resolutions in lower than in higher frequency regions. The work by Chang and Kuo [16], however, indicate that texture features are most prevalent in intermediate frequency bands. Hence, the octave band decomposition is not ideal for texture recognition. In this section, an extension of the scheme of [50] to more arbitrary decompositions is developed.

Some possible two-dimensional sub-band decompositions are given in Figure 2.6. The filter of Equation (2.6) is used and f_0 is set to the radial value of the center frequency of the pass-band. The filter orientation is set to the corresponding angle. Previously, filter orientations in the range 0° to 180° have been used [50]. The decomposition proposed

(a) (b)

Figure 3.1: Segmentation results with the texture mosaic of Figure 1.1(a) and different Gabor filter approaches, (a) with the decomposition of Jain and Farrokhnia [50] and (b) with the sub-band decomposition of Figure 2.6(d).

here will only make it possible to specify orientations from 0° to 90°.[1] To maintain some selectiveness of angles from 90° to 180°, filters whose frequency responses are the sums of the frequency responses in the range 0° to 90° and the frequency response of the orthogonal filter are used.

In the experiments, 6% of the feature vectors (see the discussion in Section 2.1.2) were used for training an LVQ classifier. The experimental results are illustrated in Figure 3.1. The classification errors were 5.4% with the decomposition of Figure 2.6(d) and 2.4% with the dyadic decomposition, indicating that the non-octave band decomposition is a good alternative. Remember that in this experiment, the dyadic decomposition was more flexible with respect to orientation. In more thorough experiments in Chapter 6, it is shown that the alternative decomposition outperforms the dyadic decompositions in several cases. When the same orientation limitations were imposed on both dyadic and non-dyadic decompositions, the non-dyadic decomposition yielded better or practically equal results in all test cases. The problem, however, remains the same for virtually all decompositions: to achieve a good segmentation quality we need a large number of sub-bands. This leads to a high computational complexity, as well as large memory requirements.

3.2 Quadrature mirror filter banks

In most approaches to multi channel texture segmentation, filter banks in which the sub-bands each *have the same number of image samples as the unfiltered image* are applied. Hence, after filtering, the amount of data is increased by a factor corresponding to the number of frequency channels in the filter bank. Sub-sampling of the frequency bands so that the total number of sub-band signal samples equals the number of signal samples in the input image can be performed without losing any information, as long as the filters are designed appropriately [133]. To achieve correct classification we should in principle not need a redundant representation. For critically sampled filter banks with

[1] The reason for this restriction is parallelism to the separable approach of Section 3.2.

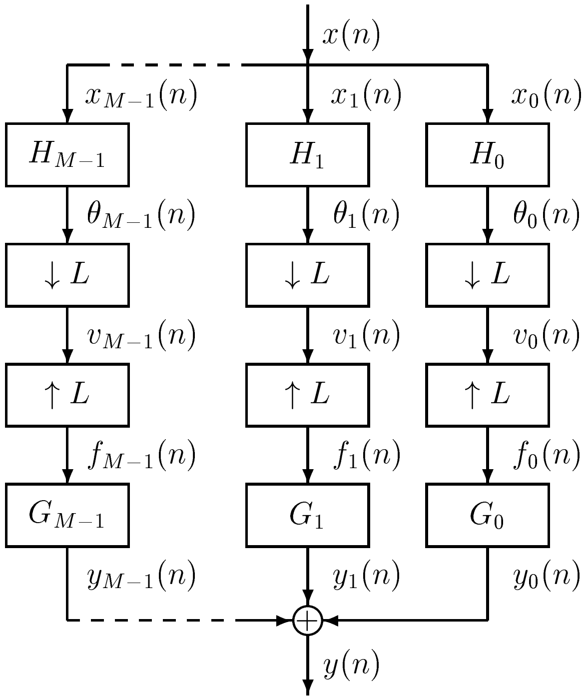


Figure 3.2: General analysis-synthesis filter bank.

perfect reconstruction, as used in image coding [96], the problem with redundancy is solved. It is examined what kind of segmentation can be obtained by operating on sub-sampled frequency channels when using critically sampled filter banks allowing perfect reconstruction of the input signal. By using this approach, tremendous computational savings over full rate approaches can be realized.

In Subsection 3.2.1 the basic theory for multi rate signal processing is reviewed and in Subsection 3.2.2 a local energy function for multi rate texture feature extraction is proposed. Computational characteristics are discussed in Subsection 3.2.3, and experimental results are given in Subsection 3.2.4.

3.2.1 Multi rate signal decomposition

Figure 3.2 shows a general *analysis - synthesis* filter bank. The symbol $\downarrow L$ represents *sub-sampling* by L, while the symbol $\uparrow L$ represents *up-sampling* by L. Sub-sampling by L is the process of discarding all but every L'th sample, while up-sampling by L is the process of inserting $L-1$ zeros between each sample.

The filter bank in Figure 3.2 is *critically sampled* if $L = M$, that is, if the sub-sampling factor is equal to the number of filters in the filter bank. A critically sampled filter bank has *perfect reconstruction* if the output of the synthesis filter bank is equal to the input to the analysis filter bank (with a possible delay). There exists an infinite class of filter banks with perfect reconstruction properties.

It is possible to implement the filtering operations of the filter banks at low sampling

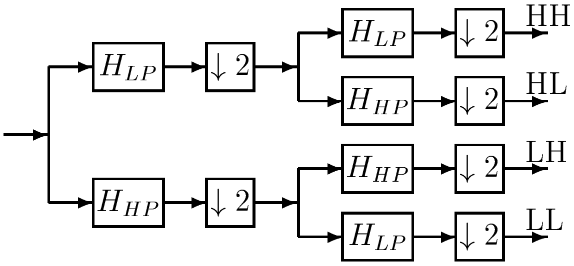


Figure 3.3: Tree-structured filter bank. The nodes correspond to filters and sub-samplers.

rate and thereby achieve big savings in computational complexity [2]. The theory of multi rate filter banks is covered in detail in [2, 133]. The further discussion is restricted to *separable* and *tree-structured* perfect and approximately perfect reconstruction filter banks. A filter bank is separable if it is possible to apply the filters in each dimension separately. A filter bank is tree-structured if it is built up as indicated in Figure 3.3 where each node in the tree is a filter bank.

Two types of filters will be used, *finite impulse response* (FIR) and *infinite impulse response* (IIR) filters. The IIR filters used are based on digital *all-pass* filters, given by

$$H_{LP}(z) = A_0(z^2) + z^{-1}A_1(z^2) \tag{3.1}$$
$$H_{HP}(z) = A_0(z^2) - z^{-1}A_1(z^2), \tag{3.2}$$

where

$$A_i(z) = \frac{a_i + z^{-1}}{1 + a_i z^{-1}}, \tag{3.3}$$

are the first order all-pass filters. First-order filters were empirically found to be sufficient. Two IIR filter bases will be used in this dissertation, these are "F_2_1_smpl" and "F_2_1_09" [46], with $a_0 = 0$ and $a_1 = \frac{1}{2}$, and $a_0 = 0$ and $a_1 = 0.6632$, respectively. See Johnston [53] for the coefficients of the FIR filters.

3.2.2 Multi rate local energy function

In Section 2.1.1, an experimental setup for multi channel filtering for texture segmentation with band-pass filters was presented. To be able to use sub-sampled filter banks successfully, a modification to the smoothing filter in the local energy function is needed. All other parts, except from the filter bank, remain the same.

Assuming no aliasing, when we decimate the signal by a factor N, a sinusoid with period M and frequency $f = 1/M$ will get the period M/N and the frequency $N/M = Nf$.[2] Hence, the smoothing filter designed for a given full rate signal will no further be

[2]Strictly speaking, restrictions for periodicity of discrete signals exist [18]. Particularly, the signals period must be a rational factor of the sampling interval. However, in this context, this issue is of purely theoretical interest.

appropriate for the same signal after sub-sampling. We remember that σ in Equation (2.2) was inversely proportional to the radial frequency f_0. Therefore the equation

$$\sigma = \frac{\sqrt{2}}{2} \frac{1}{N f_0} \tag{3.4}$$

may be used, where N is the sub-sampling factor and f_0 is the absolute value of the center frequency of the filter bank channel.

When a high-pass band of a filter bank is sub-sampled, the relation between signal frequency before and after sub-sampling is not as given above, due to aliasing [133]. However, the corresponding relationships are easily developed.

3.2.3 Computational characteristics

The filter bank basis defined by Equations (3.1), (3.2), and (3.3) has several properties that make it computationally very attractive, especially when subsampling is applied. First of all, the all-pass filters of Equation (3.3) have very few parameters. Furthermore, if subsampling is applied, only the first application of the low-pass and high-pass filters are at the full rate image and subsequent levels have sub-sampled data as input, see Figure 3.3. For example, for the decomposition of Figure 2.6(a) it can easily be shown that for an $M \times N$ image there will be only

$$4MN + 4\frac{M}{2}\frac{N}{2} + 4\frac{M}{4}\frac{N}{4} = 5.25MN \tag{3.5}$$

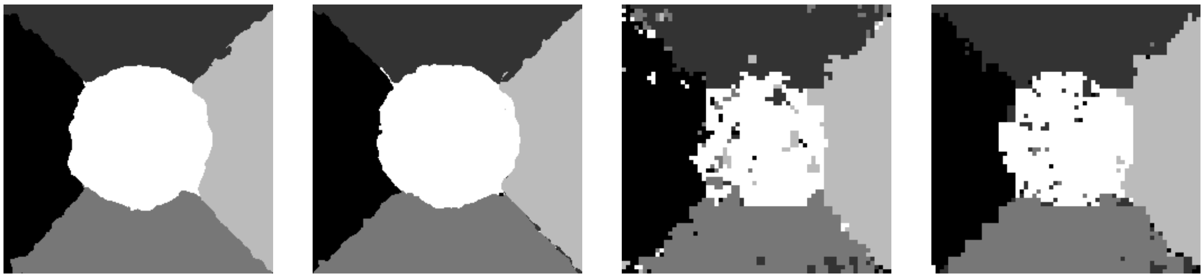
multiplications in the entire filter bank. Furthermore, for the filter basis "F_2_1_smpl", all of these multiplications are with numbers that are powers of two. Hence, these multiplications may be implemented by bitwise shifts.

The filter bases applied in this work give filter banks with good image compression capabilities [46]. If an image being examined is compressed with such a filter bank [96], only parts of the decompression have to be performed to extract the features.

Note that due to the subsampling, the complexity of this approach is significantly less than that of full rate filter approaches. With the decimated filter bank, the Gaussian smoothing filtering in the local energy function is performed on the sub-sampled dataset. It is thereby not very computationally complex. The computational characteristics of the FIR QMF filter banks are not as good as those of the IIR banks. However, with sub-sampling the complexity is still far less than that of full rate filter banks. Last, but not least, using sub-sampling implies classification with significantly reduced feature data sets.

3.2.4 Results

In order to study the effect of sub-sampling with respect to segmentation accuracy, tests with both full rate and sub-sampled filter banks have been performed. Some results are reported here and more results are given in Chapter 6. Segmentation using the perfect reconstruction IIR filter banks from Husøy [46] and the nearly perfect reconstruction FIR



(a) (b) (c) (d)

Figure 3.4: Segmentation results from experiments with the QMF filters. (a) Results from the full rate "F_2_1_09", (b) full rate "f16b", (c) critically sampled "F_2_1_09", and (d) critically sampled "f16b" filters. The test image was the texture mosaic of Figure 1.1(a) and the sub-band decomposition was the one in Figure 2.6(d).

filter banks from Johnston [53] were tested. The squaring with logarithmic second nonlinearity, see Section 2.1.1, is used. Experiments on a large number of combinations of filters, images, and sub-band decompositions have been performed [97]. This study concluded that the FIR filter "f16b" [53] and the IIR filter "F_2_1_09" were quite representative of the different classes of filters.

Examples of the segmentation of the image in Figure 1.1(a) with these filters at full rate are shown in Figures 3.4(a) and 3.4(b). We see that perfect reconstruction filters are suited to texture segmentation when no sub-sampling of the channel filters is applied (full rate). The classification errors were 2.9% in both cases. The LVQ classifier [58] was applied, using 6% of the feature vectors for training.

However, by using full rate filters we do not achieve the full potential savings in computational complexity. We remember that due to perfect or nearly perfect reconstruction properties of these filters, practically all image information is retained in critically sampled sub-bands. Is it possible to use critically sampled filter banks and achieve good segmentation? Figures 3.4(c) and 3.4(d) show the results from two experiments. The classification errors were 10.8% with the "F_2_1_09" filter and 6.2% with the "f16b" filter. We see from the results that the segmentation accuracies in the sub-sampled cases are only slightly degraded relative to the full rate results. Remember that the computational savings are tremendous. For the current test image, we have a feature set of 2 621 440 feature samples in full rate and only 65 536 samples with sub sampling. This has large implications on the computational complexity and the memory requirements.

We see that the segmentation based on critical sampling has a stair-case effect near the edges. Further experiments show, as expected, that the stair-case effect is reduced if we use a lower sub-sampling rate, but at a higher computational cost. With a sampling rate of four times the critical sampling and the filter "F_2_1_09", the classification error was 3.1% with drastically reduced stair-case effect, see Figure 3.5.

Segmentation from sub-sampled filter bank outputs show promising results when the filter banks have the perfect reconstruction property. However, when we are segmenting images, we do not necessarily need the perfect reconstruction property. What if we sub-sample filter banks without the perfect reconstruction property? Experiments [104] show

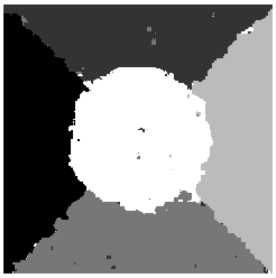


Figure 3.5: Segmentation result from four times critical sampling of the QMF filter "F_2_1_09" with with the texture mosaic of Figure 1.1(a) and the sub-band decomposition of Figure 2.6(d).

that significant degradation may occur if subsampling Gabor filters. Gabor filters do not have the perfect reconstruction property when sub-sampled. This, however, does of course not prove that non-perfect reconstruction filters yielding good segmentation do not exist.

3.3 Summary

In this chapter a new scheme for sub-band decomposition with the Gabor filter has been proposed and tested. Furthermore, the class of QMF filter banks has been introduced for multi channel filtering for texture segmentation. Several QMF and other filter banks have the prefect reconstruction property when sub-sampled, i.e., it is possible to reconstruct the input to the filter bank from the sub-sampled outputs. Using sub-sampling allows considerable savings in computational complexity. A computationally inexpensive scheme for local energy computation and classification using sub-sampling has been proposed in this chapter. The experiments have illustrated that other sub-band decompositions than the commonly used octave band decomposition may be viable. Furthermore, it is shown that it is possible to use critically sampled perfect reconstruction filter banks at the cost of only a modest degradation in segmentation quality.