A SCALING METHOD FOR SPONTANEOUS IMBIBITION IN SYSTEMS WITH DIFFERENT WETTABILITY

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ABSTRACT

Wettability is a dominant parameter governing spontaneous imbibition. However less attention has been paid to the effect of wettability on the scaling of spontaneous imbibition data. A scaling model has been developed for oil-water-rock systems with different wettability based on the fluid flow mechanisms in porous media. Relative permeability, capillary pressure, initial water saturation, and wettability are considered in the scaling model. Theoretically this scaling model is suitable for both cocurrent and countercurrent spontaneous imbibition. The experimental data of countercurrent spontaneous water imbibition at different wettability cannot be scaled using the frequently used scaling model but can be scaled satisfactorily using the scaling model developed in this study. An analytical solution to the relationship between oil recovery and imbibition time for linear spontaneous imbibition has also been derived. The analytical solution predicts a linear correlation between the oil recovery by spontaneous water imbibition and the square root of imbibition time, which has been verified against experimental data.

INTRODUCTION

Recently much attention has been paid to characterizing and scaling spontaneous water imbibition in both gas-liquid-rock \cite{1-3} and oil-water-rock systems. \cite{4-11} Li and Horne \cite{1} developed a model to correlate the imbibition rate and the recovery in gas-liquid-rock systems by considering almost all the parameters influencing spontaneous water imbibition. End-point relative permeability of the water phase and the corresponding capillary pressure can be extracted from spontaneous water imbibition data using this model. Based on this model, a scaling method has been developed for gas-liquid-rock systems with relative permeability, capillary pressure, wettability, and gravity considered \cite{2}. The imbibition model \cite{1} was also used to study the effect of initial water saturation on spontaneous imbibition theoretically \cite{3}.

Li et al. \cite{4} reported an analytical solution for linear countercurrent spontaneous imbibition. The saturation profile and the recovery as a function of time can be calculated according to this solution. A calibration coefficient \(B\) was used to match the experimental data with the theoretical prediction. Tong et al. \cite{5} verified that the spontaneous water imbibition at mixed wettability for recovery of mineral oil of different viscosities could be correlated satisfactorily by the square root of the geometric mean of the oil and water viscosities. Zhou et al. \cite{6} proposed a scaling group of dimensionless time with mobility terms
included. Babadagli\textsuperscript{7} tested the existing capillary imbibition up-scaling formulations in literature and proposed modifications to them for surfactant and polymer injection applications. Kashchiev and Firoozabadi\textsuperscript{8} reported that some of the analytical solutions for the initial stage of countercurrent flow were in good agreement with existing numerical solutions and experimental data for oil recovery from strongly water-wet cores. Civan and Rasmusson\textsuperscript{9} presented a methodology to obtain analytical solutions for imbibition waterfloods in naturally fractured oil reservoirs. 

There are many other papers on scaling the experimental data of spontaneous imbibition in fluid-fluid-rock systems with different size, shape, boundary conditions, permeability, viscosity, and interfacial tension.\textsuperscript{11-22} However it has been a challenge for a long time to scale the experimental data of spontaneous imbibition in oil-water-rock systems with different wettability. This is because wettability was not properly taken into account in the dimensionless time used to scale in some cases\textsuperscript{22} and was excluded in other cases\textsuperscript{10, 13, 23}. To this end, a scaling method was developed for such systems based on the fluid flow mechanisms in porous media. A theoretical derivation was conducted to achieve this scaling model. As is already known, relative permeability and capillary pressure are important parameters that govern spontaneous imbibition in porous media. However the two parameters are only partially considered in some existing scaling methods. For example, capillary pressure is usually considered by using the interfacial tension while another important variable influencing capillary pressure, wettability, is not taken into account. Considering this, a new dimensionless time is defined in this paper in which viscosity, permeability, porosity, initial water saturation, relative permeability, capillary pressure, and wettability are included. The new scaling method was verified using published experimental data\textsuperscript{23} of countercurrent spontaneous imbibition in oil-water-rock systems with wettability altered by using different aging times in crude oil.

THEORETICAL
In this study, we focus mainly on oil-water-rock systems with different wettability. Gravity was neglected, which is valid in many cases. Such cases include low permeability rock, small density difference between oil and water. Assuming Darcy’s Law during the process of spontaneous imbibition in a core with a specific value of initial water saturation (\(S_{wi}\)), the volumetric fluxes of the water and the oil phases in the core sample are expressed as follows:

\begin{align}
\dot{v}_w &= -\frac{k_w \cdot \partial p_w}{\mu_w} \frac{\partial x}{\partial x} \\
\dot{v}_o &= -\frac{k_o \cdot \partial p_o}{\mu_o} \frac{\partial x}{\partial x}
\end{align}

where \(\dot{v}_w\) and \(\dot{v}_o\) are the volumetric fluxes of the water and oil phases; \(k_w\) and \(k_o\) are the effective permeabilities of the water and oil phases; \(\mu_w\) and \(\mu_o\) are the viscosities of the water and oil phases; \(\rho_w\) and \(\rho_o\) the densities of the water and oil phases; \(p_w\) and \(p_o\) are the
pressures of the water and oil phases at the position $x$. From the definition of capillary pressure, the pressure of the water phase can be calculated:

$$p_w = p_o - P_c$$  \hspace{2cm} (2)

where $P_c$ is the capillary pressure. Water phase is assumed to be the wetting phase and oil phase the nonwetting phase.

Substituting Eq. 2 into Eq. 1a:

$$v_w = M_w \left( \frac{\partial p_c}{\partial x} - \frac{\partial p_w}{\partial x} \right)$$  \hspace{2cm} (3)

where

$$M_w = \frac{k_w}{\mu_w}$$  \hspace{2cm} (4)

Eq. 1b could be written as follows:

$$\frac{\partial p_o}{\partial x} = -\frac{v_o}{M_o}$$  \hspace{2cm} (5)

where

$$M_o = \frac{k_o}{\mu_o}$$  \hspace{2cm} (6)

In cocurrent spontaneous imbibition flow, the following equation applies:

$$v_w = v_o$$  \hspace{2cm} (7)

In countercurrent spontaneous imbibition flow, the following equation applies:

$$v_w + v_o = 0$$  \hspace{2cm} (8)

We first consider the cocurrent spontaneous imbibition. Because the flux of the oil phase is equal to that of the water phase, Eq. 5 could be reduced:

$$\frac{\partial p_o}{\partial x} = -\frac{v_w}{M_o}$$  \hspace{2cm} (9)

Substituting Eq. 9 into Eq. 3:

$$v_w = \frac{M_w M_o}{M_o - M_w} \frac{\partial p_c}{\partial x}$$  \hspace{2cm} (10)

We define that:

$$M_e = \frac{k_e}{\mu_e} = \frac{M_w M_o}{M_o - M_w}$$  \hspace{2cm} (11)
where $M_e$ is a coefficient referred as effective mobility, representing the combined effect of the mobilities of both the water and the oil phases on spontaneous imbibition. $k_e$ and $\mu_e$ are the effective permeability and the effective viscosity of the two phases (but considered as one phase) respectively.

It is assumed in this study that the following equation holds:
\[
\frac{\partial P}{\partial x} = \frac{P_c}{x} \tag{12}
\]

One of the cases in which Eq. 12 holds is piston-like spontaneous imbibition. The validity of Eq. 12 has been discussed in more detail in gas-liquid-rock systems by Li and Horne\(^\dagger\). Later we will show that the experimental results in oil-water-rock systems do not contradict this assumption.

Substituting Eqs. 11 and 12 into Eq. 10:
\[
v_w = M_e P_c x \tag{13a}
\]

Assuming that the distribution of $S_{wi}$ in the porous medium is uniform, the cumulative volume of the water phase imbibed into the core with $S_{wi}$ can be calculated as follows:
\[
N_{wt} = A x \phi (S_{wf} - S_{wi}) \tag{13b}
\]

where $N_{wt}$ is the cumulative volume of the water phase imbibed into the core and $A$ is the cross-section area of the core. $S_{wf}$ is the average water saturation behind the imbibition front.

The imbibition rate of the water phase $q_w$ is equal to $A v_w$ in cocurrent spontaneous imbibition. Therefore, Eq. 13a can be expressed as follows:
\[
q_w = \frac{dN_{wt}}{dt} = AM_e \frac{P_c}{x} \tag{14}
\]

Substituting Eq. 13b into Eq. 14:
\[
N_{wt}^2 = 2 P_c M_e \phi (S_{wf} - S_{wi}) A^2 t \tag{15a}
\]

In Eq. 15a, $P_c$ and $M_e$ are the capillary pressure and the effective mobility at $S_{w}$, We define $P^*_c = P_c (S_{wf})$ and $M^*_e = M_e (S_{wf})$ to distinguish them from the capillary pressure and the effective mobility at any other specific water saturation. $M^*_e$ is expressed as follows:
\[
M^*_e = \frac{k_e^*}{\mu_e} = \frac{M^*_w M^*_o}{M^*_o - M^*_w} = k \frac{k_{rw}^* k_{ro}^*}{k_{ro}^* k_{rw}^*} \frac{\mu_w}{\mu_o} \tag{15b}
\]
where $k$ is the absolute permeability of the core. $k_{ro}^*$ and $k_{rw}^*$ are the oil and water phase relative permeability at $S_{wf}$. $M_{o}^*$ and $M_{w}^*$ are the oil and water phase mobility at $S_{wf}$.

Therefore, Eq. 15a is arranged as follows:

$$N_{wt}^2 = 2P_c^* M_e^* \phi(S_{wf} - S_{wi}) A^2 t$$

(16)

Eq. 16 can be expressed as:

$$N_{wt} = c t^{1/2}$$

(17a)

Where $c$ is a constant:

$$c = A\sqrt{2P_c^* M_e^* \phi(S_{wf} - S_{wi})}$$

(17b)

According to Eq. 17a, the oil produced by spontaneous water imbibition is directly proportional to the square root of imbibition time. The theoretical prediction will be tested using the experimental data from Ma et al. Handy showed a similar relationship for gas-liquid-rock systems by neglecting gravity and assuming infinite mobility of the gas phase. Note that the representation of the constant in the Handy equation is different from Eq. 17b. Li and Horne modified the Handy equation by considering the effect of gravity. Dullien summarized some of the studies in this area. According to the Washburn equation, the imbibition rate is inversely proportional to the imbibition height in capillary tubes when gravity is neglected, which implies a linear correlation between the recovery and the square root of time. Chatzis mentioned that the imbibition rate in dry porous media is similar to that in a single capillary tube. However mathematical expressions of the correlation between recovery and time were reported only for a single capillary tube instead of a porous medium.

The recovery in terms of oil originally in place (OOIP) can be calculated based on Eqs. 17a and 17b:

$$R_{OOIP} = \frac{c}{V_p(1 - S_{wi})} t^{1/2}$$

(17c)

The recoverable recovery can be calculated as follows:

$$R_r = \frac{c}{V_p(1 - S_{wi} - S_{or})} t^{1/2}$$

(17d)

here $S_{or}$ is the residual oil saturation.

Define that:

$$t_d = \frac{M_e^* P_c^*}{\phi} \frac{S_{wf} - S_{wi}}{L_a} = \frac{kk_{re}^* P_c^*}{\phi} \frac{S_{wf} - S_{wi}}{L_a}$$

(18)
here \( k_w^* \) is the pseudo relative permeability associated with \( k_\text{ro}^* \) and \( k_\text{rw}^* \). \( t_d \) is the dimensionless time with relative permeability, capillary pressure, and wettability included. In the cocurrent spontaneous imbibition case, \( L_a \) is equal to the core length.

Substituting Eqs. 18 into Eq. 16, the following equation is obtained:

\[
\frac{R dR}{d t_d} = 1
\]  

(19)

Here, \( R \) is the recovery in the units of pore volume. The values of \( S_{wi} \) in all the cases studied in this paper were equal to zero, so \( R \) is equal to \( R_{OOIP} \).

The solution of Eq. 19 is:

\[
R^2 = 2 t_d
\]  

(20)

We can see from Eq. 20 that \( R \) is only a function of the newly defined dimensionless time. This feature shows that experimental data from spontaneous imbibition in rocks with different size, porosity, permeability, initial fluid saturation, interfacial tension, relative permeability, and wettability can be scaled to a single curve of \( R \) vs. \( t_d \).

For countercurrent spontaneous imbibition, the calculation of the effective mobility is different, which is represented as follows:

\[
M_e^* = \frac{k_e^*}{\mu_e} = \frac{M_w^*}{M_o^* + k_w^*} = \frac{k_\text{ro}^* k_\text{rw}^*}{k_\text{ro}^* + k_\text{rw}^*} \frac{\mu_w}{\mu_o} \frac{\mu_o}{\mu_w}
\]  

(21)

The derivation of Eq. 21 is similar to that for cocurrent spontaneous imbibition.

The procedure to scale the spontaneous imbibition using the new method is described briefly in the following. The recovery is first plotted vs. the square root of imbibition time. A straight line is expected from which the value of \( c \) could be obtained from linear regression analysis. Therefore the dimensionless time defined in Eq. 18 could be computed. At last the recovery is plotted vs. the new dimensionless time. According to Eq. 20, the experimental data of spontaneous imbibition in different rocks with different specific properties, including different wettability, is expected to correlate in the form of the recovery vs. the new dimensionless time.

As a comparison, one of the frequently used dimensionless time\(^{18} \) variables is expressed as follows:

\[
t_D = \frac{k}{\phi} \frac{\sigma}{\mu_w L_a^2} t
\]  

(22)
where $t_D$ is the existing dimensionless time, $\mu_m$ is the geometric mean of water and oil viscosities. The limitations for this dimensionless time are that (1) wettability must be the same, (2) relative permeability functions must be identical, (3) capillary pressure functions must be identically proportional to interfacial tension, (4) initial fluid distributions must be duplicated, and (5) gravity must be neglected. We will show later that the first four conditions can be relaxed by using the new dimensionless time (see Eq. 18).

RESULTS

The experimental data from Ma et al.\textsuperscript{23} were used to test the model (Eq. 17a or Eq. 17c) to characterize spontaneous water imbibition into oil-saturated rock and verify the scaling method (Eq. 18) proposed in this study.

Alaskan’93 Crude Oil

The countercurrent spontaneous imbibition data at different aging times, representing different wettability, for Alaskan’93 crude oil from Ma et al.\textsuperscript{23} are plotted in Fig. 1 in the form of oil recovery (OOIP) vs. square root of imbibition time. The wettability in the oil-water-rock system was altered significantly by aging the core sample with crude oil for different lengths of time. Ma et al.\textsuperscript{23} reported the relationship between wettability and aging time. Note that the data at the aging time of 48 hours were very close to those at the aging time of 24 hours and were not used in this study.

For all the wettability states, one can see from Fig. 1 that there is a good linear relationship between the oil recovery (OOIP) and the square root of imbibition time until the water imbibition reaches a critical time ($t_{wf}$). $t_{wf}$ is the time that the imbibition front reached the no-flow boundary. The results in Fig. 1 demonstrate experimentally that the oil recovery is directly proportional to the square root of imbibition time before $t_{wf}$, which is foreseen by the model (Eq. 17c). This also demonstrates the validity of the assumption that the capillary pressure gradient before $t_{wf}$ is equal to $P_c/x$.

Ma et al.\textsuperscript{23} showed that the spontaneous imbibition data for Alaskan'93 crude oil at different wettability could not be scaled using the existing dimensionless time (defined in Eq. 22), as depicted in Fig. 2.

We calculated the values of $c$ according to Eq. 17c using the linear relationship between oil recovery and the square root of imbibition time, as shown in Fig. 1. Then we calculated the dimensionless time defined in Eq. 18. The scaling results for both recovery in terms of OOIP and recoverable recovery are shown in Figs. 3 and 4 respectively. Comparing to Fig. 2 in which the existing dimensionless time (defined in Eq. 22) was used, we can see from Fig. 3 that all the experimental data of the spontaneous imbibition at different wettability are correlated satisfactorily. A good correlation curve of oil recovery vs. dimensionless imbibition time can be obtained from Fig. 3, which shows the success of the proposed scaling model to scale the experimental data of spontaneous
imbibition conducted at different wettability conditions. The scaling results shown in Fig. 4 also demonstrate an acceptable correlation between recoverable oil recovery and the new dimensionless imbibition time.

As is already known, relative permeability and capillary pressure will not be the same once wettability of the oil-water-rock system is altered significantly. Considering this observation and the results in both Fig. 3 and Fig. 4, we can see that the first three limitations of the existing dimensionless time defined in Eq. 22 can be relaxed by using the new dimensionless time defined in Eq. 18. The effect of initial water saturation is also considered in the new dimensionless time. Data are not available to confirm experimentally whether the fourth limitation of the existing dimensionless time can be relaxed or not. However Li and Horne proved the relaxation of initial water saturation constrain in gas-water-rock systems. Therefore we speculate that the fourth limitation of the existing dimensionless time may be relaxed by using the new dimensionless time in oil-water-rock systems.

**Lagrange Crude Oil**

Ma et al. also conducted countercurrent spontaneous water imbibition in a different oil-water-rock system in which the oil phase was Lagrange crude oil with different properties from Alaskan'93 crude oil. The wettability was altered by aging the core samples with Lagrange crude oil for varying lengths of time.

To further confirm the recovery model developed in this study (Eq. 17c), the experimental data (for Lagrange crude oil) of spontaneous water imbibition at different wettability from Ma et al. were plotted in Fig. 5 in the form of oil recovery (OOIP) vs. square root of imbibition time. Fig. 5 shows a good linear relationship between the oil recovery and the square root of imbibition time before \( t_{wf} \) for all the wettability states. This observation further validates the model we developed (see Eq. 17c) to characterize spontaneous water imbibition into oil-saturated rock.

As for the Alaskan'93 crude oil, Ma et al. pointed out that the spontaneous water imbibition data for Lagrange crude oil at different wettability could not be scaled using the existing dimensionless time defined in Eq. 22 (see Fig. 6). This is not surprising because of the limitations of the existing dimensionless time.

Using the linear relationship between oil recovery and the square root of imbibition time (see Fig. 5), the values of \( c \) were calculated according to Eq. 17c. The dimensionless time defined in Eq. 18 could be obtained once the values of \( c \) are available. The relationship between recovery in terms of OOIP is shown in Fig. 7 and the relationship between recoverable recovery is shown in Fig. 8. We can see from Fig. 7 that all the experimental data of the spontaneous water imbibition at different wettability are correlated reasonably except those after \( t_{wf} \).
Fig. 8 demonstrates that the recoverable oil recovery and the new dimensionless imbibition time can also be scaled satisfactorily in the case of Lagrave crude oil. A good correlation curve of oil recovery (recoverable) vs. dimensionless imbibition time can be obtained from Fig. 8. The scaling results in Fig. 8 provides further evidence that the dimensionless imbibition time proposed in this study (see Eq. 18) can scale the experimental data of spontaneous water imbibition conducted at different wettability successfully.

**DISCUSSION**

One of the important features of the proposed scaling method is that the variables, as a group, needed to calculate the dimensionless time can be determined from the experimental data of spontaneous water imbibition. Li and Horne\(^1\) showed that the capillary pressure at \(S_{wf}\) extracted from the experimental data of spontaneous water imbibition into gas saturated rock were equal to those measured using a different approach (an X-ray CT technique). The end-point relative permeability extracted from spontaneous imbibition tests were equal to those measured using a steady-state method.\(^27\)

It may be interesting to compare the relative permeability and capillary pressure extracted from spontaneous water imbibition into oil saturated rock to those measured using other methods. It may also be interesting to calculate the proposed dimensionless time using relative permeability and capillary pressure measured using methods other than spontaneous imbibition techniques.

Gravity is neglected in this paper, which is suitable in the cases studied as shown in Figs. 4 and 8. However gravity may play an important role in some cases and may not be neglected. Such cases include low interfacial tension between oil and water, weak water wetness, and high rock permeability. We have completed a study in oil-water-rock systems in which gravity is included and the results will be presented in another paper.\(^28\)

**CONCLUSIONS**

The following conclusions may be drawn from the present study:

1. An analytical model was developed to characterize spontaneous water imbibition into oil-saturated rock. This model predicts a linear relationship between oil recovery and the square root of imbibition time, which was confirmed against experimental data.

2. A dimensionless time was proposed to scale the experimental data of spontaneous water imbibition into oil-saturated rock at different wettability based on fluid flow theories.

3. Porosity, permeability, initial water saturation, relative permeability, capillary pressure, boundary condition, fluid viscosities, and wettability are included in the proposed dimensionless time.

4. The group of the parameters included in the proposed dimensionless time can be calculated from the spontaneous imbibition data.
5. The proposed scaling method works satisfactorily for the experimental data of spontaneous imbibition in oil-water-rock systems with fifteen different wettability conditions.

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REFERENCES


Fig. 1. Relationship between oil recovery and square root of imbibition time for Alaskan'93 crude oil.

Fig. 2. Scaling results using the existing dimensionless time for Alaskan'93 crude oil.
Fig. 3. Scaling results using the new dimensionless time for Alaskan'93 crude oil (recovery in the units of OOIP).

Fig. 4. Scaling results using the new dimensionless time for Alaskan'93 crude oil (recoverable oil recovery).

Fig. 5. Relationship between oil recovery and square root of time for Lagrave crude.
Fig. 6. Scaling results using the existing dimensionless time for Lagrave crude oil.

Fig. 7. Scaling results using the new dimensionless time for Lagrave crude oil (oil recovery in the units of OOIP).

Fig. 8. Scaling results using the new dimensionless time for Lagrave crude oil (recoverable oil recovery).