FLUID DISTRIBUTION IN TRANSITION ZONES
(Using a New Initial-Residual Saturation Correlation)

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ABSTRACT
The fluid distribution as a function of height in transition zones is often very complex. This may be due to movement of water-oil contact, tilting of the reservoir at some point in time, leak of fluid out of the reservoir zone or complex inflow during secondary migration. The resultant fluid distribution seen in saturation logs may be difficult to model. In this paper we address the changes in fluid distribution versus height, inferred by changes of fluid distribution due to the movement of water-oil contact only.

The experimental procedures for determining capillary pressure are based on fluid saturation monitoring by gamma absorption from centrifuge experiments. An analytical capillary pressure-saturation model was fit to the bounding imbibition capillary pressure-saturation data. The drainage-imbibition hysteresis curves were then constructed assuming that these curves have similar shape to that of the bounding imbibition curve. The imbibition hysteresis model proposed may be used to calculate fluid saturation in the reservoir due to the movement of water-oil contact. We also proposed necessary auxiliary equations to solve the new linear and four-parameter (sigmoidal type) initial-residual fluid saturation equations. Thus once the shape of the bounding-imbibition capillary pressure-saturation curve and maximum non-wetting fluid saturation are known one can easily construct any imbibition hysteresis curves that may be required.

INTRODUCTION
The potential and importance of transition zones are discussed in a number of recent and old papers\textsuperscript{1-5}. Oil transition zone is a zone above the water-oil contact where the water saturation (Sw) is above the irreducible water saturation. In many cases recovery from transition zones are not economical, however for a low permeability reservoir, entire reservoir or a substantial part of it might become a potential target for recovery.

In this paper, we present a new initial (S_{nwi}) -residual (S_{nwr}) non-wetting fluid correlation that can be used to construct imbibition capillary pressure (Pc) hysteresis curves. No attempt has been made to model imbibition relative permeability (kro/krw) curves. However if no data is available, a first estimate can be obtained by using Corey-Burdine\textsuperscript{6} correlation. No data has been collected for secondary drainage. Further capillary pressure hysteresis has been reported\textsuperscript{7-9} to be a closed loop type hysteresis curves. Tan\textsuperscript{10} notes that Killough's Pc hysteresis model\textsuperscript{12} is suitable for specific reservoir types where the imbibition and drainage curves meet at the same residual (non-wetting) fluid saturation. Kleppe et al\textsuperscript{11} criticised the formulation of Killough's model since it is based on Land's\textsuperscript{13} initial-residual non-wetting fluid correlation.
Using limited experimental data and synthetic core, Kleppe et al showed that a linear correlation exists between $S_{nwi}$ and $S_{nwr}$. Masalmeh and Oedai also found piecewise linear-constant correlation between $S_{nwi}$ and $S_{nwr}$ for low permeable carbonate samples. Fanchi et al reported experimental data on 20/40-70/100 mesh glass beads that showed a kind of S-shaped relation between $S_{nwi}$ and $S_{nwr}$. Based on experimental investigation Larsen et al concluded that Land’s $S_{nwi}$-$S_{nwr}$ correlation predicts too high $S_{nwr}$ at low values of $S_{nwi}$. Morrow and Harris also confirmed this and their experimental Pc hysteresis curves suggest that Killough's model might be suitable for unconsolidated sand but most likely will fail for consolidated material.

**THE EXPERIMENTAL PROCEDURES AND DATA PROCESSING**

Using centrifuge (Beckman L8-55/P), high melting point (29°-32°C) paraffin oil (C_{19}H_{40}) and gamma absorption technique similar to Sarwaruddin approach, we have been able to establish, maintain and calculate fluid distribution profile for liquid-wet Berea cores. The standard deviation of each saturation value was estimated to 0.01 saturation fraction after setting $\gamma$-ray counting time to 20 minutes. Note that radial and gravity effect was considered when average Pc was related to the corresponding average saturation.

According to the above procedures, we gathered a set of { $\bar{S}_w$, $\bar{P}_c$} of data points for three samples (Sample ID: Bx3, Bx4 and Bx5). For each sample, a non-linear regression was performed to smooth the data points using a weighting factor $1/\bar{P}_c^2$ and a constraint $S_{wir} \leq \bar{S}_w$. Here $S_{wir}$ is referred as irreducible wetting fluid saturation, while $\bar{S}_w$ as minimum measured average saturation at any cross-section of a given sample. We used Microsoft® Excel's solver function to minimise the $\sum (1- \bar{P}_{c,a}/\bar{P}_c)^2$. Here, $\bar{P}_{c,a}$ is analytical Pc of the following form

$$P_{c,a} = a \left( \frac{S_{w} - S_{wir}}{1-S_{wir}} \right)^{-n}$$

In Eq.3, "a", "n" and $S_{wir}$ are fitting parameters and their values are determined by regression. The best-fit analytical Pc-saturation function was used to estimate initial fluid distribution in the transition zone. The movement of water-oil contact reduces the capillary pressure at every point along the height of the transition zone. The reduction of Pc also changes the fluid saturation that may be obtained from a set of similar but different imbibition Pc-saturation curves (see Fig.1), if the reservoir in question is water-wet. The imbibition Pc curves are also called hysteresis curves and note that each of them satisfies their turning/initial wetting fluid saturation ($S_w$) point.

In this experiment, the movement of water-oil contact is simulated changing the liquid level in the centrifugation cup. After rising the liquid level, the centrifuge is again run at a set rotation. This procedures allows us to collect a large pair of initial-final fluid saturation data {$S_{wi}$,$S_{wr}$} which are basically end point data for the drainage-imbibition Pc hysteresis curves. Note that we regard a point in the sample reached to the residual saturation only if its Pc value reached -5 kPa , which is approximately -10 kPa for an equivalent water-oil system. The bounding imbibition curves were also obtained after establishment of
irreducible liquid saturation ($S_{wir}$) and each of the curves was fit to an imbibition $P_{c}$ model shown below

$$P_{c}^{I}(S_w) = \frac{C_1}{(S_{w} - S_{wi})^{n_1}} - \frac{C_2}{(S_{w2} - S_{w})^{n_2}}; \quad S_{wir} \leq S_{w} \leq (1 - S_{nwr}^{\text{max}})$$ \hspace{1cm} (2)

Here, $P_{c}^{I}(S_w)$ : Imbibition $P_{c}$, a function $S_{w}$, $S_{w}$: Wetting fluid saturation; $C_1$, $C_2$ are positive constants; $n_1,n_2$ are exponents and $S_{w1}$, $S_{w2}$ are two asymptotes. Note that for bounding imbibition $P_{c}$, $S_{w1}\leq S_{wir}$, $S_{wir}$: irreducible wetting fluid saturation and $S_{w2}\geq 1 - S_{nwr}^{\text{max}}$, $S_{nwr}^{\text{max}}$: maximum residual non-wetting fluid saturation.

In order to construct drainage-imbibition $P_{c}$ hysteresis curves, we collected some experimental drainage-imbibition $P_{c}$-saturation data including end point data $\{S_{wi},S_{wr}\}$ sets for the above samples. The imbibition $P_{c}$ model (Eq.2) is again used to fit the data. However, for each drainage-imbibition curve, the asymptotes $\{S_{w1},S_{w2}\}$ were determined honouring the corresponding initial-residual set $\{S_{wi},S_{wr}\}$. Except "$n_2$", the bounding shape parameters i.e. $C_1$, $C_2$ and $n_1$, $n_2$ are assumed to be equal for all other drainage-imbibition hysteresis curves. The exponent "$n_2$" has been used to adjust the shape of the hysteresis curves from that of the bounding imbibition curve. We recommend adjusting $n_2$ as a linear function of $S_{nwi}$. For the three samples that were used in this experiment, the average slope was found 0.43 while the intercept at $S_{nwi}=0$, may be obtained honouring the bounding $n_2$. Note that saturation boundary for the hysteresis curves (Eq.2) are $S_{wi} \leq S_{w} \leq S_{wr}$ although difference between the asymptotes $S_{w1}$, $S_{w2}$ is larger than $S_{wi}$ and $S_{wr}$.

**THE NEW INITIAL-RESIDUAL SATURATION CORRELATION**

Using the experimental procedures discussed earlier, a large number of data set $\{S_{wi},S_{wr}\}$ has been collected for four Berea samples (Bx2,Bx3,Bx4 and Bx5) whose average porosity and permeability are 22.6% and 228 md respectively. The corresponding initial-residual non-wetting fluid saturation ($S_{nwi}$, $S_{nwr}$) is plotted for these samples in Fig2. Linear and sigmoidal equations of the following form are fitted to the data by non-linear regression tool.

$$S_{nwr} = (m)S_{nwi}; \quad S_{nwr} > S_{nwi} \geq 0 \quad \ldots (3a)$$

Where, $m$ and $S_{nwi}^*$ are constants. The slope "m" is estimated to be equal to 0.15 while $S_{nwi}^*$ may be determined solving Eq.3a and Eq.3b. There might have several solutions for $S_{nwi}^*$, however, we suggest taking the minimum $S_{nwi}^*$. In case, no solution is found for $S_{nwi}^*$, one may seek a solution by slightly increasing the value of "m".

$$S_{nwr} = Y_0 + \frac{b}{1 + e^{-(S_{nwi} - S_{nwi}^*)/d}} \quad ; \quad 1 \geq S_{nwi} \geq S_{nwi}^* \quad \ldots (3b)$$

Where, $Y_0$, $b$, $d$ and $S_{nwx}$ are fitting parameters.

Apparently Eq.3b is bit complicated since it involves four parameters which are difficult to obtain. However a closer look to Eq.3b reveals that "d" is simply a scaling parameter.
which ensures that the denominator \([1 + e^{-(S_{nw} - S_{nwo})/d}]\) becomes \(\approx 1\) as \(S_{nw}\) equals to \((1 - S_{wir})\). Therefore, parameter "b" becomes \(S_{max}^{nr}\) the maximum residual non-wetting fluid saturation.

Accordingly "d" may be estimated from a functional relation such that \(d = f_2(b = S_{max}^{nr})\). The other parameters \(S_{nwo}\) and \(Y_0\) may be considered as a rock-fluid property of porous media. Hence it is also expected that \(S_{now}\) and \(Y_0\) may be written as \(S_{nwo} = f_1(S_{max}^{nr})\) and \(Y_0 = Y_0(S_{max}^{nr})\) since \(S_{max}^{nr}\) is a rock-fluid property. Considering the arguments above, equation 3b can be rewritten as

\[
S_{nwr} = Y_o \left(S_{max}^{nr}\right) + \frac{S_{max}^{nr}}{1 + e^{-\left[S_{nwr} - f_1(S_{max}^{nr})\right]/f_2(S_{max}^{nr})}} \quad \cdots (4a)
\]

With the regressed parameters obtained from Fig 2, we found the following functions for \(f_1(S_{max}^{nr})\) and \(f_2(S_{max}^{nr})\) and \(Y_0\).

\[
f_1(S_{max}^{nr}) = 0.7223 S_{max}^{nr} + 0.1815 \quad \cdots (4b)
\]

\[
f_2(S_{max}^{nr}) = 0.096 S_{max}^{nr} + 0.0432 \quad \cdots (4c)
\]

\[
Y_o = 0.0218 S_{max}^{nr} - 0.0014 \quad \cdots (4d)
\]

Now, Eq.3a and 4a can be solved using equation 4b, 4c and 4d if one has knowledge about \(S_{max}^{nr}\) either from experimental bounding imbibition curve or from other sources.

**RESULTS AND DISCUSSION**

The initial fluid distribution in the transition zone was modelled by primary drainage Pc-saturation function. The drainage Pc function was obtained in two stages. In order to capture saturation at low Pc range, we conducted centrifuge experiment at 500 rpm while data was extended to high Pc range during drainage-imbibition experiment at 1000 rpm. We also conducted centrifuge experiment at 2000 rpm to establish irreducible wetting fluid saturation \((S_{wir})\). We fit Pc-saturation data together with the experimentally determined \(S_{wir}\) to analytical Eq.1. The analytical equation was then used as our basis for initial fluid distribution function. One of the analytical drainage Pc function (Bx3) is shown in Fig.3 for demonstration purpose.

Centrifuge experiments were conducted again at 500 rpm after the establishment of \(S_{wir}\) in order to obtain the full bounding imbibition curves (positive as well as negative part) for the three samples (Bx3, Bx4 and Bx5). The imbibition Pc model, Eq.2 was fit to the bounding imbibition Pc-saturation data. The drainage-imbibition experiments provide initial-residual wetting-fluid saturation data. The analytical drainage Pc, bounding imbibition Pc and initial-residual wetting fluid saturation data and some experimental hysteresis data are gathered. However, for page limitation, one sample-Bx3 is drawn in Fig4. The drainage-imbibition hysteresis curves are also shown in Fig4.
Fig. 5 is a comparison between our new initial-residual non-wetting saturation model to the other existing models i.e. from Land\textsuperscript{13}, Kleppe\textsuperscript{11}, Masalmeh\textsuperscript{2} and Jerauld\textsuperscript{23}. In order to compare the different models, we choose the sample Bx3 and its properties i.e. $S_{nwi}^{\text{max}} = 0.2$, $S_{nwt}^{\text{max}} = 0.84$ from the bounding Pc imbibition curve.

**CONCLUSIONS**

A new initial-residual non-wetting-fluid-saturation correlation has been proposed which may be used for calculating fluid saturation distribution in transition zone due to the movement of water-oil contact.

**REFERENCES**

12. Killough, J.E.:" Reservoir Simulation With History-Dependent Saturation Functions", Trans. AIME 261 (1976) 37
22. Leverett, M.C.:" Capillary Behaviour in Porous solids", Trans. AIME (1941) 142, 152-168

**NOMENCLATURE**

**Constants**
- \( a \) = Capillary entry pressure [kPa] (Eq.1)
- \( C_1, C_2, n_1, n_2 \) = Shape parameters (Eq.2)
- \( m \) = slope, \( S_{nw} \) (Eq.3a)
- \( Y_0, b, d \) = Trapping characteristic (Eq.3b)

**Variables**
- \( k_r \) = Relative permeability
- \( S \) = saturation
- \( P \) = Pressure [kPa]
- \( \bar{P} \) = Average Pressure [kPa]
- \( P_{ni} \) = Initial Pressure [kPa]
- \( P_{wi} \) = Water Imbibition Pressure [kPa]

**Subscript:**
- \( a \) = analytical, \( C \) = capillary, \( g \) = gas, \( o \) = oil, \( w \) = wetting, \( nw \) = non-wetting, \( wc \) = connate water, \( wi \) = wetting initial, \( nwI \) = non-wetting initial, \( wr \) = wetting residual, \( nwr \) = non-wetting residual, \( wI \) = asymptote-1, \( asymptote-2 \) (Eq.2).

**Superscript:**
- \( max \) = maximum, \( min \) = minimum; \( I=I \) mb