MODELLING OF THE FULL ENVELOPE OF CAPILLARY PRESSURE CURVES FROM THE CENTRIFUGE

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ABSTRACT
The full envelope of capillary pressure curves consists of primary drainage; the bounding loops of spontaneous and forced imbibition; spontaneous drainage and secondary drainage; and scanning loops in between.

The capillary pressure curve for primary drainage and forced imbibition may routinely be determined by a high-speed centrifuge and well-known interpretation methods. However, even a shift to an inverted bucket for forced imbibition destroys the reversible, natural fluid displacement process and may create artificial, discontinuous step changes in the equilibrium condition of the core.

For the spontaneous imbibition branch of the capillary pressure curve, the measurement procedures and their interpretation may still be categorized as being in a development phase. In this paper it is demonstrated how a validated capillary pressure correlation for mixed-wet rock and an associated hysteresis scheme may provide any natural bounding and scanning loop. With the Free Water Level (FWL) always inside the core, a complete capillary pressure envelope may be generated without demounting the core, simply by stepwise increasing and/or decreasing the centrifuge frequency, while recording the FWL level by a capacitance technique.

At each point along the core, a saturation reversal process (e.g. from drainage to imbibition), will follow an individual scanning curve. For each frequency, the saturations along the scanning curves are calculated by the hysteresis scheme, and they define the FWL. The positive capillary pressure curve is important to estimate transition zones in the reservoir and to model spontaneous imbibition in a fractured reservoir.

The procedure explained in the present paper for a core in a centrifugal field is equivalent to the modelling of contact movement and transition zones in the gravitational field in the reservoir, as previously published in various papers.

INTRODUCTION
Fig. 1a shows a schematic of the bounding curves of water-oil capillary pressure \( p_c = p_o - p_w \) as a function of the water saturation \( S_w \) for a small core plug unaffected by gravity forces (i.e., with uniform saturation). The bounding imbibition curve (b) has a positive, spontaneous branch from the residual water saturation \( S_{wr} \) down to \( S_{w0i} \) followed by a negative, forced imbibition branch.
further down to residual oil saturation $S_{or}$. The bounding secondary drainage curve (c) crosses the zero capillary pressure axis at $S_{wd}$ and forms a closed loop with the bounding imbibition curve. If the imbibition process reverses before reaching $S_{or}$, a closed scanning curve is formed, similar in shape to the bounding loop, [1]. Fig. 1b shows a series of scanning loops spawned from reversal points on the primary drainage curve and closed by a second reversal from their respective residual oil saturation. The bounding loop is the largest possible scanning loop.

The entry capillary pressure into a water-wet rock is denoted by $c_{wd}$, and $c_{oi}$ into oil-wet rock, Fig. 1a. A correlation to model the bounding curves is suggested by Skjaelveland et al. [1],

$$
P_c = \frac{c_w}{S_w - S_{wr}} a_w + \frac{c_o}{1 - S_{or}} a_o,
$$

The $a$’s and $c$’s are constants and there is one set for imbibition and another for drainage. An imbibition curve from $S_{wr}$ to $S_{or}$ is modelled by Eq. 1a and the four constants ($a_{wi}$, $a_{oi}$, $c_{wi}$, $c_{oi}$), and a secondary drainage curve from $S_{or}$ to $S_{wr}$ by the constants ($a_{wd}$, $a_{od}$, $c_{wd}$, $c_{od}$). Furthermore, there are the following relations between them,

$$
c_{oi} = -\frac{c_{wi} \left(1 - S_{wi} - S_{or}\right) a_{wi}}{1 - S_{or}},
$$

$$
c_{od} = -\frac{c_{wd} \left(1 - S_{wd} - S_{or}\right) a_{wd}}{1 - S_{or}}.
$$

To estimate residual oil saturation dependent on the reversal saturation, we apply Land’s [2] expression,
\[
\frac{1}{S_{or}[1]} - \frac{1}{S_{w}[1]} = C,
\]
where \(S_{w}[1]\) is the start saturation of an imbibition process beginning on the primary drainage curve, and \(S_{or}[1]\) is the end saturation of the imbibition process, and \(C\) is Land’s trapping constant.

**Procedure**

Let \(S_{w}[k]\) denote the water saturation at the \(k\)'th reversal and let \([k]\) also label properties of the scanning curve after the \(k\)'th reversal with the convention that odd numbers denote imbibition and even numbers drainage. The asymptotes \(S_{wr}[k]\) and \(S_{or}[k]\) are used as scanning loop residual saturations and \(S_{wr}\) and \(S_{or}\) as the bounding loop residual saturations.

**First Reversal.** The historic two-phase flooding process of a reservoir rock sample is usually primary drainage from \(S_{w} = 1\). The first reversal will then be modelled by an imbibition curve \(p_{ci}[1]\) originating from the primary drainage curve \(p_{cd}[0]\) which is given by

\[
p_{cd}[0](S_{w}) = \frac{c_{wd}}{S_{w} - S_{wr}}a_{wd}, \tag{1b}
\]

The first reversal saturation \(S_{w}[1]\) is a point on both the \(p_{cd}[0](S_{w})\) and the \(p_{ci}[1](S_{w})\) graphs, that is

\[
p_{cd}[0](S_{w}[1]) = p_{ci}[1](S_{w}[1]), \tag{4a}
\]

and where explicitly, on the primary drainage curve,

\[
p_{cd}[0](S_{w}[1]) = \frac{c_{wd}}{S_{w}[1] - S_{wr}}a_{wd}, \tag{4b}
\]

and on the scanning imbibition curve,

\[
p_{ci}[1](S_{w}[1]) = \frac{c_{wi}}{S_{w}[1] - S_{wr}[1]}a_{wi} + \frac{c_{wi}}{S_{w}[1] - S_{or}[1]}a_{oi}. \tag{4c}
\]

To honor Eq. 4a, we have to choose which parameter to adjust. Considering the design constraints by Skjæveland et al. [1], we keep all \(a\)'s and \(c\)'s constant for the given rock-fluid system and adjust the water branch asymptote of the scanning curve, i.e., \(S_{wr}[1]\). The oil branch asymptote, \(S_{or}[1]\), is given by Land’s expression, Eq. 3. With this choice, the scanning curve will have a similar shape as the bounding imbibition curve since \((a_{wi}, a_{oj}, c_{wi}, c_{oi})\) are the same. The scanning curve may be considered a compression of the bounding imbibition curve between the two new asymptotes \(S_{wr}[1]\) and \(S_{or}[1]\).

In summary, the first reversal imbibition curve \(p_{ci}[1](S_{w})\) starts on the primary drainage curve \(p_{cd}[0](S_{w})\) at the reversal point \(S_{w}[1]\) and scans down towards the asymptote \(S_{or}[1]\), as shown in Fig. 1b for three values of \(S_{w}[1]\). The secondary drainage graphs curving back from the \(S_{or}\)'s are also shown, forming closed scanning loops that continue along the primary drainage curve. Also shown in the figure is the bounding imbibition curve from \(S_{wr}\), the minimum value of the \(S_{w}[1]\)'s,
back to $S_{or}$. All curves in the figure are produced with a constant set of parameters. Only the residual saturations (asymptotes) are varied.

To simulate the centrifuge experiment we also need the equation expressing the centripetal force,

$$p_c(r) = \frac{1}{2} \Delta \rho \omega^2 (r_2^2 - r_1^2),$$

where $r_1 < r < r_2$, and $r_1$ is the radius from the rotational axis to the top of the core and $r_2$ is the distance to the FWL which is assumed to be inside the core, i.e., the displaced fluid during drainage (water) is is at all times in contact with the expelled or imbibing water. In Eq. 5, the units are $p_c[\text{Pa}]$, fluid density difference $\Delta \rho[\text{kg/m}^3]$, angular frequency $\omega[\text{rad/s}] = \text{RPM} \times \frac{2\pi}{60}$, $r[\text{m}]$. Please note that this is not the usual centrifuge equation which is referenced to the top of the core.

**Spontaneous imbibition.** Primary drainage starts with the core fully saturated with water and the frequency is increased stepwise. In equilibrium at each step, the water saturations along the core is determined by Eq. 5 in combination with Eq. 4b. A subsequent frequency decrease will spawn scanning curves emanating from each position $r$ along the primary drainage curve with reversal saturations $S_{wr}[1](r)$ determined by Eq. 4a and modelled by Eq. 4c.

**General scanning loop logic.** If the process experiences a $k$-th saturation reversal while already moving along scanning curve, the asymptotes both at the start and end of the loop, $S_{wr}[k]$ and $S_{or}[k]$, are determined by closure of the scanning loops, Skjaeveland et al. [1]. By this loop logic, it is possible to model compounded centrifuge sequences and possibly estimate Land’s trapping constant and residual oil saturation.

**MODELLING EXAMPLES**

The correlation, Eq. 1a, was programmed in Maple and an example was constructed with the parameter values called ‘LAB’ set in Table 1, and with $c_{oi} = -8.56$ and $c_{od} = -6.10$ from Eqs. (2a, 2b). We also choose $r_1 = 0.0446$ m, $r_2 = 0.0938$ m, $\Delta \rho = 223$ kg/m$^3$. The parameters are shifted arbitrarily to higher values in Case 1 and to lower values in Case 2, Table 1. The parameters iterated on to model spontaneous imbibition are those shown in Table 2.

| Table 1 ‘LAB’ parameter set to create synthetic ‘laboratory’ saturation profiles and shifted parameter example sets for Case 1 and 2. |
|---|---|---|---|
| Parameter | Units | ‘LAB’ | Case 1 | Case 2 |
| $a_{wi}$ | [-] | 0.25 | 0.30 | 0.20 |
| $a_{oi}$ | [-] | 0.50 | 0.60 | 0.40 |
| $a_{wd}$ | [-] | 0.25 | 0.30 | 0.20 |
| $a_{od}$ | [-] | 0.50 | 0.60 | 0.40 |
| $c_{wi}$ | [kPa] | 10.00 | 11.00 | 8.00 |
| $c_{wd}$ | [kPa] | 7.00 | 8.00 | 5.00 |
| $S_{or}$ | [-] | 0.10 | 0.13 | 0.10 |
| $S_{wr}$ | [-] | 0.21 | 0.28 | 0.18 |
| $S_{w0i}$ | [-] | 0.50 | 0.50 | 0.50 |
| $S_{w0d}$ | [-] | 0.70 | 0.70 | 0.70 |
We now assume that it is possible to measure the saturation profile along the core for each step in imbibition frequency, for example by the method suggested by Spinler and Baldwin [3]. The ‘LAB’ dataset is used to generate synthetic ‘laboratory’ profiles and Case 1 and 2 show two example sets of startpoint parameters for automated adjustment of the correlation parameters back to the ‘LAB’ set.

Figures 2a and 2b show the saturation profile along the core for series of decreasing frequencies during the imbibition sequence. The fully drawn curves represent synthetic ‘laboratory’ data and the point-marked lines called ‘composed’ profiles are the results from automatic adjustment of the correlation parameters in Case 1 and 2 to make a fit to the ‘laboratory’ data. The radii are $r_1 = 0.0446$ m and $r_2 = 0.0938$ m. In this case the free water level has to be considered inside the core and $r_2$ is set to equal to the radius of the free water level, chosen at $r = 0.093$ m.

![Saturation Profile - Case 1](image)

**Figure 2a** Composed (adjusted) saturation profiles, Case 1 and those from the synthetic ‘lab’ data. Water saturation as a function of radial position along the core for 500 to 4500 rotations per minute. The curves for each speed are displayed in the same color. The thicker lines with no points represent the ‘lab’ data and the lines with dots the composed data.
There is good agreement between the profiles generate by the ‘lab’ data and the ‘composed’ profiles.

The optimization package Microsoft Excel Solver is used to adjust the values of the parameters of the capillary pressure correlation to make it fit the the ‘laboratory’ data. The value of the capillary pressures along the core is known from Eq. 5. The corresponding saturations $S_w$ are given by $p_c[1](S_w)$, Eq. 4c. To optimize the parameters of Eq. 1a, Eq. 4c needs to be solved with respect to water saturation. This is done in an Excel spreadsheet by the Visual Basic code in Appendix A.1. An error is calculated as the sum of the difference squared between the saturations from the adjusted correlation and the ‘laboratory’ data. And the Solver is invoked to minimize this error.

Figure 3 demonstrates how the invoked Solver adjusts the correlation to the artificial ‘laboratory’ data by minimizing the error, for a selected frequency.
DISCUSSION
Several papers suggest procedures to measure the positive part of the imbibition curve by centrifuge.

Torsaeter [4] presented interpreted centrifuge experiments going stepwise up and down in rotational frequency without dismounting. The core was at all times in contact with the displaced water and the varying FWL was recorded along the core. The same equation was used to interpret drainage and imbibition mode results to render the saturation at the inlet end \( r_1 \) from the measurements of the average water saturation. However, inclusion of hysteresis makes the average saturation also dependent on the reversal saturations along the primary drainage curve, not just the frequency (or \( p_c(r_1) \)).

Spinler and Baldwin [3] suggest a generalized chronological sequence to produce complete capillary pressure scanning curves. First they centrifuge with a FWL contacting the plug to produce the primary drainage curve. Then the oil phase (octodecane) was frozen by lowering the temperature from 27 deg Celcius to ambient 23 deg and saturation distribution along the core was recorded by magnetic resonance imaging. Then core holder was inverted and the plug centrifuged in the non-wetting fluid. The procedure was repeated to reach a relatively uniform saturation distribution throughout the plug to prepare for imbibition. However, this preparatory procedure may destroy the natural history of saturation change in the reservoir and the fluid pressure continuity.

Preconditioning the plug to uniform saturation as an initial condition for the imbibition process may be questionable since the whole transition zone in the reservoir is compressed into the length of the plug and saturations along the plug depend on both the capillary pressure through the
centripetal force equation, Eq. 5, and the reversal saturation along the primary drainage curve and its associated trapped residual oil saturation predicted by Land’s equation, Eq. 3.

Fleury et al. [5] solve the problem of proper conditioning plug for imbibition by artificially moving the FWL outside the plug, keeping it at constant position by a capacitance-based level detector combined with a Pumping While Centrifuging system and a PID controller. The imbibition starts from a truly uniform saturation equal to the residual (irreducible) water saturation $S_{wr}$. All reversal saturations are equal to this value and the residual (trapped) oil saturations $S_{or}$ are independent of their position along the core.

The approach of Fleury et al. [5] is a consistent, well-defined procedure for measuring the positive imbibition curve. It is limited, however, to bounding curves without any measure of hysteresis effects and scanning curves and loops. Fig. 4 shows capillary pressure imbibition curves resulting from 51 points along the core.

Both for Torsaeter’s [4] and Fleury et al.’s [5] methods the displaced fluid during drainage (water) is at all times in contact with the expelled or imbibing fluid (water) as required by the method suggested in the present paper. However, the scanning curves originating from each position along the primary draining curve, when the frequency is reduced, should also be considered.

![Capillary Pressure Imbibition Curve](image.png)

**Fig. 4 –** Single (blue) capillary imbibition curve from an initial uniform water saturation profile equal to residual water saturation $S_{wr}$ along the core at the end of the primary drainage. FWL is placed outside the core. Capillary pressure as a function of water saturation.

The optimization procedure applies an Excel sheet in combination with the Solver and the VB-program in the Appendix in the following loop:

1. Estimate a set of parameters of the capillary pressure correlation.
2. Calculate the capillary pressure from Eq. 5 for a selected series of radii.
3. Use the VisualBasic-program to find the corresponding saturation from $p_{ci}[1](S_w)$, Eq. 4c.
4. Calculate an error, (i.e. sum of squared differences) between the observed saturations f.ex. by the method of Spinler and Baldwin [3] and the calculated saturations.
5. Minimize the error by letting the Solver find a better estimate of the parameters, and return to 2. above.

There are several choices to be made in selecting the error to be minimized:

1. The number and spacing of the positions to calculate the saturation values
2. The error function could be constructed by summing all the imbibition centrifuge speeds, as shown in Table 2.
3. The FWL could be recorded as suggested by Fleury et al. [5], and the error function set equal to the summed differences between measured and calculated average saturations for each rotational speed.

<table>
<thead>
<tr>
<th>Avg. optimized</th>
<th>‘LAB’ data (given)</th>
<th>Case 1 results</th>
<th>Case 2 results</th>
</tr>
</thead>
<tbody>
<tr>
<td>parameter</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>(S_{wr})</td>
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<td>0.20</td>
<td>0.23</td>
</tr>
<tr>
<td>(S_{sr})</td>
<td>0.10</td>
<td>0.12</td>
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<td>0.25</td>
<td>0.21</td>
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</tr>
<tr>
<td>(a_{oi})</td>
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<td>0.57</td>
<td>0.43</td>
</tr>
<tr>
<td>(c_{wi})</td>
<td>10.0</td>
<td>10.99</td>
<td>7.99</td>
</tr>
<tr>
<td>(S_{w0i})</td>
<td>0.50</td>
<td>0.61</td>
<td>0.66</td>
</tr>
</tbody>
</table>

The general hysteresis loop logic presented by Skjaeveland et al. [1] makes it possible to go up and down in centrifuge speed to create a suite of closed scanning loops. This may increase the possibility of estimating correlation parameters more accurately, for example Land’s constant and the residual oil saturation.

**CONCLUSIONS**

The positive imbibition process of a centrifuge experiment is in general a composite result from a infinite number of hysteresis curves or branches spawned from the water saturation profile prevailing at the end of the primary drainage process, except if the FWL artificially is placed outside the core and the whole core is at residual water saturation \(S_{wr}\) when imbibition starts. In this case it would not be possible to determine Land’s constant from the scanning curves.

Specifically, a capillary pressure correlation for mixed-wet rock is demonstrated to model the centrifuge positive capillary imbibition process by stepwise reduction in frequency and a VisualBasic macro to facilitate the use of the Microsoft Excel Solver to adjust the parameters of the correlation to fit the observations.
REFERENCES


Appendix

A.1 Visual Basic Code ComputeS for Excel

Function ComputeS(Pc As Double) As Double
'Compute saturation S given value Pc of capillary pressure
'Iterative method: bisecting interval

Dim S As Double
Dim i As Integer
Dim Err As Double

'Input Parameters for computing Pc, specified in Data sheet
Swr = Worksheets("Data").Cells(4, 3).Value
aw = Worksheets("Data").Cells(5, 3).Value
cw = Worksheets("Data").Cells(6, 3).Value
Sor = Worksheets("Data").Cells(7, 3).Value
ao = Worksheets("Data").Cells(8, 3).Value
S1 = Worksheets("Data").Cells(9, 3).Value

'S1 = S_w0, replaces co as input paramater, S1 > 0.
\[ S11 = \frac{(S1 + Sor - 1)}{(Sor - 1)} \]
\[ S22 = \frac{(Swr - S1)}{(Swr - 1)} \]

\[ co = -cw \ast (S11 ^ ao) \ast (S22) ^ (-aw) \]

Parameters for iteration procedure (bisecting interval)

\[ tol = \text{Worksheets}("Data").Cells(13, 3).Value \]
\[ maxit = \text{Worksheets}("Data").Cells(14, 3).Value \]

'Start values for iterations

\[ Smin = Swr \]
\[ Smax = 1 - Sor \]

'Start iterations

\[ i = 0 \]
\[ \text{Do While } i < \text{maxit} \]
\[ i = i + 1 \]

\[ S = 0.5 \ast (Smax + Smin) \]

\[ SS1 = \frac{(S - Swr)}{(1 - Swr)} \]
\[ SS2 = \frac{(1 - S - Sor)}{(1 - Sor)} \]

\[ \text{ComputedPc} = cw \ast SS1 ^ (-aw) + co \ast SS2 ^ (-ao) \]

\[ Err = Pc - \text{ComputedPc} \]

If \( Err < 0 \) Then
\[ DErr = -Err \]
Else
\[ DErr = Err \]
End If

If DErr > tol Then

If \( Err < 0 \) Then
\[ Smin = S \]
Else
\[ Smax = S \]
End If

Else

\[ \text{ComputeS} = S \]
\[ \text{Exit Do} \]

End If
Loop

' If i > maxit - 1 Then
'    MsgBox "NB!! no convergence"
' End If

End Function