Abstract
The measurement of three-phase displacement involves varying two independent saturations. Therefore, recording the relative permeability for all possible three-phase displacement combinations in the reservoir becomes impractical. A common practice utilizes two-phase data to estimate three-phase relative permeability in the reservoir simulator. Most three-phase oil relative permeability models used in commercial simulators do not contain the flexibility of tuning with measured experimental data. In this paper, a new empirical three-phase oil relative permeability model is proposed to offer a tuning flexibility to improve the simulator prediction. The new model is based on saturation-weighted interpolation between the two-phase values, similar to the Baker model, but with a simple tuning coefficient that allows adjustment to match the experimental measurements.

The paper discusses factors that are essential for accurate reservoir simulation modeling. The presented experimental data shows the inter-relationship between phase relative permeability, trapped gas and residual oil saturation. Also, a comparison of this new three-phase oil relative permeability model with standard industry models is given.

Introduction
The laboratory measurements of three-phase flow experiments are costly and time consuming, which have limited the number of these experiments actually reported. In most reservoir engineering calculations, particularly in reservoir simulation code, the processes involving three-phase flow are approximated by the three-phase relative permeability models or correlations based upon two-phase oil-water, oil-gas and/or water-gas relative permeability laboratory measurements.

Three-phase permeability core flooding data exist for some of the giant carbonate fields in Saudi Arabia. Due to unreasonable prediction results provided by several three-phase reservoir simulation studies in Saudi Aramco, a further investigation of the industry standard three-phase relative permeability models based on two-phase data was performed. This work concluded that none of the available industry standard correlations provided three-phase results close to laboratory results measured on the Saudi Aramco core samples. As a result, an alternative model was developed to provide three-phase relative permeability curves that closely match the available three-phase relative permeability data; thus, providing significantly improved reservoir simulation predictions in three-phase flow situations.

Review of Three-Phase Oil Relative Permeability Correlations used in Commercial Simulators
The most commonly used three-phase oil relative permeability correlations are the Stone$^{1,2}$ models. The Stone I model was introduced in 1970 based on the channel flow theory that assumes in any flow channel there is mostly one mobile fluid. The water and gas relative permeability will be a function of its own saturation. The oil relative permeability depends on the two-phase oil / water and gas/oil relative permeability corresponding to the effective water and gas saturations:

\[
k_{ro} = \frac{k_{rew}k_{rog}k_{row}}{k_{ro,max}(1-S_{we})(1-S_{ge})}
\]  

(1)

The Stone I model effective phase saturations ($S_{oc}$, $S_{ge}$ and $S_{we}$) are calculated from the following equations:
A correlation was proposed by Fayers and Matthews\textsuperscript{3} to estimate residual oil saturation after water and gas flood:

\[ S_{om} = \lambda S_{orw} + (1 - \lambda)S_{org} \]  
(5)

\[ S_{om} = S_{orw} - 0.5S_g \quad \text{(with trapped gas)} \]  
(6)

\[ \lambda = \frac{S_g}{1 - S_{wc} - S_{org}} \]  
(7)

Due to the difficulty in estimating the three-phase residual oil saturation (\(S_{om}\)), the modified Stone II model by Aziz and Settari\textsuperscript{4} is more commonly used in the industry:

\[ k_{ro} = (k_{row} + k_{ro max} k_{rw})(k_{rog} + k_{ro max} k_{rg}) - k_{ro max} (k_{rw} + k_{rg}) \]  
(8)

A saturation-weighted interpolation between oil-water and oil-gas two-phase relative permeability data was proposed by Baker (1988)\textsuperscript{5}. It assumes that the end-points of two-phase data are the same as in the three-phase system. The Segregated model in Eclipse\textsuperscript{TM} and Line model in Chears\textsuperscript{TM} are also variations of this saturation-weighted approach:

\[ \begin{align*}
    k_{ro} &= \frac{(S_w - S_{wc})k_{row} + (S_g - S_{gr})k_{rog}}{(S_w - S_{wc}) + (S_g - S_{gr})} \\
    k_{row} &= \frac{S_{ orgas}}{S_{org}} k_{row} \\
    k_{rog} &= \frac{S_{org}}{S_{org}} k_{rog}
\end{align*} \]  
(9)

In these models, \(k_{row}\) and \(k_{rog}\) are calculated as a function of the grid cell oil saturations.

Heiba\textsuperscript{6} suggested the Line model where the oil iso-permeabilities can be approximated by straight lines connecting between the two-phase relative permeability at irreducible water saturation. The gas and water saturations would be proportional to the gas and water saturations at the three-phase oil relative permeability. The \(S_g(k_{ro})\) and \(S_w(k_{ro})\) are found by iteration to satisfy the proportional function.

**Proposed Modified Baker Model**

When the Baker correlation was used to estimate the three-phase oil relative permeability in this study, it was found that its estimates were lower than \(k_{rog}\) and the laboratory measured \(k_{ro}\). In order to attain a higher \(k_{ro}\), the \(k_{rog}\) was evaluated at \(S_g\) rather than \(S_o\) and \(k_{rog}\) is evaluated at oil saturation. In addition, it is proposed to introduce a matching parameter (\(\alpha\)) to match the laboratory data. As suggested by Blunt\textsuperscript{7}, the weighting function of \(k_{rog}\) is also set to decrease at the second order of three-phase mobile oil saturation (\(S_{om}\)). If it is not determined experimentally, \(S_{om}\) interpolates linearly between \(S_{orw}\) and \(S_{org}\). The following modified Baker correlation is proposed:

\[ k_{ro} = \frac{(S_w - S_{wc})k_{row} + S_{om}^2 \alpha(S_g - S_{gr})k_{rog}}{(S_w - S_{wc}) + S_{om}^2 \alpha(S_g - S_{gr})} \]  
(10)

The mobile oil saturation (\(S_{on}\)) is calculated by the relationship:

\[ S_{on} = \frac{(S_g - S_{om})}{(1 - S_{wc} - S_g - S_{om})} \]  
(11)

The three-phase residual oil saturation (\(S_{om}\)) used for calculating \(S_{on}\) is determined by the expression:

\[ S_{om} = \frac{S_{orw} + S_g S_{org}}{S_w + S_g} \]  
(12)
Testing of New Modified Baker Model with Published Data

The data sets of Donaldson\cite{5} / Dean and Oak\cite{6} were selected for the testing of the proposed modified Baker correlation. The comparisons and iso-permeabilities were reported by Delshad\cite{7} and Pejic\cite{8}, respectively. The error of deviation was calculated according to the following equation:

\[ \sum_{i=1}^{n} \left( k_{r\exp,i} - k_{r\text{calc},i} \right)^2 \]  

(13)

The modified Baker correlation performs better than all the other four correlations mentioned above as shown in Table 1. The three-phase iso-permeabilities are shown in Figures 1 and 2.

Three-Phase Relative Permeability Flow Effects

Core laboratory experiments were conducted to obtain three-phase relative permeability data. The experimental data indicates four major flow effects that should be considered in order to realistically simulate oil and gas recovery processes. Those are:

(a) gas relative permeability hysteresis, (b) effect of trapped gas saturation on residual oil saturation, (c) oil relative permeability in presence of trapped gas, and (d) water relative permeability in the presence of trapped gas.

(a) Gas Relative Permeability Hysteresis

The injected gas in the oil drainage cycle displaced the original oil leaving irreducible oil saturation after gas flooding (S_{org}) in the laboratory core samples. Subsequently, an oil imbibition cycle will take place as oil displaces the gas. The laboratory experiments show that a trapped gas saturation remains in the core plugs following the oil imbibition cycle. This trapped gas needs to be modeled by a gas hysteresis correlation to simulate the reduction in gas movement and recovery (Figure 3).

Gas drainage relative permeability data from three gas flood experiments were fitted with equations, as shown in Figure 4. Gas imbibition relative permeability was derived based on the conceptual hysteresis model by Carlson\cite{9} since no experimental measurements are available. Carlson’s model assumes that the total gas in the core during displacement of gas by oil can be separated into immobile (trapped) gas and mobile free gas.

Land\cite{10,11} empirically demonstrated a relationship between the maximum non-wetting phase (gas) saturation and the value of the trapped gas saturation. Based on the experimental data from this study, a trapped gas model was developed (S_{gt} = 0.4 S_{max}), as shown in Figure 5. The drainage curve at free flowing gas saturation can be utilized to generate the imbibition boundary curve. Most commercial simulators possess the option to model gas hysteresis with drainage and imbibition curves.

(b) Effect of Trapped Gas on Residual Oil Saturation

The residual oil saturation is less in the presence of trapped gas than in the absence of trapped gas as observed in ten waterflood experiments. The reduction can be correlated by the equation \[ \Delta S_{or} = 0.48 S_{gt} \] (Figure 6).

In the three-phase laboratory experiments, it was observed that residual oil saturation in the presence of trapped gas following waterflooding was higher than residual oil saturation after gas displacement (S_{org}). Both these residual oil saturations were less than residual oil saturation (S_{orw}) after waterflooding the oil column without introduction of gas.

(c) Oil Relative Permeability in Presence of Trapped Gas

Laboratory data indicates that the presence of trapped gas usually increases the oil relative permeability, which can positively impact oil mobility, but it may not increase the ultimate oil recovery (Figure 7). Oil relative permeabilities by waterflooding with trapped gas were measured under reservoir and pseudo reservoir conditions.

The effect of hysteresis on the oil relative permeability is shown in Figure 8. Both oil drainage (Figure 9) and imbibition (Figure 10) relative permeabilities can be determined from laboratory measurements from gas flood / centrifuge and gas displacements by oil. The oil-gas hysteresis can be modeled with drainage and imbibition k_{rog} curves.

(d) Water Relative Permeability in the Presence of Trapped Gas

The experimental data presented in Figure 11 shows a consistent trend in the reduction of relative permeability to water by waterflooding in the presence of trapped gas. Laboratory measurements from four composite cores under reservoir conditions using reservoir fluids indicate that the water relative permeability depends on trapped gas saturation as well as water saturation. The presence of trapped gas significantly reduces water relative permeability, which positively impacts oil recovery by slowing down the water front velocity. The water relative permeability (k_{rw}) with presence of trapped gas can be modeled by shifting S_{gt} in the normalized water saturation (S_{wn}):

\[ S_{wn} = \frac{S_w - S_{wc} - A S_{gt}}{1 - S_{wc} - S_{orw}} \]  

(14)

It is obvious that three-phase relative permeability correlations are needed to simulate these complex recovery processes. The accuracy of the residual oil saturation modeled in the reservoir simulator strongly affects the ultimate oil recovery.
Testing of Oil Three-Phase Relative Permeability Correlations with Recent Laboratory Data

The recent laboratory measurements of three-phase oil relative permeability under reservoir conditions with trapped gas saturation of 0.181 were used to calibrate the three-phase relative permeability correlations. The three-phase oil relative permeability starts from two-phase oil-gas \( k_{rog} \) (Figure 12) and terminates at the model calculated residual oil saturation. The Stone I, Segregated, Baker, and Line correlations calculate very similar oil relative permeabilities; however, these correlations do not match the available experimental data. The modified Baker correlation introduces a matching parameter that can be used to obtain a reasonable match with the laboratory data, as demonstrated in Figure 12.

The Stone II algorithm provided a high residual oil saturation of 0.56 in presence of the trapped gas saturation of 0.181, as depicted in Figure 13. Consequently, at oil saturations lower than 0.56 in the oil flooded gas area, the oil will be immobile in the reservoir simulator. Oil will move and accumulate near the gas area to the threshold oil saturation of 0.56. After that oil can start to move into the gas flooded area again. The Stone II model yields pessimistic and unrealistic simulation results.

The Baker and Segregated models use the minimum of either the residual oil saturation after gas flood \( S_{org} \) or residual oil saturation after waterflood \( S_{orw} \). The Line model residual oil saturation is a linear interpretation of \( S_{org} \) and \( S_{orw} \). The residual oil saturation \( S_{om} \) in Stone I and modified Baker can match either the laboratory measurements or as a weighted function of \( S_{org} \) and \( S_{orw} \). Figure 13 presents a comparison of the residual oil saturation \( S_{om} \) at trapped gas saturation \( S_{gt} \) of 0.181 for the different three-phase models.

Conclusions

1) In three-phase recovery processes, four major relative permeability effects are identified. The two most important of these four factors are three-phase oil relative permeability and the three-phase residual oil saturation.

2) The proposed modified Baker model can be calibrated with laboratory measurements and it also yields better predictability than available three-phase oil relative permeability correlations in commercial reservoir simulators.

3) In this study, the Stone II three-phase relative permeability correlation produced unreasonable and unreliable residual oil saturations.

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Symbols, Abbreviations and Units

- \( A \): tuning parameter for \( krw \) with trapped gas to match laboratory data, \( A = 0.0 \sim 1.0 \) (dimensionless)
- \( k_{rg} \): two-phase gas relative permeability in oil/gas (dimensionless)
- \( k_{ro} \): three-phase oil relative permeability (dimensionless)
- \( k_{rog} \): two-phase oil relative permeability in oil/gas (dimensionless)
- \( k_{row} \): two-phase oil relative permeability in oil/water (dimensionless)
- \( k_{romax} \): two-phase oil relative permeability measured at connate water saturation (dimensionless)
- \( krw \): two-phase water relative permeability in oil/water (dimensionless)
- \( S_g \): gas saturation (fraction)
- \( S_{ge} \): effective gas saturation used in Stone I model (fraction)
- \( S_{gmax} \): maximum gas saturation in grid cell (fraction)
- \( S_r \): residual gas saturation (fraction)
- \( S_{gr} \): trapped gas saturation (fraction)
- \( S_o \): oil saturation (fraction)
- \( S_{oe} \): effective oil saturation used in Stone I model (fraction)
- \( S_{om} \): three-phase residual oil saturation (fraction)
- \( S_{om} \): three-phase mobile oil saturation (fraction)
- \( S_{org} \): two-phase residual oil saturation in oil/gas (fraction)
- \( S_{orw} \): two-phase residual oil saturation in oil/water (fraction)
- \( S_w \): water saturation (fraction)
- \( S_{wc} \): connate water saturation (fraction)
- \( S_{we} \): effective water saturation used in Stone I model (fraction)
- \( S_{we} \): normalized water saturation (fraction)
- \( \Delta S_{or} \): reduction in residual oil saturation at a given trapped gas saturation compared to \( S_{or} \) at \( S_{gt} = 0 \) (fraction)
- \( \alpha \): matching parameter for modified Baker (dimensionless)
- \( \lambda \): \( S_{om} \) averaging weighting factor (dimensionless)
References

Table 1: Deviation ($\sum$DEV) of oil relative permeability models

<table>
<thead>
<tr>
<th>Correlation</th>
<th>Donaldson &amp; Dean</th>
<th>Oak</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1966</td>
<td>1991</td>
</tr>
<tr>
<td>Stone I</td>
<td>0.10632</td>
<td>0.16746</td>
</tr>
<tr>
<td>Stone II</td>
<td>0.09949</td>
<td>0.27085</td>
</tr>
<tr>
<td>Segregated</td>
<td>0.12932</td>
<td>0.06860</td>
</tr>
<tr>
<td>Baker</td>
<td>0.12932</td>
<td>0.06804</td>
</tr>
<tr>
<td>Line</td>
<td>0.14378</td>
<td>0.06810</td>
</tr>
<tr>
<td>Modified Baker</td>
<td>0.08850</td>
<td>0.06271</td>
</tr>
</tbody>
</table>

Figure 1 – Modified Baker model ($\alpha$=2.7) prediction of Donaldson & Dean experiment

Figure 2 - Modified Baker model ($\alpha$=-0.3) prediction of Oak experiment

Figure 3 – Gas relative permeability hysteresis model
Sgn = \frac{(S_g - S_{gc})}{(1 - S_{gc} - S_{wi} - S_{org})}, \text{ fraction}

\begin{align*}
S_{gmax}, \text{ fraction} \\
S_{gt}, \text{ fraction}
\end{align*}

\begin{align*}
\text{▲ Rock Group 2} \\
\star Rock Group 3 \\
\text{Model}
\end{align*}

Figure 4 - Gas drainage relative permeability

\begin{align*}
S_{org} = (1 - S_g - S_{wi} - S_{org}), \text{ fraction}
\end{align*}

\begin{align*}
\text{▲ Well 1} \\
\star Well 2 \\
\bigstar Well 3 \\
\blacklozenge Well 4 \\
\text{Model}
\end{align*}

Figure 5 - Trapped gas saturation ($S_{gt}$) versus maximum gas saturation ($S_{gmax}$)

\begin{align*}
S_{gt} = 0.0 \\
S_{gt} = 0.211 \\
S_{gt} = 0.0
\end{align*}

\begin{align*}
S_{org} = (1 - S_g - S_{wi} - S_{org}), \text{ fraction}
\end{align*}

Figure 6 - Reduction in $S_o$ (after 5 PV injections) versus trapped gas saturations

\begin{align*}
\text{▲ Well 1} \\
\star Well 2 \\
\bigstar Well 3 \\
\blacklozenge Well 4 \\
\text{Model}
\end{align*}

Figure 7 - Oil relative permeability by waterflood with and without $S_{gt}$

\begin{align*}
\text{▲ Well 1} \\
\star Well 2 \\
\bigstar Well 3 \\
\blacklozenge Well 4 \\
\text{Model}
\end{align*}

Figure 8 - Oil relative permeability $k_{org}$ hysteresis

\begin{align*}
S_{org} = (1 - S_g - S_{wi} - S_{org}), \text{ fraction}
\end{align*}

Figure 9 - Drainage oil relative permeability ($k_{org}$)
Figure 10 - Imbibition oil relative permeability ($k_{org}$)

Figure 11 - Water relative permeability by waterflood with and without $S_{gt}$

Figure 12 - Comparison of calculated $k_{ro}$ by three-phase models with laboratory measurements

Figure 13 - Comparison of residual oil saturations calculated by three-phase models