SPE 20742

General Coning Correlations Based on Mechanistic Studies
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ABSTRACT

The coning of water can impact well productivity and increase water treatment requirements. Coning correlations are often used to model water-breakthrough time and water cut due to coning. Most existing coning correlations are based on a steady state approximation so that the prediction of water-breakthrough time and initial water cut development are generally unreliable. By first determining the key reservoir, production, and well completion parameters through a theoretical analysis, improved correlations were developed. They have been extensively compared with numerical simulation results to validate their general applicability. The correlations simplify the procedure in the study of the effects of production rate, vertical and horizontal permeabilities, well completion location, physical properties of fluids, relative permeabilities, aquifer thickness, completion interval size, and drainage radius on coning dynamics.

The new correlations can be used in a stand-alone program or in reservoir simulation. In field-scale reservoir simulation, correlations are often employed to model sub-scale flow behavior near a well. Hence, technical issues in implementing correlations into a finite difference simulator are also discussed. In addition, the new correlations are extended to multi-layered, nonhomogeneous reservoir models.

1. INTRODUCTION

The coning of water (i.e., the production of water from an underlying aquifer) can impact well productivity and increase water treatment requirements. Engineers use correlations or numerical simulation to model water-breakthrough time and water cut due to coning. Although numerical simulation is a more rigorous means to model water coning, engineers often use simple correlations for a preliminary estimate of coning phenomena because, compared with numerical simulation, they are simple and easy to use.

In addition, coning correlations are employed in field-scale reservoir simulation. In many reservoir simulation studies, the grid size is so large that the local coning phenomena around a wellbore cannot be resolved in finite difference simulation. Many approximate methods have been devised to overcome this shortcoming. For instance, special pseudo-relative permeability curves for well blocks, local grid refinements around wells, or coning correlations are often employed in field-scale modeling. Though local grid refinement is the most rigorous method of the three, it is also the most expensive for a large model with many wells. A pseudo-relative function method is cumbersome to implement and requires pre-simulation to develop the curves. As a result, coning correlations are still commonly used in reservoir simulation.

Coning correlations have been studied by many investigators (e.g., Muskat, Arthur, Meyer and Gardner, Schols, Chappelar and Hirasaki, Wheatley, Hoyland, Papatzacos, & Skjaeveland, and Abass). Nevertheless, all current methods include some crude assumptions, which limit their applicability in a wide range of problems. Many correlations employ a steady state approximation, and adopt the concept of critical rates. Because the steady state approximation cannot hold for cone development in general operating conditions, especially in an early stage of produc-
3. FORMULATION

An extensive analysis of governing equations and boundary conditions provides a logical approach to determine characteristic variables and important independent variables.

In domains I, II, and III of Figure 1, fluid flow is governed by Darcy’s law. Hence, the governing equations become

\[ \mathbf{u}_w^{(1)} = -\frac{K}{\mu_w} \nabla p^{(1)} \]  
\[ \mathbf{u}_o^{(2)} = -\frac{K_{krw}}{\mu_w} \nabla p^{(2)} \]  
\[ \mathbf{u}_o^{(3)} = -\frac{K_{krw}}{\mu_o} \nabla p^{(3)} \]

The relative permeability of water at \( S_w = 1 - S_{or} \) and that of oil at \( S_o = 1 - S_{we} \) are included in equations (2) and (3), respectively. Note that \( k_{ro} = 1 \) for most cases. The incompressibility condition of fluids can be expressed as follows:

\[ \nabla \cdot \mathbf{u}_w^{(1)} = \nabla \cdot \mathbf{u}_o^{(2)} = \nabla \cdot \mathbf{u}_o^{(3)} = 0 \]

The matching boundary conditions between domains I and II are given by

\[ \mathbf{u}_w^{(1)} = \mathbf{u}_w^{(2)} \]  
\[ p^{(1)} = p^{(2)} \]

and between domains II and III by

\[ \mathbf{u}_w^{(2)} = \mathbf{u}_o^{(3)} \]  
\[ p^{(2)} = p^{(3)} + g \Delta \rho f_c(r) \]

Here, \( f_c \) is the cone shape function which describes the height of the cone. Velocity is always continuous at domain interfaces, whereas pressure tends to jump due to density differences. In equation (8), the pressure jump at the cone front is linearly proportional to the cone height and the density difference between oil and water.

The constant aquifer potential, \( \Phi \), at the drainage boundary can be described by

\[ \Phi = \Phi_0 = p_0 - \rho_wgz, \quad \text{for } r = r_e \text{ and } -h_a \leq z < 0 \]

and the no-flow boundary condition at the drainage boundary in the oil zone becomes

\[ u_n = 0, \quad \text{for } r = r_e \text{ and } 0 \leq z \leq (h_a + h_p + h_I) \]

Here, \( u_n \) is the normal component of fluid velocity at the boundary. Furthermore, the boundary condition at the completion becomes

\[ q = 2\pi r_w \int_{h_p}^{h} u_n \, dz \]

Since the Darcy velocity is an apparent (filter) velocity, the actual cone front velocity is larger than the Darcy velocity.
The actual front velocity can be readily calculated from the material balance at the front.

\[ u_{\text{front}} = \frac{u^{(2)}_{w}}{\phi(S_{w}^{(2)} - S_{w}^{(3)})} \]  

(12)

We chose as characteristic variables \( l_c = h_0, p_c = 100 \ \text{psia} \), \( u_c = p_c k_c k_{ro}/l_c \mu_o \), \( k_c = k_v, t_c = l_c / u_c \), and \( q_c = l_c^2 / t_c \), and nondimensionalized the governing equations and boundary conditions with respect to these characteristic variables. Henceforth, a tilde, ~, denotes a dimensionless variable.

Without loss of generality, a coordinates system is chosen in the principal directions of \( \tilde{K} \) so that the off-diagonal elements of \( \tilde{K} \) vanish. Then \( \tilde{K} \) has two non-zero diagonal elements.

\[ \tilde{K} = \begin{pmatrix} k_h & 0 \\ 0 & k_v \end{pmatrix} \]  

(13)

Further, when the velocity components are rescaled as

\[ \tilde{u}_i = \frac{u_i}{\sqrt{k_i}} \]  

(14)

and the components of the position vector are rescaled as

\[ \tilde{x}_i = \frac{x_i}{\sqrt{k_i}} \]  

(15)

the governing equations (1)-(3) can be simplified as

\[ \tilde{u}^{(1)}_w = -m_1 \tilde{\nabla} \tilde{p}^{(1)} \]  

(16)

\[ \tilde{u}^{(2)}_w = -m_2 \tilde{\nabla} \tilde{p}^{(2)} \]  

(17)

\[ \tilde{u}^{(3)}_o = -\tilde{\nabla} \tilde{p}^{(3)} \]  

(18)

Here,

\[ m_1 = \frac{\mu_o}{k_{ro} \mu_w} \]  

(19)

\[ m_2 = \frac{k_{rw} h_0}{k_{ro} \mu_w} \]  

(20)

The equation of continuity becomes

\[ \tilde{\nabla} \cdot \tilde{u}^{(1)}_w = \tilde{\nabla} \cdot \tilde{u}^{(2)}_w = \tilde{\nabla} \cdot \tilde{u}^{(3)}_o = 0 \]  

(21)

The matching conditions between domains are given by, between domains I and II,

\[ \tilde{u}^{(1)}_w = \tilde{u}^{(2)}_w \]  

(22)

\[ \tilde{p}^{(1)} = \tilde{p}^{(2)} \]  

(23)

and between domains II and III by

\[ \tilde{u}^{(2)}_w = \tilde{u}^{(3)}_o \]  

(24)

\[ \tilde{p}^{(2)} = \tilde{p}^{(3)} + C_g \tilde{f}_c \]  

(25)

where

\[ C_g = \frac{g \Delta p l_c}{p_c} \]  

(26)

The boundary conditions at the drainage radius, \( \tilde{r} = \tilde{r}_e \), are expressed as

\[ \tilde{q} = \tilde{q}_0, \ \text{for} \ -h_o \leq \tilde{z} < 0 \]  

(27)

and

\[ \tilde{u}_n = 0, \ \text{for} \ 0 \leq \tilde{z} \leq (1 + \tilde{h}_p + \tilde{h}_1) \]  

(28)

In addition, the boundary condition at the completion is given by

\[ \tilde{q} = 2\pi \tilde{r} \int_{\tilde{r}_c} \tilde{u}_n d\tilde{z} \]  

(29)

4. CHARACTERISTIC VARIABLES AND IMPORTANT INDEPENDENT VARIABLES

Even in the simplified problem described in Section 2, there are many independent physical variables: directional permeabilities \( (k_v, k_h) \), relative permeabilities \( (k_{ro}, k_{rw}) \), porosity \( (\phi) \), densities \( (\rho_o, \rho_w) \), viscosities \( (\mu_o, \mu_w) \), residual oil saturation \( (S_{or}) \), and connate water saturation \( (S_{wc}) \). In addition, the geometrical description involves drainage radius \( (r_e) \), wellbore radius \( (r_w) \), aquifer thickness \( (h_a) \), formation thickness \( (h_t + h_p + h_o + h_a) \), and top and bottom position of the completion \( (h_o, h_o + h_p) \). The boundary conditions include production rate \( (q) \), wellbore pressure \( (P_w) \), and aquifer pressure \( (P_0) \) at the drainage radius. Because there are so many independent variables, it is important to identify a small set of variables that mainly govern coning dynamics.

In Section 3 the governing equations and boundary conditions were nondimensionalized with respect to characteristic variables: \( l_c = h_0, p_c = 100 \ \text{psia} \), \( u_c = p_c k_c k_{ro}/l_c \mu_o \), \( k_c = k_v, t_c = l_c / u_c \), and \( q_c = l_c^2 / t_c \). Here, the distance between the bottom of the well completion and the original oil-water contact line was considered the most relevant length scale for water breakthrough. The vertical permeability was also chosen as a characteristic variable, which controls vertical motion of the water cone. We had the freedom to choose either a characteristic pressure or a characteristic velocity from operating conditions. For simplicity, we chose a characteristic pressure from a typical pressure drop between the wellbore and reservoir pressure. All other characteristic variables were then determined from the governing equations and boundary conditions.

The velocity components and coordinates were rescaled with nondimensionalized permeabilities as in equations (14) and (15). Fortunately, in the rescaled and nondimensional domain the permeability, \( \tilde{K} \), does not directly appear in equations (16) to (18). It is imbedded in the geometrical description of the problem. Furthermore, porosity only changes the front velocity as shown in equation (12).
domain that characterize coning development. In the governing equations the controlling parameters are the mobility ratios, \( m_1 \) and \( m_2 \), as defined in equations (19) and (20). In the boundary conditions the production rate, \( q \), the gravitational force due to density difference, \( C_p \), and the ratio of actual front velocity and Darcy velocity, \( r_f = \phi(S_w^{(2)} - S_w^{(3)}) \), are independent variables. The geometrical description is determined by wellbore radius, \( r_w \), drainage radius, \( r_e \), reservoir conditions. The ratio of actual front velocity and Darcy velocity, \( r_f \), only adjusts the front location based on material balance, which is independent of coning dynamics. A coning correlation, developed at a specific value of \( r_f \), can be easily generalized by adjusting the time scale for a reservoir with a different \( r_f \).

Since \( r_w < 1 \) and \( r_e > 1 \) in most reservoirs, the wellbore radius and the drainage radius can be excluded in studying coning dynamics. In a gravitationally stabilized production, the oil production from the zone above the completion is minimal, which implies that \( h_t \) is not important in this study. We first assume that \( \tilde{h}_a >> 1 \) and \( \tilde{h}_p << 1 \) and develop correlations based on this simple geometrical description. Later the correlations are adjusted to include the effects of small \( \tilde{h}_a \) and large \( \tilde{h}_p \). Given these conditions, three important controlling parameters are identified: mobility ratio, \( m = m_2 \); gravitational force, \( C_p \); and production rate, \( q \). Correlations are developed based on these three parameters.

5. PRELIMINARY STUDY OF CONING DYNAMICS BY A BOUNDARY INTEGRAL METHOD

Water coning phenomena are dominated by the water front motion, which carries a sharp saturation change as in a Buckley-Leverett displacement process. In a conventional finite difference method, the location of water fronts can be indirectly tracked from the dispersed saturation distribution. Due to its numerical dispersity and finite difference approximation, it is rather cumbersome to study the detailed dynamics of moving fronts by a finite difference method. It is always helpful to have a numerical method that can conveniently describe the detailed dynamics of a water front near a well completion. Consequently we developed a boundary integral method to study the dynamics of water fronts.

The boundary integral method has been successfully applied in a variety of fields: solid mechanics, low-Reynolds-
interval, $h_p$, were added in the correlations. A base model was constructed with the physical properties in Table 1 and straight-line relative permeability curves. With a finite difference simulator, coning phenomena were systematically investigated to determine their dependencies on physical parameters and geometrical and boundary conditions. In addition, the effects of relative permeability, oil viscosity variation, and a small drainage radius were also investigated. The correlations are expressed with nondimensional and rescaled variables, as defined in Section 2. For simplicity, the tilde and hat for dimensionless and rescaled variables are omitted in this section.

6.1 Water-Breakthrough Time

$q, C_g$, and $m$ dependency

Instead of modeling water-breakthrough time ($t_{wb}$), we chose to model the average cone development velocity, which is the reciprocal of water-breakthrough time.

$$u = \frac{1}{t_{wb}}$$ (30)

The average cone development velocity can be expressed as a general functional.

$$u = u(q, C_g, m)$$ (31)

Using a finite difference simulator, breakthrough time is computed for models with various $q$ and $C_g$. The mobility ratio, $m$, is kept constant ($m = 4.146$). The simulation results are plotted in Figure 4. The figure clearly shows similarity in shape among the curves with different production rates. Hence, a similarity transformation is proposed.

$$u = f(q)g(C_{gr})h(m, q)$$ (32)

where

$$C_{gr} = C_g/C_{gm}$$ (33)

$$C_{gm} = f_1(q)$$ (34)

Regression of the curves in Figure 4 yields

$$C_{gm} = f_1(q) = 0.597q + 0.0484$$

Velocity at $C_g = C_{gm}$ is plotted in Figure 5. From these results, functions $f(q)$ and $g(C_{gr})$ are determined.

$$f(q) = 0.61490q - 2.32377q^2 + 6.93017q^3,$$

$$f(q) = 1.1977q - 0.13422,$$  for $0.4179 < q < 0.58$

$$g(C_{gr}) = 26.8977 - 69.9435C_{gr} + 47.8454C_{gr}^2,$$

$$g(C_{gr}) = 9.0060 - 15.9050C_{gr} + 7.8366C_{gr}^2,$$

$$g(C_{gr}) = 0.01309 + 1.003x^{-3.0016(C_{gr} - 1)},$$  for $1.144 < q < 3$

The next step was to model the mobility dependency of the correlation. In the above equation of $u$, it is assumed that the functional dependencies of $C_g$ and $m$ are separable, and $f(q)$ and $g(C_{gr})$ are developed with a constant mobility ratio ($m = 4.146$). Subsequently the mobility effect on water-breakthrough time was examined by changing mobility and production rate for the model. The results are depicted in Figure 6. Because the effect of mobility ratio on water breakthrough time is strongly coupled with production rate, the following equation is proposed to represent the curves in Figure 6,

$$h(m, q) = h_1(q)(1.0 - e^{-h_2(q)m}) + h_3(q)m$$ (35)

and the equation of $u$ is also modified as

$$u = f(q)g(C_{gr})h(m, q)h(4.146, q)$$ (36)

From regression of the curves, we obtain

$$h_1(q) = 1.4103 - 2.0228q,$$  for $0.2074 < q < 1.1144$

$$h_1(q) = -0.000012085 - 161.9204e^{-1.7126q},$$  for $1.1144 < q < 3$

$$h_2(q) = 0.49724 + 0.78129q,$$  for $0.2074 < q < 0.4179$

$$h_2(q) = 1.22017 - 0.94544q,$$  for $0.4179 < q < 3$

$$h_3(q) = 0.54307 - 0.68135e^{-1.0937q},$$  for $0.2074 < q < 1.1144$

$$h_3(q) = 0.02545 + 1.130e^{-1.143q},$$  for $1.144 < q < 3$

For extreme values of $q$, $h(m, q)$ is limited by

$$h(m, q) = h(m, 0.2074),$$  for $q < 0.2074$

$$h(m, q) = h(m, 3),$$  for $q > 3$

$h_p$ and $h_a$ dependency

The average cone development velocity is also dependent on the perforation interval and aquifer thickness. A large completion interval for a given production rate delays breakthrough time. When a line sink is distributed over a large interval, some portion of the sink is far from the oil-water contact line. Therefore, the total pulling force (singularity strength) of the sink becomes smaller, compared with a well with a small completion interval and the same production rate. A thin aquifer restricts water flow to prolong water-breakthrough time. These effects of perforation interval and aquifer thickness were added to the correlations.

For simplicity of modeling, simple functional dependencies of $u$ on $h_a$ and $h_p$ are proposed.

$$u = \alpha_1(h_p)\alpha_2(h_a)f(q)g(C_{gr})h(m, q)h(4.146, q)$$ (37)

The functionals of $\alpha_1$ and $\alpha_2$ are determined from simulation results with various aquifer thickness and perforation intervals.

The simulation results for $h_p$ dependency are modeled by the following equations.

$$\alpha_1(h_p) = 1, \text{ for } h_p \leq 0.1428$$
\( \alpha_1(h_p) = 0.1965 - 0.1175h_p + 2.0752e^{-6.4807h_p}, \)
\[ \text{for } 0.1428 < h_p \leq 0.857 \]
\( \alpha_1(h_p) = 0.01315 + 0.8039e^{-2.5602h_p}, \)
\[ \text{for } 0.857 < h_p \]

Water flow restriction due to the aquifer size is modeled by
\( \alpha_2(h_a) = 1, \) \[ \text{for } h_a > 1.428 \]
\( \alpha_2(h_a) = 0.97979 - 0.7901e^{-6.2028h_a}, \) \[ \text{for } 0 \leq h_a \leq 1.428 \]

### 6.2 Water Cut Prediction

Next, correlations to predict water cut projections after water breakthrough were developed. Upon examining water cut performance for various cases, a single functional form with an independent variable, time, and three coefficients was devised to represent water cut performance.

\[
\text{Water Cut} = (a + bt')(1 - e^{-ct'})
\]  
\[ t' = t - t_{wb} \]

Note that time, \( t \), is a nondimensionalized time w.r.t. \( t_e \).

The three coefficients \( a, b, \) and \( c \) are dependent on the controlling parameters.

\( a = a(q, C_g, m, h_p, h_a) \)  
\( b = b(q, C_g, m, h_p, h_a) \)  
\( c = c(q, C_g, m, h_p, h_a) \)

\( q, C_g, \) and \( m \) dependency

We first examine the coefficient dependency on \( q \) with the assumption of

\[ a \propto a_1(q) \]  
\[ b \propto b_1(q) \]  
\[ c \propto c_1(q) \]

while other variables are fixed \( (m = 4.146, C_g = 0.426, h_p = 0.142, \) and \( h_a = 2.143) \). Coefficients \( a, b, \) and \( c \) versus production rates are tabulated in Table 2, and the results are regressed with polynomials and exponential functions.

\( a_1(q) = (1.0 + 0.058245q)(0.755788 - 0.49752e^{-5.16409q}) \)
\[ \text{for } q \leq 0.8527 \]
\( b_1(q) = 3.9975 \cdot 10^{-4} - 1.42251 \cdot 10^{-8}e^{-21.0453q}, \]
\[ \text{for } q > 0.8527 \]
\( c_1(q) = 4.29719 \cdot 10^{-3} - 1.25198 \cdot 10^{-1}q + 2.400593q^2, \]
\[ \text{for } q \leq 0.0974 \]
\( c_1(q) = 2.93319 \cdot 10^{-1} - 5.45448q + 26.6589q^2, \]
\[ \text{for } 0.0974 < q \leq 0.162 \]
\( c_1(q) = -7.72884 \cdot 10^{-2} + 1.32738q - 2.39082 \cdot 10^{-1}q^2, \]
\[ \text{for } 0.162 < q < 2.776 \]
\( c_1(q) = 1.76507, \) \[ \text{for } 2.776 < q \]

Secondly the coefficient dependencies on \( C_g \) were investigated. Simulations with various \( q \) and \( C_g \) were made and the water cut performance was regressed. The results are plotted in Figures 7, 8, and 9. In these figures the curves appear to have a similar shape for different production rates. So it seems feasible to represent the curves with a single curve by normalizing \( C_g \) with \( C_{gm} \) as in modeling water-breakthrough time. Using this conjecture, we propose the following functional forms for the parameters:

\[ a \propto a_1(q) \frac{a_2(C_{gr})}{a_2(C_{gr,base})}a_3(m) \]  
\[ b \propto b_1(q) \frac{b_2(C_{gr})}{b_2(C_{gr,base})}b_3(m) \]  
\[ c \propto c_1(q) \frac{c_2(C_{gr})}{c_2(C_{gr,base})}c_3(m) \]

Since \( a_1, b_1, \) and \( c_1 \) were determined with constant \( C_g (= 0.426), \)
\[ C_{gr,base} = \frac{C_g}{C_{gm}} \]

The \( C_g \) dependency of the coefficients are obtained as follows. For \( C_{gr} \leq 1.0, \)
\[ a_2(C_{gr}) = -0.01446 + 1.0942e^{-0.90527C_{gr}} \]
\[ b_2(C_{gr}) = 45.8569 - 45.5209e^{-0.101826C_{gr}} \]
\[ c_2(C_{gr}) = 1.18127 - 2.77058C_{gr} + 4.35034C_{gr}^2 - 2.56542C_{gr}^3 \]

For \( C_{gr} > 1.0, \)
\[ a_2(C_{gr}) = 1.21555 - 0.80422e^{-4.1248C_{gr}} \]
\[ b_2(C_{gr}) = 0.077663 + 4.7436e^{-5.6002C_{gr}} \]
\[ c_2(C_{gr}) = 0.012916 + 0.18722e^{-19.5342C_{gr}} \]

Thirdly the mobility ratio dependent terms, \( a_3, b_3, \) and \( c_3 \) were successively determined from simulation results. For \( m > 4.146, \)
\[ a_3(m) = 1.097858 - 0.332619e^{-0.31419m} \]
\[ b_3(m) = 0.859829 + 0.256246e^{-0.12555m} \]
\[ c_3(m) = 1.65041 - 1.26192e^{-0.15555m} \]

For \( m \leq 4.146, \)
\[ a_3(m) = 1 - (0.3785q - 1.565)(e^{-0.086q+3.57} - e^{(0.086q+3.57)m}) \]
\[ b_3(m) = 1 + (-0.59618 + 1.57235q^2)(m_r - 1)^2, \text{ for } q \leq 0.663 \]
\[ b_3(m) = 1. + (1.62q - 0.979)(m_r - 1)^2, \text{ for } 0.663 < q \leq 1.14 \]
\[ b_3(m) = 1. + 0.867(m_r - 1)^2, \text{ for } 1.14 < q \]
\[ c_3(m) = 0.6m_r + 0.4 \]

where \( m_r = m/4.146 \)

\( h_p \) and \( h_a \) dependency

(a) perforation interval
As in the previous subsection 6.1 of water-breakthrough time, a modified production rate was introduced to model the effect of perforation interval.

\[ q^* = q \gamma_p(h_p) \]  

(48)

The function \( \gamma_p \) is given by

\[ \gamma_p = 0.32364 + 1.03274e^{-2.8S_{0.48}h_p}, \quad \text{for } 0.1429 \leq h_p \]

\[ \gamma_p = 1, \quad \text{for } h_p < 0.1429 \]

Numerical simulation results reveal that the coefficient \( c \) can be modeled with the modified production rate as

\[ c = c(q^*, C_g, m) \]  

(49)

Coefficient \( a \) cannot be modeled only with \( q^* \), especially for large \( h_p \), so that a modifier \( \alpha_a \) is introduced.

\[ a = \alpha_a \cdot a(q^*, C_g, m) \]  

(50)

Here,

\[ \alpha_a = 1.0, \quad \text{for } h_p \leq 0.7667 \]

\[ \alpha_a = 0.966 - 0.3766(h_p - 0.857), \quad \text{for } h_p > 0.7667 \]

Finally, as coefficient \( b \) is not successfully modeled with \( q^* \), \( b \) is directly modeled with a modifier \( \alpha_b \) without \( q^* \).

\[ b = \alpha_b \cdot b(q, C_g, m) \]  

(51)

For \( h_p \leq 0.1429 \),

\[ \alpha_b = 1.0 \]

and for \( h_p > 0.1429 \),

\[ \alpha_b = 6.1872 - 5.2816e^{-0.1619h_p} \]

(b) aquifer size

The effect of aquifer size is modeled with a modifier \( \beta \). It is noted that the modifier is dependent on the perforation interval as well as aquifer thickness. The following functionals are thus proposed to characterize the effect of aquifer thickness on water cut development.

\[ a = \alpha_a(h_p)\beta_a(h_a, h_p)a_1(q^*) \frac{a_2(C_{gr})}{a_2(C_{gr, base})}a_3(m) \]  

(52)

\[ b = \alpha_b(h_p)\beta_b(h_a, h_p)b_1(q) \frac{b_2(C_{gr})}{b_2(C_{gr, base})}b_3(m) \]  

(53)

\[ c = \beta_c(h_a, h_p)c_1(q^*) \frac{c_2(C_{gr})}{c_2(C_{gr, base})}c_3(m) \]  

(54)

For \( h_a > 1.714 \),

\[ \beta_a = \beta_b = \beta_c = 1.0 \]

For \( h_a \leq 1.714 \),

\[ \beta_a = 1. + c_1e^{-c_2h_a} \]

6.3 Relative Permeability and Oil Viscosity Variation

The correlations have been developed with the assumption of straight-line relative permeability curves and constant fluid properties. In some reservoirs, relative permeability curves can be very different from straight lines, and oil viscosity can also change as pressure changes. Numerical simulation results showed that relative permeability and oil viscosity variations have substantial impact on breakthrough time, but minimal impact on water cut development after water breakthrough. We thus investigated the effect of nonstraight-line relative permeability curves and oil viscosity variation on water-breakthrough time.

In general, if relative permeabilities do not differ too much from a straight line, the end-point relative permeabilities at \( S_w = S_{wc} \) and \( S_a = 1 - S_{or} \) determine coning phenomena. The correlations were developed with the assumption that the actual shape of relative permeabilities are not important in coning dynamics. Nevertheless, when oil relative permeability becomes very small, almost practically immobile at \( S_w << 1 - S_{or} \), the shape effect of relative permeability curves cannot be neglected.

Since the effect of relative permeability is mostly due to the change in oil mobility, a modifier for the mobility ratio is devised to include this effect.

\[ m^* = \alpha_m m \]  

(55)

This modified mobility ratio is used only in the correlations of breakthrough time, not in those of water cut projections. When the mobility ratio is reduced according to the theoretical front saturation of the fractional flow curve, the agreement between the correlations and numerical simulation results improves significantly.

In some reservoir fluids, oil viscosity varies as pressure is reduced. Since there is a noticeable pressure drop around a wellbore, oil viscosity variation can affect breakthrough time as well. This can also be modeled with equation (55) by adjusting the mobility ratio.

6.4 Boundary Conditions at the Drainage Radius

The effect of the drainage radius on coning dynamics was...
studied with various well models. Numerical simulation results clearly showed that coning dynamics generally becomes less dependent on boundary conditions at the drainage radius as the nondimensionalized and rescaled production rate, \( \tilde{q} \), increases. However, in a model with a low production rate, the development rate of water-cut after water breakthrough is directly dependent on the size of the drainage domain.

To include the effect of the drainage radius in the correlations, the breakthrough time \( t_{wb} \) and the coefficients \( b \) and \( c \) in equation (36) are modified as

\[
\begin{align*}
    t_{wb}^* &= \eta_{twb} t_{wb} \\
    b^* &= \eta_b b \\
    c^* &= \eta_c c
\end{align*}
\]  

(56)

Here,

\[
\begin{align*}
    \eta_{twb} &= 1, \quad \text{for } q > 0.441 \text{ or } r_e > 10 \\
    \eta_{twb} &= 1 - w_1 (1 - \left(\frac{r_e}{10}\right)^2), \quad \text{for } q \leq 0.441 \text{ and } r_e \leq 10 \\
    \eta_b &= w_2, \quad \text{for } r_e \leq 14 \text{ or } q \leq 1 \\
    \eta_b &= w_2 (1 - w_3) + w_3, \quad \text{for } r_e > 14 \text{ and } q > 1 \\
    \eta_c &= 1, \quad \text{for } q > 0.441 \\
    \eta_c &= 1 - w_1 (1 - w_3), \quad \text{for } q \leq 0.441 \\
    w_1 &= 5.12 (0.1945 - q^2) \\
    w_2 &= \left(\frac{14}{r_e}\right)^2 \\
    w_3 &= (1 - e^{-2q})
\end{align*}
\]

For simplicity, the tilde and hat for dimensionless and rescaled variables are omitted in the above equations. Note that coefficient \( a \) is independent of \( r_e \).

7. COMPARISON WITH FINITE DIFFERENCE SIMULATION RESULTS

In this section the correlations for water-breakthrough time and water cut profile after breakthrough are compared with simulation results from finite difference simulation. As previously mentioned, the development of the correlations are intended to mimic finite difference simulation results. A comparison with field data may not be meaningful due to large uncertainties in field properties and measurements. It was thus assumed that a comparison with finite difference simulation solely determines the accuracy of correlations.

7.1 Breakthrough Time

The first example is the model reported in Chappelear and Hirasaki. Since the oil and water viscosities are identical in this model, it is a favorable displacement process \( (m = 0.25) \). In Table 3, the predictions of the correlations are compared with simulation results. In the first group, the isotropic permeability is varied from 100 md to 750 md, and in the second group the vertical permeability is changed, while the horizontal permeability is kept constant at 750 md. In the last group the characteristic length \( (h_0) \) is changed from 20 ft to 80 ft. These conditions represent a wide range of physical properties in a reservoir.

The dimensional analysis in Section 3 shows that as the permeability of an isotropic reservoir decreases, the characteristic production rate, \( q_e \), decreases. The rescaled production rate, \( \tilde{q} \), increases as a result, which warrants early water-breakthrough. In the second case, when only the vertical permeability increases, the rescaled production rate \( \tilde{q} \) does not change, but the characteristic time increases. So the breakthrough time becomes linear to the characteristic time scale. In the last case, as the distance between the oil-water contact line and the bottom of perforation interval increases, the characteristic length increases accordingly. This creates a smaller rescaled production rate and also decreases the rescaled aquifer thickness and perforation interval. This combined effect prolongs water-breakthrough time. In Table 3, the estimate of water-breakthrough time from the correlations agrees very well with finite difference simulation results for all the different cases tested.

The correlations were also tested with the base reservoir model whose physical properties are summarized in Table 1. Straight line relative permeability curves and constant oil viscosity were used in this study. In Table 4 breakthrough time calculated by the correlations are compared with simulation results for various production rates, distance between the original oil-water contact line and the bottom of perforation interval, perforation interval size, and aquifer thickness. The correlations provided very satisfactory results in which the discrepancy error is less than 15 percent.

In Tables 3 and 4, the correlations can predict water-breakthrough time for models with a wide range of physical and geometrical parameters. It proves that the scaling and nondimensionalization processes adopted in the development of the correlations are theoretically consistent, which ensures the general applicability of the correlations.

7.2 Water Cut Projection

A base model was constructed with the physical properties in Table 1, the production rate, \( q = 500 \text{ bbls/day} \), and the reservoir geometry of \( h_0 = 35 \text{ ft} \), \( h_f = 5 \text{ ft} \), and \( h_a = 75 \text{ ft} \). By using the base model, the prediction of the new correlations were compared with simulation results. The results are shown in Figure 10 together with the prediction of the Chappelear-Hirasaki correlation. Since the distance between the original oil-water contact line and the perforation interval is small, water breaks through early, followed by a very slow increase in water-cut for a long period. The new correlations represent the simulation results extremely well, whereas the Chappelear-Hirasaki cor-
relation predicts an instant water breakthrough, and the predicted water cut is 10 to 20 percent lower than the simulation results.

In order to validate the scaling and characteristic variables in the correlations, the correlations were tested with a model which is geometrically twice as large as the previous one. The results are drawn in Figure 11. In this enlarged model, water breaks through early as well, but the stabilized water-cut is much smaller than the previous case. This phenomenon is well predicted by the new correlations. The Chappelear-Hirasaki correlation completely misses the trend of simulation results. This comparison also validates the scaling and nondimensionalization technique employed in Sections 2 and 3.

The correlations were also examined with reservoir models with different perforation intervals and aquifer thickness. A model with \( h_p = 40 \text{ ft} \) and \( h_a = 20 \text{ ft} \) was constructed. The completion interval \( (h_p) \) is larger than the characteristic length \( (h_0) \) for this model. The comparison between simulation results and correlations is depicted in Figure 12. The new correlations represent simulation results very satisfactorily, while the Chappelear-Hirasaki correlation consistently predicts water cut 20 to 30 lower percent than the simulation results.

In Figure 13 the water cut performance from the correlations and simulation results are plotted for various production rates. The distance between the bottom of the perforation interval and the original oil-water contact line is 35 ft in this model. Because the characteristic length \( h_0 \) is relatively small, we expect an immediate water breakthrough and a quick stabilization of water cut. This is confirmed by simulation results in Figure 13. The correlations also reveal the same characteristics, and the agreement between the simulation and correlations appears to be very good. A model with twice the size of the previous one was also investigated. The comparison is shown in Figure 14. The water cut performance is very similar to the case in Figure 13, because they are basically identical except the geometrical size.

8. IMPLEMENTATION IN A FINITE DIFFERENCE SIMULATOR

The coning correlations were developed based on a simple physical model described in Section 2. In order to apply the correlations in a finite difference simulator, they should be extended to a more general physical model. Fortunately it can be accomplished by employing appropriate approximations. In implementing the coning correlations into a finite difference simulator, four main technical issues have to be resolved: (1) the effect of changes in the production rate, (2) the evaluation of effective physical properties (i.e., permeability, porosity, and saturations) for a multi-layered reservoir model, (3) the distribution of produced fluids among layers in a multi-layered reservoir model, and (4) the effect of the well-block size.

Production rate changes

Often the production rate of a well varies, depending on the operational conditions and market demands of oil. In reservoir simulation, the production history is usually incorporated as realistically as possible. In Section 5, the coning correlations were developed based on a constant liquid production rate. It is necessary to show here how the correlations can be utilized in reservoir simulation with varying production rates.

For a fixed rate of total fluid production, the water saturation change in the drainage volume can be derived as

\[
\Delta S_w V \phi = \int_0^t q(1 - \text{WCUT}(t, q)) dt \quad (59)
\]

Here, \( \text{WCUT} \) is the water cut from the correlations; \( \Delta S_w \) is the averaged water saturation change in the drainage domain; \( V \) is the total volume; and \( \phi \) is the porosity.

In most applications, when the production rate is changed, the cone quickly alters its shape in order to attain a new quasi-steady shape under the current production rate. Therefore, we assume that the cone shape is dependent only on the current production rate and the fluid saturation in the drainage domain. This assumption allows us to extend the coning correlations to a well with varying production rates.

From current and initial cell saturations, one can readily obtain the water saturation change in a drainage domain around a producing well. The time needed to achieve the water saturation change with the current production rate can be computed from the above equation. Let us denote the calculated time by \( t' \). Substituting \( q \) and \( t' \) into the coning correlations will readily yield the estimate of current water-cut.

Effective physical properties

Instead of deriving different coning correlations for a multi-layered, heterogeneous reservoir model, we conjecture that the current coning correlations can be still applicable if a heterogeneous model can be mapped into a homogeneous model by effective physical properties.

In general, vertical flow goes through each layer sequentially, and horizontal flow comes in parallel from all the layers. From this physical intuition, an arithmetic average was chosen as an effective horizontal permeability, and a harmonic average as an effective vertical permeability.

Models with the detailed layer heterogeneities and with effective permeabilities were studied by a finite difference method. The water cut performance generally agreed with each other to confirm the above simple conjecture. There-
fore, the correlations can be adequately utilized for multi-layered models, if effective permeabilities are employed. Other physical properties are not very important as long as they are not too different among layers. An arithmetic average appears to be proper for other physical properties.

**Distribution of liquids among layers**

The coning correlations calculate oil and water production rates. The completion block in finite difference simulation may not contain enough water to meet the water production of the correlations. A mechanism should be devised to produce water from the grid blocks which contain enough water to satisfy the water-oil ratio from the correlations.

This difficulty can be circumvented if production fluids are allocated based on the potential difference and availability of fluids from the completion cells as well as from all the cells below the completion. The production of each layer is proportional to the potential difference between well-block and well bore and also the productivity index of the layer. In essence, the simulator produces fluids from all the layers in which the well is located, independent of the actual location of completion intervals. Because one does not have enough information in distributing liquids from a coarse grid simulation, this is the only logical way of fluid distribution.

**Well-block size**

The coning correlations are weakly dependent on the drainage radius. If the drainage radius is large, the cone can be established without interference from the presence of the drainage boundary. The effect of a small drainage radius is included in the coning correlations through modifiers \( \eta_{wb}, \eta_{w}, \) and \( \eta_c \) in equations (56) to (58).

If a well-block size is larger than twice the drainage radius, the actual drainage radius is of no significance in applying the coning correlations. The well block can be considered the drainage boundary in computing saturation changes in the drainage domain. When a well-block size is smaller than twice the drainage radius, all the neighboring cells within the drainage radius should be included in computing the saturation change in equation (59).

**9. CONCLUSIONS**

A theoretical analysis was employed to determine the characteristic variables and key reservoir, production, and well completion parameters in water coning. By using a systematic approach to identify functional dependencies of the key parameters, new coning correlations were developed. They consist of two parts: one for water breakthrough time estimation and the other for water-cut development after water breakthrough.

The new coning correlations were extensively compared with numerical simulation results to validate their general applicability. For all the models tested, the water-cut projections from the correlations were consistently in good agreement with simulation results. In comparison, the Chappelear-Hirasaki correlation considerably deviated from the simulation results in many tested cases.

The new correlations can be used in a stand-alone program or in reservoir simulation. With a stand-alone program of the correlations, one can study the effects of production rate, vertical and horizontal permeabilities, well completion location, physical properties of fluids, aquifer thickness, completion interval size, and drainage radius on coning dynamics. The correlations can also be utilized in optimizing production schemes and planning construction of water treatment facilities.

The correlations can be incorporated in a reservoir simulator for field-scale simulation studies. In field-scale reservoir simulation, correlations are often employed to model sub-grid scale flow behavior near a well. In regard to implementing correlations into a finite difference simulator, four technical issues were resolved: (1) the effect of changes in the production rate, (2) the evaluation of effective physical properties for a multi-layered reservoir model, (3) the distribution of produced fluids among layers, and (4) the effect of the well-block size.

**ACKNOWLEDGEMENT**

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**NOMENCLATURE**

\[ C_g : \quad \frac{g \Delta \rho}{p_c} \]
\[ C_{gm} : \quad \text{defined in equation (33)} \]
\[ C_{gr} : \quad \text{defined in equation (34)} \]
\[ f_c : \quad \text{cone shape function} \]
\[ g : \quad \text{gravity} \]
\[ h_0 : \quad \text{distance between the original oil-water contact and the bottom of perforation interval} \]
\[ h_a : \quad \text{aquifer thickness} \]
\[ h_p : \quad \text{perforation interval size} \]
\[ K : \quad \text{permeability tensor of the porous media} \]
\[ k_c : \quad \text{characteristic permeability} \]
\[ k_h : \quad \text{horizontal permeability} \]
\[ k_r : \quad \text{relative permeability} \]
\[ k_v : \quad \text{vertical permeability} \]
\[ l_c : \quad \text{characteristic length} \]
\[ m : \quad \text{mobility ratio} \]
\[ p : \quad \text{pressure} \]
\[ p_c : \quad \text{characteristic pressure} \]
REFERENCES


Table 1. Physical Properties of the Base Model

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_o )</td>
<td>6.526 (cp)</td>
</tr>
<tr>
<td>( \mu_w )</td>
<td>0.57 (cp)</td>
</tr>
<tr>
<td>( \rho_o )</td>
<td>0.849</td>
</tr>
<tr>
<td>( \rho_w )</td>
<td>1.122</td>
</tr>
<tr>
<td>( k_{rw} )</td>
<td>0.372</td>
</tr>
<tr>
<td>( k_{ro} )</td>
<td>1.00</td>
</tr>
<tr>
<td>( \phi )</td>
<td>0.28</td>
</tr>
<tr>
<td>( S_{wco} )</td>
<td>0.08</td>
</tr>
<tr>
<td>( S_{wco} )</td>
<td>0.14</td>
</tr>
<tr>
<td>( k_h )</td>
<td>2100 (md)</td>
</tr>
<tr>
<td>( k_e )</td>
<td>1050 (md)</td>
</tr>
</tbody>
</table>

Table 2. Coefficients \( a, b, \) and \( c \) vs. Production Rate.

<table>
<thead>
<tr>
<th>Production Rate</th>
<th>( a )</th>
<th>( b ) (( \times 10^{-4} ))</th>
<th>( c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.106</td>
<td>0.476</td>
<td>2.26</td>
<td>0.0648</td>
</tr>
<tr>
<td>0.162</td>
<td>0.521</td>
<td>3.64</td>
<td>0.111</td>
</tr>
<tr>
<td>0.170</td>
<td>0.564</td>
<td>3.76</td>
<td>0.146</td>
</tr>
<tr>
<td>0.213</td>
<td>0.611</td>
<td>3.31</td>
<td>0.198</td>
</tr>
<tr>
<td>0.426</td>
<td>0.715</td>
<td>4.33</td>
<td>0.472</td>
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<tr>
<td>0.853</td>
<td>0.788</td>
<td>3.87</td>
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</tr>
<tr>
<td>1.705</td>
<td>0.831</td>
<td>3.80</td>
<td>1.493</td>
</tr>
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</table>

Table 3. Breakthrough Time Comparison With Simulation Results: the Model in Chappelear and Hirasaki (1973).

<table>
<thead>
<tr>
<th>$k_h$ (md)</th>
<th>simul. (days)</th>
<th>corr. (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>750</td>
<td>11.4</td>
<td>11.1</td>
</tr>
<tr>
<td>300</td>
<td>8.0</td>
<td>7.1</td>
</tr>
<tr>
<td>150</td>
<td>6.04</td>
<td>6.1</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>$k_h$ (md)</th>
<th>simul. (days)</th>
<th>corr. (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>75.</td>
<td>26.8</td>
<td>22.3</td>
</tr>
<tr>
<td>750.</td>
<td>101.5</td>
<td>110.9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$k_h$ (md)</th>
<th>simul. (days)</th>
<th>corr. (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>11.4</td>
<td>11.1</td>
</tr>
<tr>
<td>2000</td>
<td>177.8</td>
<td>192.0</td>
</tr>
</tbody>
</table>

Table 4. Breakthrough Time Comparison With Correlation: the Base Model, $k_e/k_h = 0.5, k_h = 2100$ md.

<table>
<thead>
<tr>
<th>$q$ (bbls/day)</th>
<th>$k_e$ (ft)</th>
<th>$h_e$ (ft)</th>
<th>$t_w$(simul) (days)</th>
<th>$t_w$(corr) (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>35</td>
<td>40</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>1000</td>
<td>70</td>
<td>10</td>
<td>150</td>
<td>11</td>
</tr>
<tr>
<td>1345</td>
<td>95</td>
<td>10</td>
<td>200</td>
<td>37</td>
</tr>
<tr>
<td>2000</td>
<td>95</td>
<td>25</td>
<td>200</td>
<td>29</td>
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</tbody>
</table>

Figure 1. Schematic Diagram of Coning Phenomena

Figure 2. Interface Shape Calculated by a Boundary Integral Method.

Figure 3. The Position of the Cone Apex Calculated by a Boundary Integral Method

Figure 4. Cone Development Velocity: $m=1.146$, $hp=0.142$, $la=2.143$
Figure 5. Cone Development Velocity at Cg= Cgm

Figure 6. The Effect of Mobility Ratio on Cone Development Velocity

Figure 7. Parameter a vs. Cg.

Figure 8. Parameter b vs. Cg.

Figure 9. Parameter c vs. Cg.

Figure 10. Comparison of Correlations and Simulation Results:
q = 500bblday, ho = 35ft, bp = 6f, ha = 75f, kV = 1050md, Kh = 2100md
Figure 11. Comparison of Correlations and Simulation Results:
q = 500 bbl/day, ho = 70ft, hp = 10ft, ha = 150ft, Kv = 1000md, Kh = 2100md

Figure 12. Comparison of Correlations and Simulation Results:
q = 500 bbl/day, ho = 35ft, hp = 40ft, ha = 20ft, Kv = 1000md, Kh = 2100md

Figure 13. Comparison of Correlations and Simulation Results:
ho = 35ft, hp = 5ft, ha = 75ft, Kv = 1050md, Kh = 2100md

Figure 14. Comparison of Correlations and Simulation Results:
ho = 70ft, hp = 10ft, ha = 150ft, Kv = 1050md, Kh = 2100md