

FYS 610 Many-particle quantum mechanics

Exercises for 24 February 2017

PROBLEM 12: *Schwartz*, problem 5.3.

PROBLEM 13:

- Find the energy-momentum tensor, $\mathcal{T}^{\mu\nu}$, from Noether's theorem for the free electromagnetic field. Is this $\mathcal{T}^{\mu\nu}$ symmetric in μ and ν ?
- Write out the energy density, $\mathcal{E} = \mathcal{T}^{00}$, and the momentum density, $\pi^i = \mathcal{T}^{0i}$, in terms of the electric and magnetic fields, \mathbf{E} and \mathbf{B} .
- For *any* conserved energy-momentum tensor, $\mathcal{T}^{\mu\nu}$ with $\partial_\mu \mathcal{T}^{\mu\nu} = 0$, we can define another tensor by:

$$\tilde{\mathcal{T}}^{\mu\nu} = \mathcal{T}^{\mu\nu} + \partial_\lambda K^{\lambda\mu\nu},$$

where $K^{\lambda\mu\nu} = -K^{\mu\lambda\nu}$, but otherwise arbitrary. Show that we also have $\partial_\mu \tilde{\mathcal{T}}^{\mu\nu} = 0$.

- Show that if one chooses:

$$K^{\lambda\mu\nu} = -F^{\lambda\mu} A^\nu,$$

the resulting $\tilde{\mathcal{T}}^{\mu\nu}$ is symmetric in μ and ν . Also show that the total energy and momentum found from $\tilde{\mathcal{T}}^{\mu\nu}$ and $\mathcal{T}^{\mu\nu}$ are the same.

- Write out the modified energy and momentum densities based on $\tilde{\mathcal{T}}^{\mu\nu}$. Do you recognize the results?

[This is roughly a corrected version of Problem 9, *Schwartz*, Problem 3.3.]

PROBLEM 6: Once more. See problems for 10.02 2017.