Per Amund Amundsen
Department of Mathematics and Natural Science
UNIVERSITY OF STAVANGER

Set 7. Exercises for 6 October 2017

Problem 39: Exam problem 1, 2014 spring, part i.

Problem 40: *Goldstein*, exercise 3.14.

Problem 41: A particle of mass m and angular momentum ℓ moves around a fixed center of force in a potential given by:

$$V(r) = -\frac{k}{r} + \frac{h}{2r^2}$$

for some positive constants k and h.

a) Find the transformed radial equation for $1/r = u(\theta)$, where θ is the polar angle, and prove, *e.g.* by substitution, that it has solution:

$$r = \frac{c}{1 + e\cos(\beta\theta)} ,$$

where c, β and e are positive constants.

b) Express c and β in terms of known quantities, and describe the orbit for the different possible values of e.

c) For what values of e and β is the orbit closed?

Problem 42: A particle of mass m, initial velocity v_0 and impact parameter s is scattered in the field of a repulsive potential $V(r) = h/2r^2$, where $h > 0$.

a) Show that the orbital equation has the solution:

$$r = \frac{r_t}{\cos(\beta\theta)} ,$$

if we chose coordinates such that $\theta = 0$ at the turning point, $r = r_t$. Express β in terms of known quantities.

b) Find an expression for the scattering angle, Θ, in terms of known quantities. Verify that $\Theta = 0$ if $h = 0$ and that $\Theta = \pi$ for $s = 0$ and $h > 0$.

c) Find the scattering cross section $\sigma(\theta)$.

Problem 43: The conditions are the same as in Problem 42, except that the potential is now *attractive*, so $h = -|h|$.

a) Show that for large enough s the solution in Problem 42a remains valid, with $0 < \beta < 1$. What is the smallest impact parameter, s_c, where this is the true?

b) Find an expression for the scattering angle, Θ, in terms of known quantities and verify that $\Theta < 0$. For which range of s do we have *orbiting*, *i.e.* $|\Theta| > 2\pi$.

c) Show that for $s = s_c$ the solution for the orbit is $r = c/\theta$, if we chose the coordinates such that $\theta = 0$ for $r = \infty$ (*i.e.* long before the collision). Thus the particle is spiraling forever toward the singularity, never coming out again.

d) Solve the orbital equation for $s < s_c$, and show that the particle behaves qualitatively in the same manner as in the previous point.