UNIVERSITETET I STAVANGER

Institutt for matematikk og naturvitenskap

Suggested solutions, FYS 500 — Classical Mechanics Theory 2017 fall

Set 13 for 16–17 November 2017

Problem 77:

The relativistic equation og motion for the Lorentz force law, $\mathrm{d}p^\mu/\mathrm{d}\tau = qF^\mu{}_\nu u^\nu$ in the absence of an electric field takes the form $\mathrm{d}\mathbf{p}/\mathrm{d}t = q\mathbf{v}\times\mathbf{B}$, or:

$$\begin{aligned}
\dot{p}_x &= \frac{\mathrm{d}p_x}{\mathrm{d}t} = qv_yB_0 = \frac{qB_0}{\gamma m}p_y \\
\dot{p}_y &= \frac{\mathrm{d}p_y}{\mathrm{d}t} = -qv_xB_0 = -\frac{qB}{\gamma m}p_x \\
\dot{p}_x &= \frac{\mathrm{d}p_z}{\mathrm{d}t} = 0\,.
\end{aligned}$$

This is identical to the non-relativistic case, except that $\mathbf{p}$ here is the relativistic momentum. The equation for the motion in the z-direction then yields $p_z = \gamma m v_z = p_z^0$, a constant. Now, by observing that

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{p}^2 = \mathbf{p}\cdot\dot{\mathbf{p}} = \gamma m\mathbf{v}\cdot q\mathbf{v}\times\mathbf{B} = 0\,,$$

by the properties of the vector product, we see that $\mathbf{p}^2 = \gamma^2m^2v^2$ is a constant of motion, which means that v and hence γ are constants. Then also v_z is constant, and the same must be thus true for the magnitude of the transverse velocity, $v_\perp = \sqrt{v^2 - v_z^2}$. Differentiating the equation for p_x we then find:

$$\frac{\mathrm{d}^2p_x}{\mathrm{d}t^2} = \frac{qB_0}{\gamma m}\frac{\mathrm{d}p_y}{\mathrm{d}t} = -\left(\frac{qB_0}{\gamma m}\right)^2 p_x\,.$$

This is the harmonic oscillator equation, with solution:

$$p_x(t) = p_\perp\cos(\omega_c t + \delta) \qquad \omega_c = \frac{qB_0}{\gamma m}\,.$$

Here $p_\perp$ and δ are constants of integration. The frequency ω_c is the relativistic version of the cyclotron frequency. Furthermore, $p_y = \dot{p}_x/\omega_c = -A\cos(\omega_c t+\delta)$, so we find $p_x^2+p_y^2 = p_\perp^2 = \gamma^2m^2v_\perp^2$, or $p_\perp = \gamma m v_\perp$. Thus the transverse motion is circular. The period is $T = 2\pi/\omega$, and since for circular motion we have $v_\perp = 2\pi r_0/T$, where r_0 is the radius of the orbit, we find $r_0 = Tv_\perp/2\pi = v_\perp/\omega_c = p_\perp/qB_0$.

Problem 78:

Exam problem 2, 2015 fall. See separate solution sheet.

Problem 79:

Exam problem 2, 2016 fall. See separate solution sheet.

Problem 80:

Exam problem 2, 2017 spring. See separate solution sheet.

Problem 81:

Exam problem 1, 2015 fall. See separate solution sheet.

Problem 82:

Exam problem 2, 2014 spring. See separate solution sheet.

Problem 83:

Exam problem 1, 2014 fall. See separate solution sheet.

Problem 81:

Exam problem 1, 2013 fall. See separate solution sheet.