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Set 10. Exercises for 27 October 2017

Problem 56:

a) Find the inertia tensor for a homogeneous cube of mass M and side length a relative to the center of mass. Let the three edges meeting at a corner be aligned with the coordinate axes.

b) Repeat the calculation above for a corner point. [Hint: See *Goldstein* p. 194].

c) Calculate the moment of inertia about an axis parallel to the z-axis through the center of mass and through the corner.

Problem 57:

a) Find the distance of the center of mass from the apex of a cone of circular cross section, with height H and radius R at the base.

b) Find the moment of inertia of this cone about its symmetry axis. Use cylindrical polar coordinates with z-axis along the symmetry axis.

c) Find the moment of inertia about an axis perpendicular to the symmetry axis through the apex. What is the moment of inertia about a parallel axis through the center of mass?

Problem 58: *Goldstein*, exercise 5.3.

Problem 59 Exam problem 2, 2015 spring.

Problem 60 Exam problem 1, 2016 fall.

Problem 61: *Goldstein*, exercise 5.13. Show first that the inertia of a thin rod about an axis through the center of mass and perpendicular to the rod is $I = ml^2/12$. Assume that the rods have uniform density, ρ. [Only an integral representation for the time is required, the integral is not elementary.]

Problem 62: A symmetric top of mass M and principal moments of inertia I_1, $I_2 = I_1$ and I_3 is set spinning on a horizontal table with angular momentum $\omega_3 = \omega$ about its symmetry axis, which we take as the z-axis in the body-fixed coordinate system. This axis is inclined with respect to the plane of the table. The distance to the center of mass of the top from the table, measured along the symmetry axis, is R. Find the torque, $\mathbf{N}$ of the gravitational force with respect to the point of contact between the top and the table. Assuming that this torque is weak, so that ω_1 and ω_2 remain small, show that the top will spin with constant angular velocity around the normal to the table. The acceleration of gravity is $\mathbf{g}$. [Hint: ω remains approximately constant.]