FACULTY OF SCIENCE AND TECHNOLOGY

COURSE: FYS500 — Classical Mechanics and Field Theory
DURATION: 09.00 – 13.00 (4 hours).
DATE: 1/3 2017
RESOURCES: Approved pocket calculator
SET: 2 problems on 2 pages

Universitetet i Stavanger

Each part of a problem counts equally, and most of them can be solved at least partly independently of the other parts.

Problem 1

a) Show that in standard cylindrical coordinates ρ, ϕ, z, the velocity of a particle satisfies:

$$\dot{\mathbf{r}}^2 = \dot{\rho}^2 + \rho^2\dot{\phi}^2 + \dot{z}^2 \,.$$

A particle of mass m slides without friction under the influence of gravity on the upper surface of a semi-cylinder of radius a resting with its flat side on a horizontal plane. The acceleration of gravity is g.

b) Write down the Lagrangian for the system when the particle is constrained to move on the surface of the semi-cylinder, using the angle from the upward vertical direction, ψ, and z as coordinates. Also find the conjugate canonical momenta and the Hamiltonian. What are the conserved quantities of the system?

c) Find the resulting equations of motion. Solve the equation for the motion in the z-direction, and show that the equation for ψ leads to a first integral that can be written:

$$\dot{\psi}^2 + \frac{2g}{a}\cos\psi = C \,,$$

where the constant C shall be expressed in terms of known quantities. [Do not try to solve this equation.]

In the following consider the case where particle starts from rest at $\psi = \psi_0 > 0, z = 0$.

d) Find the constraining reaction force, $F_r(\psi)$, needed for the particle to remain at the cylinder surface. You can choose to use either Newton's second law of motion directly or Hamilton's principle with a Lagrangian multiplier.

e) What is the condition on $F_r(\psi)$ for the particle to remain on the semi-cylinder?. Find the angle $\psi = \psi_c$, where the particle leaves the surface, in the limit $\psi_0 \to 0$. Also find the angular velocity when it leaves the cylinder.

f) Describe qualitatively the motion of the particle after it has left the cylinder. What is the velocity as it hits the horizontal plane? Neglect air resistance.

g) Find the angle of impact of the particle when it hits the horizontal plane.

PROBLEM 2

Two spherical lumps of putty collide and stick together. The first lump initially moves in the x direction, with velocity v, the second is at rest. Neglect gravity.

a) Consider first a central collision of lumps with masses m_1 and m_2 and radii a_1 and a_2, respectively. Explain why momentum conservation applies and find the magnitude, v' and the direction of the single lump after the collision. Also calculate the energies before and after the collision, and show that energy is not conserved. What has happened to the lost energy?

b) Introduce center of mass coordinates, $\mathbf{R}$, and relative coordinates, $\mathbf{r}$, and show that $v' = \dot{X}$ (in standard notation). What is the energy after the collision in the center of mass frame?

c) Show that the moment of inertia of a homogenous sphere of mass m and radius a about its center of mass is:

$$I_0 = \frac{2}{5} m a^2 \,.$$

In the following we shall assume that the lumps are equal, $m_1 = m_2 = m$ and $a_1 = a_2 = a$, and that they just hit each other tangentially, with an impact parameter $s = 2a$, but still stick together after the collision.

d) Find the moment of inertia of a sphere with respect to an axis which is tangential to its surface, and use this to find the moment of inertia, I, of the combined lump of the two touching spheres.

e) Show that after the collision the compound lump remain at rest in the center of mass, but is rotating with an angular velocity that shall be determined. Also calculate the energy loss, and explain why it must be less than for a central collision.

f) Repeat the calculation in part a) above in the case of a relativistic central collision. Explain why four-momentum conservation applies in this case, and use it to find the rest mass and velocity of the combined lump after the collison.