FACULTY OF SCIENCE AND TECHNOLOGY

COURSE:	FYS500 — Classical Mechanics and Field Theory
DURATION:	09.00 – 13.00 (4 hours).
DATE:	23/11 2016
RESOURCES:	Approved pocket calculator
SET:	2 problems on 2 pages

Universitetet i Stavanger

Each part of a problem counts equally, and most of them can be solved at least partly independently of the other parts. **The candidate is referred to the formula at the end!**

PROBLEM 1

a) Show that the moment of inertia of a uniform spherical ball of mass M and radius a about an axis through its center is:

$$I = \frac{2}{5}Ma^2 \,.$$

A ball is rolling in the radial direction near the bottom of a hemispherical bowl of radius $b > a$. The acceleration of gravity is g.

b) Show that the rolling condition leads to:

$$\omega a = \Omega b$$

where $\omega = \dot{\phi}$, with ϕ as the rotation angle of the ball about its center, and $\Omega = \dot{\Phi}$, where Φ is the angular position of the center of the ball about the center of the bowl. Let the bottom of the bowl be at $\Phi = 0$.

c) Make a sketch of a cross-section of the bowl and the ball through the centers of both for $\Phi \neq 0$. Show that the height of the center of the ball above the bottom of the bowl is:

$$z = b - (b-a)\cos\Phi \,.$$

d) Use Chasle's (Euler's) theorem to write down the Lagrangian for the rolling ball, using Φ as generalized coordinate. Also find the Hamiltonian.

e) Find the equation of motion for Φ. Solve it in the case of small oscillations of the ball about the bottom of the bowl. Find the frequency of oscillations and the constants of integration if the ball is released at $t = 0$ with $\Phi(0) = \Phi_0$ and $\dot{\Phi}(0) = \Omega_0$.

PROBLEM 2

We shall study the motion of a particle of charge q and mass m which moves in crossed constant electric and magnetic fields, $\mathbf{E} = E_0\,\mathbf{j}$ and $\mathbf{B} = B_0\,\mathbf{k}$

a) Write down the Cartesian components of the Lorentz force law for the particle. Also show that the magnetic field does no work on the particle.

b) Show that the scalar potential $\Phi = -\mathbf{E} \cdot \mathbf{r}$ and the vector potential $\mathbf{A} = \frac{1}{2}\mathbf{B} \times \mathbf{r}$ give rise to the correct fields. Write Φ and the components of $\mathbf{A}$ in Cartesian coordinates, x, y and z.

c) Write down the Lagrangian for the particle and verify that you obtain the correct equations of motion. Find the canonical momenta, and identify a cyclical coordinate and the corresponding constant of motion. Are there other constants of motion?

d) Solve the equations of motions in the case $E_0 = 0$ with the initial conditions $\mathbf{r}(0) = [x_0, 0, 0]$ and $\dot{\mathbf{r}}(0) = [0, v_0, w_0]$. Describe the motion. Also calculate the kinetic energy of the particle, and check that it is independent of B_0.

e) Solve the equations of motions in the case $E_0 \neq 0$ with the same initial conditions with $w_0 = 0$. Show that the motion can be described as the motion in a circle drifting with constant velocity in the xy-plane. [Hint: One approach is to differentiate the equation of motion for y.]

f) Write down the equations of motion for the corresponding relativistic problem, and explain the difference to the non-relativistic case. Explain why the relativistic problem can be solved in a very similar manner to the solution in d) above if $E_0 = 0$, but the solution for $E_0 \neq 0$ is much more difficult than in e). [You are not asked to actually solve the equations.]

A formula that may prove useful:

The Lagrangian for a particle of mass m and charge q in an electromagnetic field:

$$L = \frac{1}{2}m\dot{\mathbf{r}}^2 - V(\mathbf{r}) - q\,\Phi(\mathbf{r}) + q\,\dot{\mathbf{r}} \cdot \mathbf{A}(\mathbf{r})\,,$$

where $V(\mathbf{r})$ is the non-electromagnetic potential, $\Phi(\mathbf{r})$ the electrostatic potential and $\mathbf{A}(\mathbf{r})$ the magnetic vector potential.