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BOOK REVIEW

Richard W. Allmendinger, Nestor Cardozo, and Donald M. Fisher: Structural Geology Algorithms: Vectors and Tensors

New York: Cambridge University Press, 1st Edition, December 2011. Paperback ISBN 978-1-10-740138-9. 302 pages. $50

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When I received the request by Mathematical Geosciences to review this book, I was in the midst of teaching an upper-level undergraduate course in Structural Geology and Tectonics. As is also the case in my graduate-level Active Tectonics course, students are expected to use linear algebra to evaluate stress and strain to gain first-order understanding of faulting and distributed deformation in the Earth. However, students taking these courses come with a wide range of, and often insufficient, prior knowledge and preparation in mathematics. Thus, I provide those students that ask for supplementary reading with a collection of copied book chapters and lecture notes from courses I have taken some time ago, always wishing that there were a compact book that covered those topics relevant to geological and geophysical applications. This text entitled "*Structural geology algorithms: vectors and tensors*" is that long-wished-for book.

In less than 300 pages, Richard Allmendinger and his colleagues Nestor Cardozo and Donald Fisher provide a unified treatment of linear algebra and computational methods starting from the basics. Not only does this text provide all the fundamental concepts needed in a wide range of mathematical operations used in structural geology and related fields, but it also provides short computer functions (in the accessible and widely used Matlab® programming environment) to illustrate the underlying algorithms. While the book is focused on applications in structural geology, the material is relevant for students in other areas of geology and geophysics, including mineralogy, seismology and geodesy. This book is meant to serve as a lab book accompanying an advanced course in structural geology, but I also strongly recommend this book as a valuable basic resource for many Earth science students and professionals throughout their career.

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Structural geology is focused on the description and mechanical analysis of structures in the lithosphere like faults, folds, fabrics, and other forms of localized and distributed deformation. Structural geologists seek efficient and quantitative characterization and visualization of the geometry of geologic structures, they hope to quantify the strain experienced by deformed rocks, they aim to gain understanding of the forces that cause deformation, and they develop equations that describe the constitutive properties of rocks and faults. The mathematical framework for these efforts is linear algebra. Vectors, tensors and coordinate transformations underlie most of the graphical methods and mathematical manipulations we employ to solve structural geology problems. Often students learn the mechanical operations employing these mathematical principles in graphical methods (such as the manipulation of orientation data on a stereonet or the construction of a graphical Mohr circle representation of stress or strain), but they do not understand the underlying mathematical principles and calculations. This book provides this framework and the mathematical operations needed to apply linear algebra to structural geology problems.

The best way to learn mathematical concepts and methods is to apply them in actual computations. Thus the book includes examples and exercises which often draw on short program functions provided in the text and online. Well-commented and easily digested Matlab routines allow the student to follow and apply the mathematical operations and the functions that are easily modified and integrated to solve problems. The functions are drawn in part from programs written by the authors that are in wide use in the structural geology community. At least in my research group, we now mostly rely on Matlab to solve problems and thus I appreciate the reliance on this computing environment.

The structure of the book is generally parallel to the treatment of topics in standard structural geology textbooks, divided into 12 chapters. The first chapter previews the basic objectives and problem-solving needs in structural geology, including the description, visualization and quantification of geologic structures and their evolution due to deformation. The chapter also introduces linear algebra as the computational framework to solve these problems and Matlab as the convenient computing syntax to carry out related calculations. Spherical projections used to plot orientation data on stereonets and to develop projections that produce flat maps of the Earth's spherical surface are explained in some detail. Finally, a detailed three-page table summarizes all the functions provided, where they appear in the book and how they relate to each other. All remaining chapters are complemented by exercises allowing the student to apply what has been learned and deepen the understanding of the material.

The next four chapters introduce the basic components of linear algebra in less than one hundred pages. Chapter 2 summarizes coordinate systems, scalars and vectors and how they can be used to represent geologic features. Chapter 3 introduces transformations (translations and rotations) of coordinate axes and vectors and their relevance to structural analysis. Chapter 4 illustrates the utility of matrix operations and notations in related computations. Indicial notations, summation conventions and elementary matrix operations are clearly and succinctly explained and accompanied by code snippets carrying out the needed calculations. Chapters 2 through 4 develop the background for Chap. 5, which is dedicated to the concept of tensors, adding second-order tensors used to describe stress and strain and other physical quantities to the already introduced lower order scalars and vectors. The nature of tensors, their notation, transformations and fundamental properties are efficiently explained and the important concept of eigenvalues and eigenvectors is introduced.

Chapters 6 through 11 apply linear algebra to the treatment of stress and strain that underlie much of structural geology. The three-dimensional state of stress is most easily understood as a tensor quantity and Chap. 6 provides a succinct treatment of the notation, transformations and applications used to evaluate stress. Many geologic textbooks introduce the Mohr circle as a somewhat magical graphical representation of the relationship of various stress components, while the underlying principles are so much easier explained when recognizing the tensor properties of stress. A detailed exploration of methods focused on deducing the state of stress from orientations of fault and slip vector orientations serves as an illustrative example of the utility of tensor transformations. The next four chapters are devoted to the introduction of deformation, infinitesimal strain, finite strain, and progressive strain histories. Again, these topics are introduced in a succinct but comprehensive fashion that readers should appreciate. I particularly enjoyed the consideration of tectonic geodetic measurements of deformation in this treatment, which includes an introduction to least-squares fitting of GPS displacement measurements and a nice pair of exercises exploring the deformation field of a large subduction zone earthquake in Chile.

Chapter 11 introduces the interesting and relatively complex problems associated with the development of kinematic models of tectonic deformation, with a focus on various types of models of fault-related folds, which is a research application for which the authors have also written fully developed software programs available to the research community (available at http://www.geo.cornell.edu/geology/faculty/RWA/programs/ and http://www.ux.uis.no/~nestor/work/programs.html).

The final chapter deals with the very important topic of measurement uncertainties and error analysis. Unfortunately, following just two pages introducing this topic and introducing the propagation of errors, the reader is referred to other sources to learn more about statistics and error analysis. Instead, the chapter focuses on the relatively specialized topic of uncertainties in balanced cross sections and one particular type of fold–fault model introduced in Chap. 11. I suspect that most undergraduate students will not be able to follow this treatment and it would have been good to explore the importance of how to formally treat uncertainty and errors in structural analyses focusing on the more fundamental topics of stress and deformation introduced earlier. Maybe this can be considered for a future edition of this valuable text.

The book is well crafted and relatively free of typos and errors. The authors have posted an errata with some corrections at http://www.geo.cornell.edu/geology/faculty/RWA/structural-geology-algorith/.

I highly recommend this book to all structural geology students and practitioners, as well as to earth scientists from a wide range of fields, who will benefit from this clear introduction of the principles and application of linear algebra in the analysis of commonly encountered vector and tensor quantities.