Flex2D v. 5.2

2D flexural modelling

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Flex2D is free for academic, non-profit purposes. If you want to use the program for commercial purposes, please contact me at nestor.cardozo@uis.no.

Referencing this program

Please reference the program as: Flex2D by Nestor Cardozo.

Credits

The plots in Flex2D rely on the SM2DGraphView Framework by Snowmint Creative Solutions. Formulas and algorithms are from Hetenyi (1946) for the case of an elastic beam of constant thickness, and Bodine (1981) for the case of an elastic beam of variable thickness. I would like to thank German Ojeda for sharing Matlab scripts for Bodine's solution. A short description is given in the Formulas section.

Introduction

The displacement profile of the surface of the Earth under crustal loads (i.e. mountain chains), can be reasonably reproduced by an elastic or flexural model. In this model, the uppermost layer of the Earth (i.e. the elastic lithosphere) responds to crustal loads as an elastic beam, floating in a weaker, fluid-like foundation (i.e. the astenospheric mantle) (Turcotte and Schubert, 1982; Watts, 2001).

This simple flexural model provides insight into how tectonic loads (i.e. mountain chains) and sedimentary basins are linked, and how the crust and mantle support loads. The use of the flexural model has allowed geologists to understand the regional variations of strength of the lithosphere, and the implications that these variations have for mountain building, sedimentary basin formation, and earthquakes (Watts, 2001).

Flex2D is a program that implements the elastic flexural model. Crustal loads can be entered and plotted, the profile of deformation produced by these loads can be computed and visualized, and the effect of changes in model parameters can be evaluated. Geologists often talk about the link between tectonic loads and sedimentary basins, but they rarely do the math. Perhaps, because implementing the flexural model is tedious, specially if done in a spreadsheet. **Flex2D** makes flexural modeling easy.

Conventions

There are four parameters that define a load column in **Flex2D**. These parameters can be entered in the table of the *Loads* view (Figure 1):

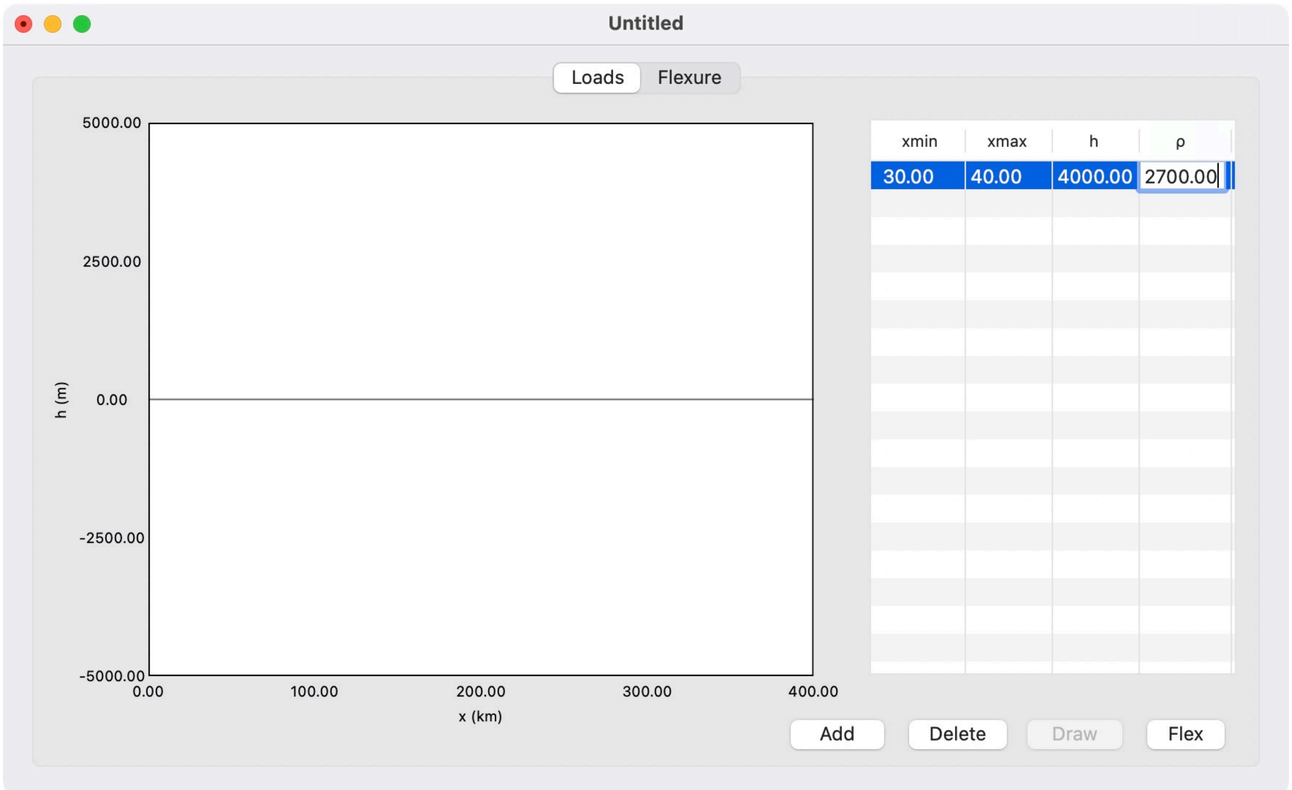


Figure 1. The *Loads* view and entering a load column

For each load column, it is necessary to input:

1. *xmin* = The x coordinate at the left side of the column in kilometers.
2. *xmax* = The x coordinate at the right side of the column in kilometers
3. *h* = The column height in meters
4. *ρ* = The column density in kg/m^3.

The *x* coordinate at the right side of the column should be higher than the *x* coordinate at the left side of the column, otherwise the load is not plotted and not included in the computation. **Please notice that when the broken beam solution is selected, loads with *x* coordinates lower than zero are not considered in the computation.** The load height can be positive or negative. Notice that the *x* coordinates (distance) are given in km while the *y* coordinates (height) are given in meters. The load density *ρ* should be positive. The meaning of each column in the *Loads* table can be observed by placing the mouse over the column labels (each label has a help tip associated to it).

There are five parameters that define the flexural model:

1. *E* = The Young Modulus of the elastic lithosphere in GPa ($1e^9$ Pa).
2. ν = The Poisson ratio of the elastic lithosphere. This parameter is dimensionless and it should be ~0.25.
3. The elastic thickness in km. This is the thickness of the outmost layer of the Earth that behaves elastically (i.e. the elastic lithosphere). This thickness can be constant or variable along the profile (see the Interface section).
4. ρ_f = Density of foundation in kg/m^3. This is the difference between the density of the mantle and the density of the material filling the basin.
5. x_{int} = The *x* interval of computation in km. In **Flex2D** the profile is divided in intervals of length x_{int}. The deflection is computed at these intervals.

These parameters can be set through the *Inspector* panel (see the Interface section).

In the table of the *Flexure* view, the resultant topography *t* is reported in meters above sea level, and the downward displacement *u* in meters (Figure 2). Positive and negative values of *u* indicate downward and upward displacement, respectively. The meaning of each column in the table of the *Flexure* view can be observed by moving the mouse over the column labels (each label has a help tip associated to it).

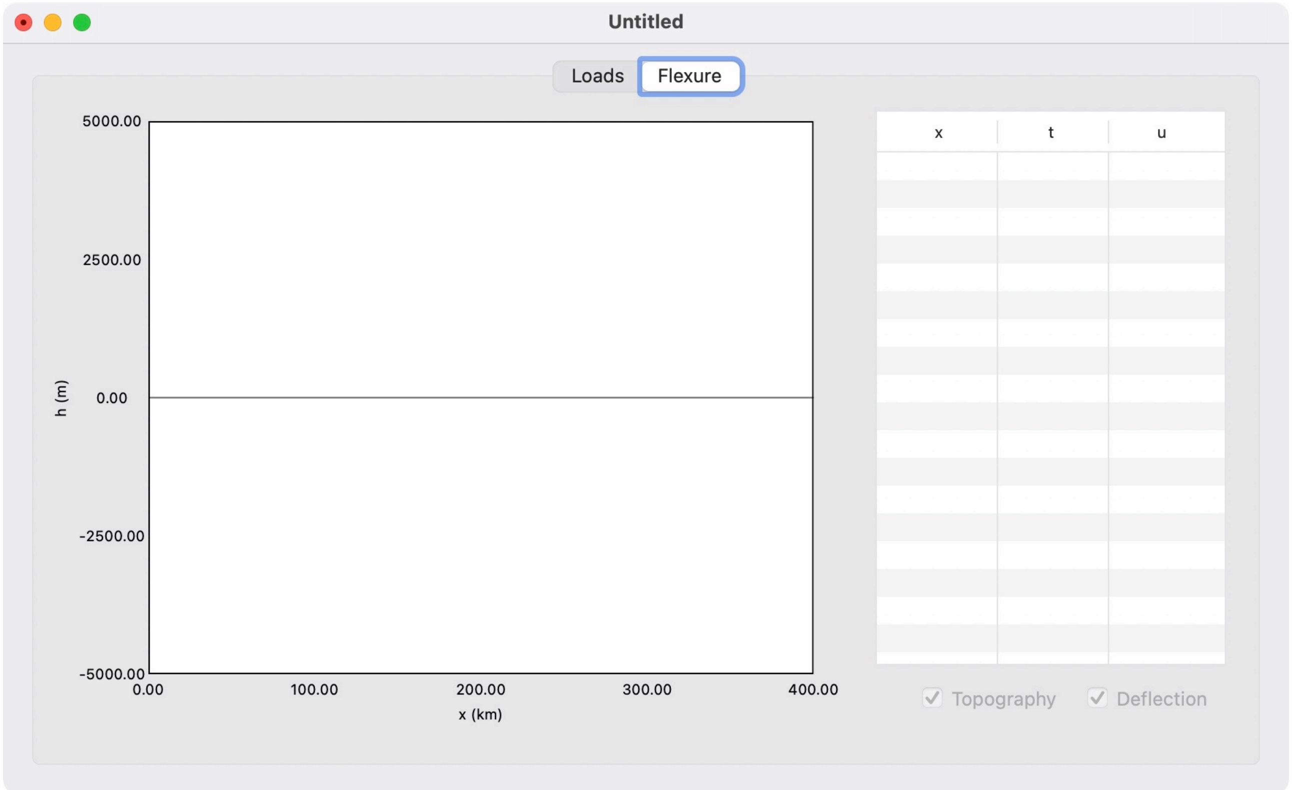


Figure 2. The *Flexure* view

Density of the foundation

- If the resultant depression is not filled with any material, ρ_f should be equal to the density of the mantle ρ_m.
- If the resultant depression is filled with water of density ρ_w, ρ_f should be equal to (ρ_m - ρ_w).
- If the resultant depression is filled with sediment of average density ρ_s, ρ_f should be equal to: (ρ_m - ρ_s).

Infinite or broken beam:

There are two types of solutions in **Flex2D**: Infinite beam, and broken beam. In the infinite beam solution, the elastic beam extends infinitely along the *x* axis. In the broken beam solution, the beam has a free end at *x* = 0, and extends infinitely in the positive *x* direction. Since the free end of the broken beam is at *x* = 0, loads with *x* coordinates lower than zero are not considered in this case. The type of beam, infinite or broken, can be set through the *Inspector* panel (see the Interface section). By default, **Flex2D** uses the infinite beam solution.

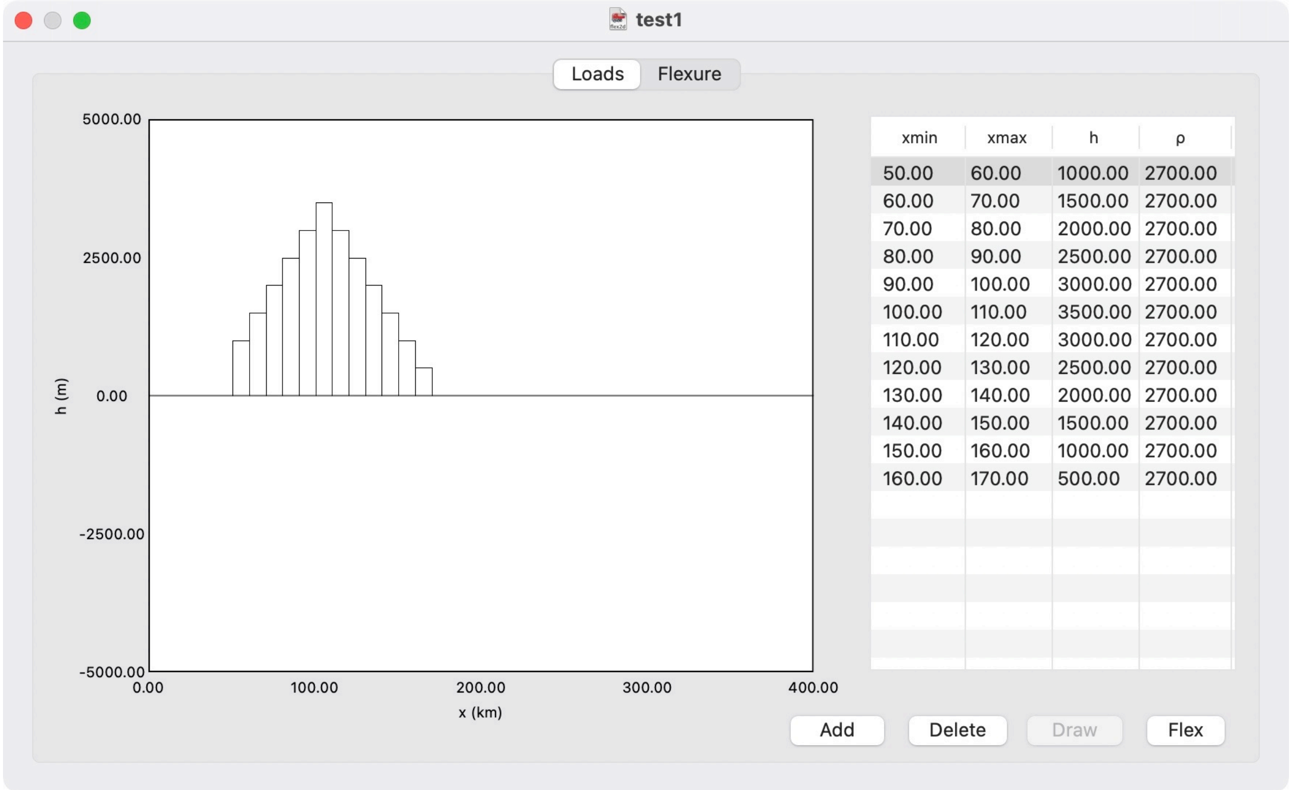
The elastic thickness can be set constant or variable and its value(s) can be edited in the *Inspector* panel (see the Interface section). By default, **Flex2D** uses a constant elastic thickness.

Interface

Flex2D consists of two views: 1. A *Loads* view, and 2. A *Flexure* view. You can choose any of these views by either clicking the tabs on top of the window, using the *Modules* menu, or typing ⌘1 or ⌘2 for the *Loads* or the *Flexure* view, respectively. **Flex2D** is a document-based application. You can open as many documents as you want; each will contain its own analysis.

The *Loads* view

In the *Loads* view (Figure 3) you can enter and visualize the loads. You can add or remove loads by either clicking the *Add* or *Delete* buttons, choosing the *Add* or *Delete* submenus in the Loads menu, or typing ⌘= (add) or ⌘- (delete). For a description of the parameters needed for defining a load see the conventions section. When entering the loads, you can undo (*Undo* submenu in the *Edit* menu or ⌘Z) or redo (*Redo* submenu in the *Edit* menu or ⇧⌘Z) changes.



xmin	xmax	h	ρ
50.00	60.00	1000.00	2700.00
60.00	70.00	1500.00	2700.00
70.00	80.00	2000.00	2700.00
80.00	90.00	2500.00	2700.00
90.00	100.00	3000.00	2700.00
100.00	110.00	3500.00	2700.00
110.00	120.00	3000.00	2700.00
120.00	130.00	2500.00	2700.00
130.00	140.00	2000.00	2700.00
140.00	150.00	1500.00	2700.00
150.00	160.00	1000.00	2700.00
160.00	170.00	500.00	2700.00

Figure 3. The *Loads* view with a collection of load columns

You will notice that the program by default automatically draws the loads if they have been entered correctly (see the Conventions section). This behavior can be changed in the *Preferences* panel.

Adding loads from the pasteboard or a text file

If you find annoying entering loads in the *Loads* table, you can import the loads from the pasteboard or a text file. Open a text editor or a spreadsheet program. Enter the loads as in **Flex2D**. If in a text editor, use spaces, commas, or tabs to separate the entries of a load, and the return key to start a new load. When finished, you can either copy the contents of the file and paste them in to the *Loads* table, or save the file as a text file (txt extension). Then you can drag the file into the *Loads* table. If entered correctly, the loads will be added to the Loads table.

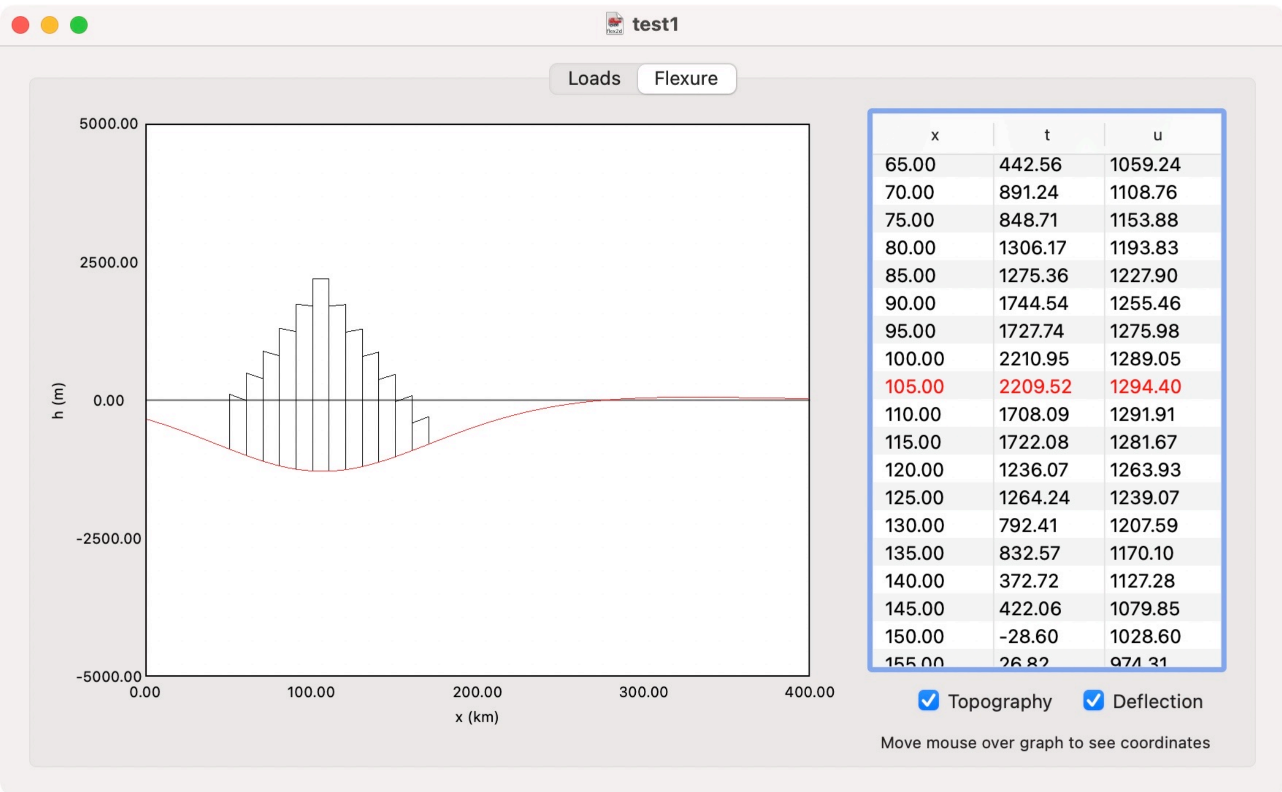
You can save the loads by either choosing the *Save* submenu in the *File* menu, or typing ⌘S. The program will create a file with extension *flex2d*.

Flexing the profile

Once the loads are entered, you can flex the profile by either clicking the *Flex* button, choosing the *Flex* submenu in the *Units* menu, or typing ⇧⌘F. The program will automatically switch to the *Flexure* view and show the deformed profile. Notice that **Flex2D** does not automatically update the *Flexure* view according to changes in the *Loads* view. To update the *Flexure* view you will need to flex the profile.

The *Flexure view*

Before applying the loads, the *Flexure* view is not very exciting. It just shows a blank graph and a table without data (Figure 2). When you flex the profile (click the *Flex* button, or choose the *Flex* submenu), the program automatically chooses the *Flexure* view. Both, the deformed topography (by default black) and the displacement profile (by default red) are shown in a graph and in a table (Figure 4). In the table, the rows for the maximum (basin axis) and minimum (forebulge) deflection are shown in red and blue respectively.



x	t	u
65.00	442.56	1059.24
70.00	891.24	1108.76
75.00	848.71	1153.88
80.00	1306.17	1193.83
85.00	1275.36	1227.90
90.00	1744.54	1255.46
95.00	1727.74	1275.98
100.00	2210.95	1289.05
105.00	2209.52	1294.40
110.00	1708.09	1291.91
115.00	1722.08	1281.67
120.00	1236.07	1263.93
125.00	1264.24	1239.07
130.00	792.41	1207.59
135.00	832.57	1170.10
140.00	372.72	1127.28
145.00	422.06	1079.85
150.00	-28.60	1028.60

Figure 4. The *Flexure* view after flexing the profile

You can toggle on and off the topography and deflection curves using the buttons below the table. You can move the mouse over the *Flexure* graph to see its coordinates.

Changing parameters and editing plots

You can set the model parameters, edit the graphs, or change the properties of lines in the plots using the *Inspector* panel (*Inspector* submenu in the *Tools* menu, or key combination ⇧⌘I). The model parameters can be set in the *Parameters* view, the plots parameters can be changed in the *Plot* view, and the properties of lines can be set in the *Lines* view of the Inspector (Figure 5).

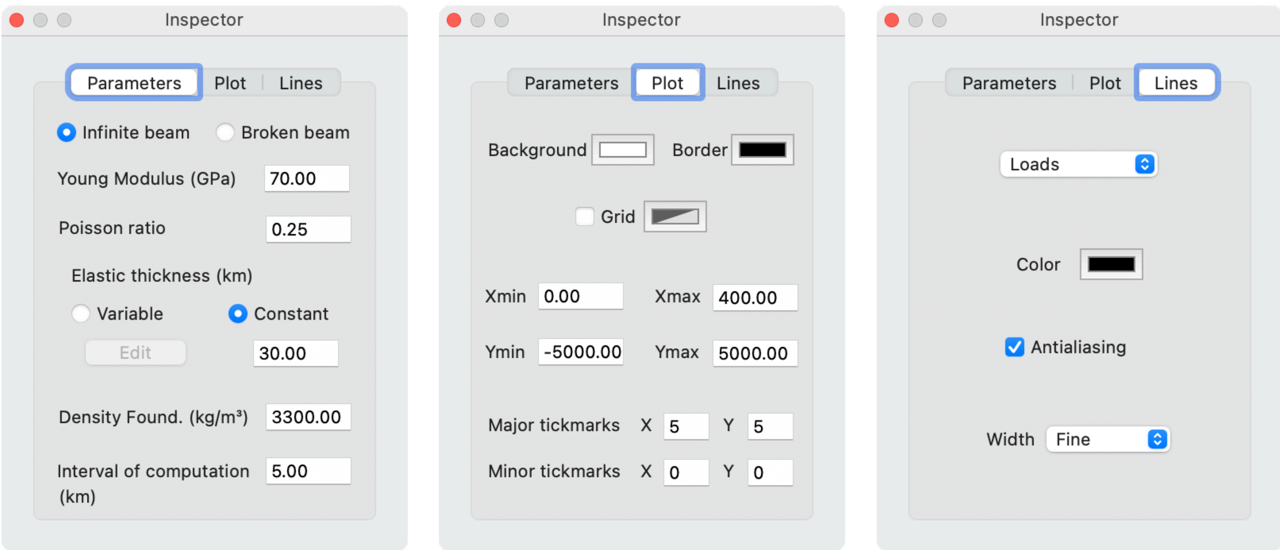
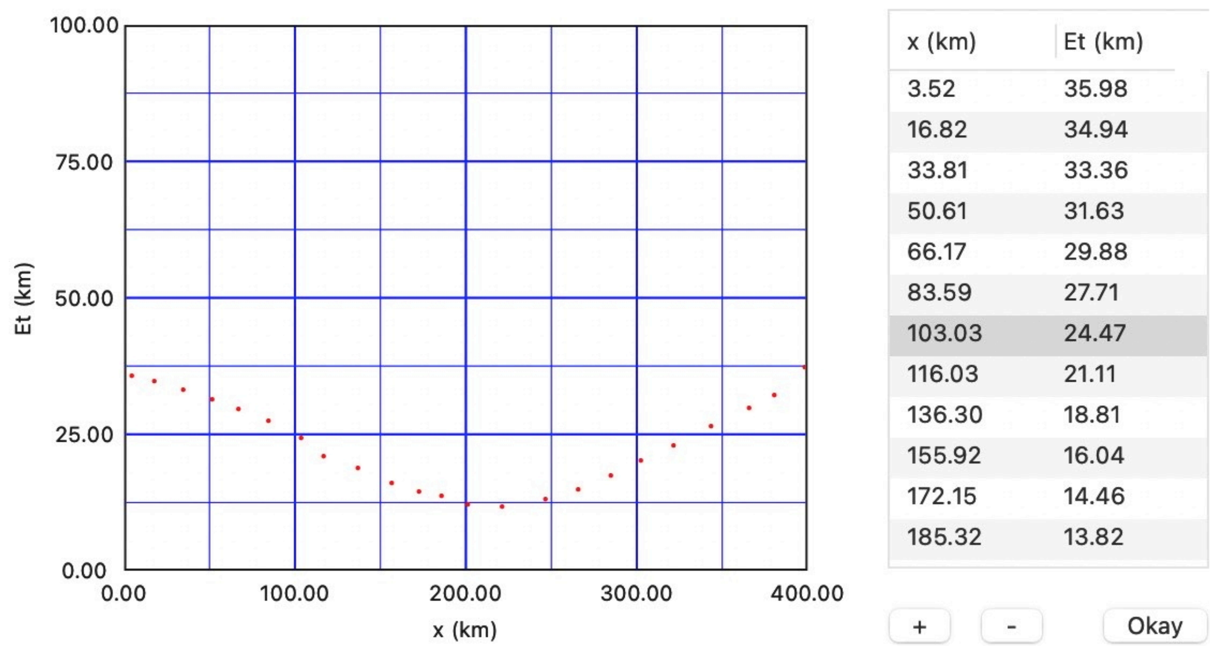


Figure 5. The *Inspector* panel and the *Parameters* (left), *Plot* (middle), and *Lines* (right) views

In the *Parameters* view, you can set the type of solution: Infinite or broken beam. The infinite beam solution uses an elastic beam that extends infinitely along the *x* axis. The broken beam has a free end at $x = 0$, and extends infinitely in the positive *x* direction. When the broken beam solution is selected, loads with *x* coordinates lower than zero are not considered. **Flex2D** uses by default the infinite beam solution.

In the *Parameters* view, you can also specify the model parameters: i. The Young Modulus, Poisson's ratio, and elastic thickness of the lithosphere, ii. the density of the foundation, and iii. the *x* interval at which the deflection is computed. The density of the foundation is the difference between the density of the mantle and the density of the material filling the basin. You can set this value to simulate: i. No material filling the resultant depression, ii. water filling the resultant depression, or iii. sediments filling the resultant depression (see Conventions section). By default the density of the foundation is the density of the mantle: 3300 kg/m³. This corresponds to the case of no material filling the basin.

The elastic thickness can be constant or variable. By default **Flex2D** uses a constant elastic thickness. If you choose the constant elastic thickness option (Figure 5, left), you can set the elastic thickness in the corresponding text field. If you choose variable elastic thickness, you can input the elastic thickness values along the profile by clicking the *Edit* button (Figure 5, left). This will open a sheet where you can enter the elastic thickness by either: (i) clicking on the graph, (ii) entering points along the profile in the table, or (iii) dropping in the table a text file with 2 columns corresponding to the *x* locations and values of elastic thickness (Figure 6).



x (km)	Et (km)
3.52	35.98
16.82	34.94
33.81	33.36
50.61	31.63
66.17	29.88
83.59	27.71
103.03	24.47
116.03	21.11
136.30	18.81
155.92	16.04
172.15	14.46
185.32	13.82

Figure 6. Defining a variable elastic thickness along the profile

The equations for the deflection of an infinite or broken beam of constant thickness are from Hetenyi (1946), and those for beams of variable thickness are from Bodine (1981). A short description of the equations is given in the Formulas section.

The *Inspector* is a great way to see the effect of changing model parameters. Input the loads, flex the profile, open the *Inspector* and in the *Parameters* view change the elastic thickness (Figure 5, left). The program will automatically update the flexure graph and table for the new elastic thickness. The *Inspector* can also be used to change the extent (horizontal or vertical) of the model (Figure 5, middle).

Editing your graphs for publications or presentations is very easy. Experiment with both the *Plot* and *Lines* views of the *Inspector*. Notice that you can choose the line (loads, topography or deflection) to be modified using the first pop-up button in the *Lines* view (Figure 5, right). Changes in the *Inspector* affect the selected document. The settings in the Inspector are saved to flex2d files. However, only the Input loads and the Inspector settings are saved. After opening a flex2d file, you will have to flex the profile to see the results.

Saving plots

You can save the plots produced by **Flex2D** as pdf files. Just choose the *Save Plot as PDF* submenu in the *File* menu.

Copy and paste

You can drag plots to vector programs, and the content of the tables to text editors and spreadsheets. You can also drag these elements to the Finder to make a pdf of a plot, or a text clipping of the selected data. You can also copy and paste table data (e.g. to a spreadsheet) using the copy (⌘C) and paste (⌘V) submenus.

Preferences panel

You can set whether or not the program draws the loads automatically or open a new document at launch, through the *Preferences* panel (Figure 7).

Formulas

In the *flexural* model, the uppermost layer of the Earth responds to crustal loads as an elastic beam, floating in a weaker, fluid-like foundation (i.e. the astenosphere) (Turcotte,

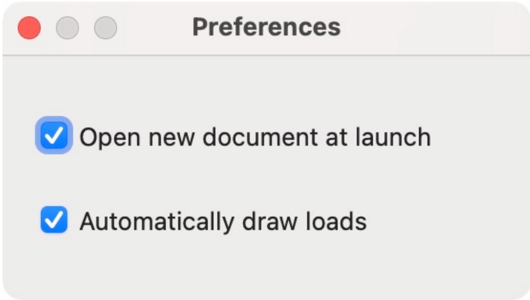


Figure 7. The *Preferences* panel

1982; Watts, 2001). The solution for this problem is the same as the solution for an elastic beam on an elastic foundation. If the thickness of the beam is constant, the solutions for and infinite or broken beam are given by Hetenyi (1946):

Infinite beam

The infinite beam extends infinitely along the *x* axis. The procedure to compute the deflection produced by a load column on the beam is as follows:

1. Use the flexural rigidity D to express the strength of the lithosphere (i.e. elastic beam):

$$D = \frac{ETe^3}{12(1-\nu^2)} \quad (1)$$

where E is the Young's modulus, ν is the Poisson's ratio, and Te is the elastic thickness.

2. Invent a term $D_{(\alpha,x)}$:

$$D_{(\alpha,x)} = \exp(-x/\alpha)\cos(x/\alpha) \quad (2)$$

where x is the horizontal coordinate and α is a length parameter given by:

$$\alpha = \left[\frac{4D}{\rho_f g}\right]^{\frac{1}{4}} \quad (3)$$

where ρ_f is the density of the foundation. This is the difference between the density of the mantle (ρ_m) and the density of the material filling the basin. g is gravity.

3. The deflection u of any point along the beam is equal to:

a. If the point is under the load column:

$$u = \frac{q}{2k}(2 - D_{(\alpha,a)} - D_{(\alpha,b)}) \qquad (4)$$

b. If the point is to the left of the load column:

$$u = \frac{q}{2k}(D_{(\alpha,a)} - D_{(\alpha,b)}) \qquad (5)$$

c. If the point is to the right of the load column:

$$u = -\frac{q}{2k}(D_{(\alpha,a)} - D_{(\alpha,b)}) (6)$$

where $\frac{q}{k} = h\frac{\rho}{\rho_f}$, h is the height of the load column, and ρ is the load density. a and b are measured as absolute distances from the point to the left and right borders of the load column, respectively.

Broken beam

The broken beam has a free end at *x* = 0 and extends infinitely in the positive *x* direction.The deflection of this beam is obtained by first considering the forces that are generated during bending of an infinite beam, and then "nulling" the bending moment and shear force at the free-end *x* = 0. The procedure is described in detail by Hetenyi (1946):

1. Introduce new terms $A_{(\alpha,x)}$, $B_{(\alpha,x)}$ and $C_{(\alpha,x)}$:

$$A_{(\alpha,x)} = \exp(-x/\alpha)(\cos(x/\alpha) + \sin(x/\alpha)) \qquad (7)$$

$$B_{(\alpha,x)} = \exp(-x/\alpha)\sin(x/\alpha) \qquad (8)$$

$$C_{(\alpha,x)} = \exp(-x/\alpha)(\cos(x/\alpha) - \sin(x/\alpha)) \qquad (9)$$

2. Compute the moment and shear force at *x* = 0:

$$M_0 = \frac{q\alpha^2}{4}(B_{(\alpha,a')} - B_{(\alpha,b')}) \qquad (10)$$

$$Q_0 = \frac{q\alpha}{4}(C_{(\alpha,a')} - C_{(\alpha,b')}) \qquad (11)$$

where a' and b' are the distances from *x* = 0 to the left and right side of the load column, respectively.

3. Compute the moment and shear force required to cancel the moment and shear force at $x = 0$:

$$M_C = -\frac{q\alpha^2}{2}[2(B_{(\alpha,b')} - B_{(\alpha,a')}) + (C_{(\alpha,a')} - C_{(\alpha,b')})] \quad (12)$$

$$Q_C = q\alpha[(B_{(\alpha,b')} - B_{(\alpha,a')}) + (C_{(\alpha,a')} - C_{(\alpha,b')})] \quad (13)$$

To simplify the equations, let's call the second terms of Eqs. 12 and 13 aa and bb:

$$aa = 2(B_{(\alpha,b')} - B_{(\alpha,a')}) + (C_{(\alpha,a')} - C_{(\alpha,b')}) \quad (14)$$

$$bb = (B_{(\alpha,b')} - B_{(\alpha,a')}) + (C_{(\alpha,a')} - C_{(\alpha,b')}) \quad (15)$$

4. The deflection at any point along the beam is:

a. If point is under the load column:

$$u = \frac{q}{2k}[(2 - D_{(\alpha,a)} - D_{(\alpha,b)}) + bbA_{(\alpha,x)} - aaB_{(\alpha,x)}] \quad (16)$$

b. If point is to the left of the load column:

$$u = \frac{q}{2k}[(D_{(\alpha,a)} - D_{(\alpha,b)}) + bbA_{(\alpha,x)} - aaB_{(\alpha,x)}] \quad (17)$$

c. If point is to the right of the load column:

$$u = \frac{q}{2k}[(D_{(\alpha,b)} - D_{(\alpha,a)}) + bbA_{(\alpha,x)} - aaB_{(\alpha,x)}] \quad (18)$$

Principle of superposition

For an infinite or broken beam, the displacement profile is computed for each load column. The total displacement profile is the sum of the displacement profiles of the load columns.

Variable elastic thickness

For a beam of variable thickness, the solution for and infinite or broken beam are given by Bodine (1981). These solutions are numerical and are based on finite differences. The solution for an infinite beam is given by Bodine in his program FMDEPYE, and for a broken beam in his program FMDEPYEC. The reader is referred to Bodine (1981) for further details about these programs.

For the same elastic thickness and load distribution, the Hetenyi and Bodine solutions don't give exactly the same results. This is because Hetenyi's solution is analytical while Bodine's solution is numerical, and also because in Hetenyi's solution loads are entered as columns, while in Bodine's solution loads change continuously along the profile. This last difference will be less marked as the width of the columns decreases.

References

Bodine J.H. 1981. Numerical computation of plate flexure in marine geophysics. Lamont Doherty Geological Observatory of Columbia University. Technical Report 1.

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