The Cauchy problem for Linear ODEs with constant coefficients

Abstract. What else can be said about a venerable and basic subject like Linear ODEs with constant coefficients? The issue is that there is almost no analysis in it and a pair of algebraic spectacles help to reveal more symmetries in the topic. The purpose of the talk, intended for a wide audience of non specialists, is to solve the Cauchy problems for linear ODEs of order r all at once by solving once and for all the *generic Cauchy problem*

$$\begin{cases} y^{(r)} - e_1 y^{(r-1)} + \ldots + (-1)^r e_r y & = & f \\ \\ (D^i y)(0) & = & a_i \end{cases} \tag{1}$$

where $0 \leq i \leq r-1$, a_i are the inital data and f may be thought of as an analytic function.

Each concrete Cauchy problem can be deduced from the general solution we shall propose to (1). As an application we shall rediscover the Euler formula $\exp(\sqrt{-1}t) = \cos t + \sqrt{-1}\sin t$, we shall obtain the ultimate generalization of $\cos^2 t + \sin^2 t = 1$ and learn how to compute the exponential of a matrix avoiding the use of Jordan Normal form (in collaboration with Inna Scherbak, Tel Aviv).

Besides its elegance, this approoach is interesting because of its relationships with the theory of symmetric functions, the intersection theory on complex Grassmannians (Schubert Calculus), the Boson-fermion correspondence between the bosonic and fermionic representation of the Heisenberg Algebra, the vertex operators which are known to be the discrete counterpart of those arising in the Skyrme model of self-interacting meson-like fields, and the celebrated Kadomtsev–Petviashvili (KP) equation that arose for the first time in the plasma physics. These motivations and relationships will not be treated in the talk, but possibly postponed to further discussions.