Stress concentration

- Transition of cross sections → High stresses
- More abrupt transition → Higher stresses

Abrupt change
Stress “flow lines” crowd
High stress concentration

Smotherer change
“Flow lines” less crowded
Lower stress concentration
- Elementary stress equations don’t apply in stress concentrations.

\[ \sigma_{\text{avg}} = \frac{P}{A} = \frac{P}{(b-d)h} \]

\[ \sigma_{\text{max}} = K_c \sigma_{\text{avg}} \]

Stress concentration factor from charts

Figure 6.1 Rectangular plate with hole subjected to axial load.
(a) Plate with cross-sectional plane.
(b) Half of plate with stress distribution.
Stress concentration factor $K_c$

- Obtained experimentally, analytically, etc
- Published in charts
- Geometric property
- Very important in *brittle* materials
- In ductile materials:
  - Important in *fatigue* calculation.
  - Important if safety is critical.
  - Localized yielding hardens material (strain hardening).
  - Redistributes stress concentration.
Stress Concentrations for Plate with Fillet

Figure 6.3  Stress concentration factor for rectangular plate with fillet.  (a) Axial Load.  [Adapted from Collins (1981).]
Stress Concentrations for Plate with Fillet (cont.)

Figure 6.3 Stress concentration factor for rectangular plate with fillet. (b) Bending Load. [Adapted from Collins (1981).]

Text Reference: Figure 6.3, page 223
Stress Concentrations for Plate with Hole

Figure 6.2  Stress concentration factor for rectangular plate with central hole.  (a) Axial Load.  [Adapted from Collins (1981).]

Text Reference: Figure 6.2, page 222
Stress Concentrations for Plate with Hole (cont.)

Figure 6.2 Stress concentration factor for rectangular plate with central hole. (b) Bending. [Adapted from Collins (1981).]

Text Reference: Figure 6.2, page 222
Figure 6.5 Stress concentration factor for round bar with fillet. (b) Bending. [Adapted from Collins (1981).]
Stress Concentrations for Bar with Fillet (cont.)

Figure 6.5 Stress concentration factor for round bar with fillet. (c) Torsion.
[Adapted from Collins (1981).]
Stress Concentrations for Bar with Groove (cont.)

Figure 6.6 Stress concentration factor for round bar with groove. (b) Bending.
[Adapted from Collins (1981).]
Stress Concentrations for Bar with Groove (cont.)

Figure 6.6  Stress concentration factor for round bar with groove.  (c) Torsion.  
[Adapted from Collins (1981).]
Fracture mechanics

- All materials contain cracks.
- If crack is bigger than critical dimension, it propagates \[ a > 2a \text{, where} \]

\[
a = \frac{1}{\pi} \left( \frac{K_{ci}}{Y\sigma_{nom}} \right)^2
\]

Fracture toughness, from tables

Geometric correction factor

Figure 6.8 Three modes of crack displacement. (a) Mode I, opening; (b) mode II, sliding; (c) mode III, tearing.

- Fracture mechanics predict mode I crack propagation \textbf{if crack size} > 2a, where
# Yield Stress and Fracture Toughness Data (@room Temperature)

<table>
<thead>
<tr>
<th>Material</th>
<th>Yield Stress, $S_y$</th>
<th>Fracture Toughness, $K_{1c}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Metals</strong></td>
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<tr>
<td>Aluminum alloy</td>
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<tr>
<td>2024-T351</td>
<td>47 ksi, 325 Mpa</td>
<td>33 ksi, 36 Mpa</td>
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<tr>
<td>Aluminum alloy</td>
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<tr>
<td>7075-T651</td>
<td>73 ksi, 505 Mpa</td>
<td>26 ksi, 29 Mpa</td>
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<td>Alloy steel 4340</td>
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<td></td>
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<tr>
<td>tempered at 260°C</td>
<td>238 ksi, 1640 Mpa</td>
<td>45.8 ksi, 50.0 Mpa</td>
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<tr>
<td>Alloy steel 4340</td>
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<td></td>
</tr>
<tr>
<td>tempered at 425°C</td>
<td>206 ksi, 1420 Mpa</td>
<td>80.0 ksi, 87.4 Mpa</td>
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<td>Titanium alloy</td>
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<tr>
<td>Ti-6Al-4V</td>
<td>130 ksi, 910 Mpa</td>
<td>40-60 ksi, 44-66 Mpa</td>
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<tr>
<td><strong>Ceramics</strong></td>
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<tr>
<td>Aluminum oxide</td>
<td>2.7-4.8 Mpa</td>
<td>3.0-5.3 Mpa</td>
</tr>
<tr>
<td>Soda-lime glass</td>
<td>0.64-0.73 Mpa</td>
<td>0.7-0.8 Mpa</td>
</tr>
<tr>
<td>Concrete</td>
<td>0.18-1.27 Mpa</td>
<td>0.2-1.4 Mpa</td>
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<tr>
<td><strong>Polymers</strong></td>
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<tr>
<td>Polymethyl methacrylate</td>
<td>0.9 Mpa</td>
<td>1.0 Mpa</td>
</tr>
<tr>
<td>Polystyrene</td>
<td>0.73-1.0 Mpa</td>
<td>0.8-1.1 Mpa</td>
</tr>
</tbody>
</table>
Stress intensity factor

• Recall critical crack length = 2 \( a \).

\[
a = \frac{1}{\pi} \left( \frac{K_{ci}}{Y \sigma_{nom}} \right)^2
\]

\( K_{ci} \) = Fracture toughness, from tables

\( Y \) = Geometric correction factor

• So fracture toughness is

\( K_{ci} = Y \sigma_{nom} \sqrt{\pi a} \)

• If the actual crack length is \( 2x \), the **stress intensity factor** is defined as

\( K_i = Y \sigma_{nom} \sqrt{\pi x} \)

• Crack will propagate if \( K_i > K_{ci} \), or \( 2x > 2a \).

• Safety factor against crack propagation is \( K_{ci} / K_i \).