Optimal Filter-Bank Design for Multiple Texture Discrimination

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Abstract

An important category of texture features are features extracted by filters together with a filter response energy measure. Most approaches to texture filtering use filter banks which are selected by some heuristic criteria and which are not optimal with respect to the given data. Some approaches to optimal filter design have been presented. A few approaches to multi-texture optimal filter bank design have also been presented, but they are all restricted to very small sets of free parameters. In this paper a new technique for multiple texture optimal design of a bank of general finite impulse response (FIR) filters is presented. A closed form optimal solution is obtained by modeling the feature extraction for the textures.

1 Introduction

Texture recognition is an important topic in image processing due to the fundamental importance of texture in several computer vision applications [1]. Several methods for texture discrimination have been proposed, and one important group use filters together with filter response energy measures as feature extractors. A typical system setup is illustrated in Fig. 1. Commonly the filter, $h(m,n)$, has been selected to be a Gabor filter or a filter derived from the wavelet transform [2]. Most approaches to feature extraction with filters use a bank of filters (multichannel filters) which is more or less determined irrespective of the actual textures to be discriminated, thus there is no guarantee that the bank of filters will be able to work properly for a specific set of textures. Another unpleasant consequence of most filter bank approaches is that they are computationally demanding.

For these reasons, the design of *optimal* filters for texture discrimination is gaining increased interest. Designing filters yielding optimal feature extraction is generally difficult, and many works have therefore concentrated on designing filters with a small number of free parameters [3, 4, 5, 6], and a small number of parameters yields few degrees of freedom. A general form FIR filter generally has many more free parameters than the above approaches, and some basically random search filter optimization approaches have been presented [7, 8, 9].

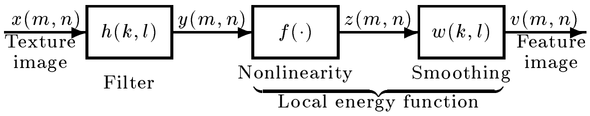

Figure 1: The experimental setup.

A few *two-texture closed form* FIR filter optimization techniques have also been presented [10, 11]. While two-texture problems may be interesting in inspection, the majority of texture segmentation and classification problems typically involve more than two textures. Hence, extensions to multiple texture cases are desirable. In this paper we will extend the closed form optimization techniques [10, 11] to be applicable for multiple texture segmentation and classification with a bank of filters.

The straightforward approach for generalizing two-texture optimization approaches to solutions for more than two textures is to design one filter for each possible pairing of two textures in the problem. This is a viable approach as long as the number of textures is moderate, but as the number of textures increases, the number of induced filters gets very large, e.g. 10 textures give 45 filters and 20 textures give 190 filters. Hence, we need another way of handling such problems. Building on the closed form two-texture FIR filter optimization approaches [11], a hierarchical feature extraction scheme will be proposed, where each filter separates one *group* of textures from another, then the textures within the groups are separated by subsequent filters recursively. The problems that need to be solved are

1. How to design a filter for optimal feature separation between two *groups* of textures. This issue is addressed in Section 2.

2. How to group the textures allowing robust feature extractors to be constructed. This issue is addressed in Section 3.

Finally, experimental results are presented Section 4 and a summary is given in Section 5.

2 Filter design

How do we design a filter for optimal feature separation between two groups of textures? In order to answer this question, we first derive a model for the feature extraction with the system of Fig. 1 in Subsection 2.1 and then we derive the optimization equations in Subsection 2.2.

2.1 Feature extraction model

Denote texture number i by $x_i(m,n)$, where m and n are the indices to the texture images. Filtering $x_i(m,n)$ by the filter $h(m,n)$ yields

$$\begin{aligned} y_i(m,n) &= h(m,n) * x_i(m,n) \\ &= \sum_{k=0}^{M_h-1} \sum_{l=0}^{N_h-1} h(k,l) x_i(m-k, n-l), \end{aligned} \quad (1)$$

where $*$ denotes two-dimensional convolution and $h(m,n)$ is an $M_h \times N_h$ filter. This operation corresponds to the first block of Fig. 1. We define

$$\mathbf{h} = \begin{bmatrix} h(0,0) \\ \vdots \\ h(0, N_h-1) \\ \vdots \\ h(M_h-1, N_h-1) \end{bmatrix}, \quad (2)$$

i.e. the vector formed by lexicographically ordering of the rows of $h(m,n)$, and

$$\mathbf{x}_i(m,n) = \begin{bmatrix} x_i(m,n) \\ \vdots \\ x_i(m, n-N_h+1) \\ \vdots \\ x_i(m-M_h+1, n-N_h+1) \end{bmatrix}. \quad (3)$$

Then, the output at pixel (m,n) for texture i, Eq. (1), may be rewritten

$$y_i(m,n) = \mathbf{h}^T \mathbf{x}_i(m,n). \quad (4)$$

Similarly, the smoothing operation may be defined as

$$v_i(m,n) = \mathbf{w}^T \mathbf{z}_i(m,n),$$

where $w(m,n)$ is the smoothing filter. This formulation may easily be extended to filter masks of any shape, but for simplicity, we only consider rectangular masks here.

Using this vector formulation for the filtering operations, it may easily be shown that the expected feature value at the output of the filter, defined by $\mu_{v_i} = E\{v_i(m,n)\}$, may be written as

$$\mu_{v_i} = (\mathbf{h}^T \mathbf{R}_{x_i x_i} \mathbf{h}), \quad (5)$$

where

$$\mathbf{R}_{x_i x_i} = E\left\{\mathbf{x}_i(m,n)\mathbf{x}_i^T(m,n)\right\}. \quad (6)$$

By writing the elements of $\mathbf{R}_{x_i x_i}$ explicitly and assuming wide sense stationarity, it is readily seen that $\mathbf{R}_{x_i x_i}$ is constructed from the two-dimensional autocorrelation function of texture x_i. The derivative of μ_{v_i} is given by [12]

$$\frac{\partial \mu_{v_i}}{\partial \mathbf{h}} = \frac{\partial(\mathbf{h}^T \mathbf{R}_{x_i x_i} \mathbf{h})}{\partial \mathbf{h}} = 2\mathbf{R}_{x_i x_i}\mathbf{h}. \quad (7)$$

If the textures $x_1(m,n) \cdots x_I(m,n)$ are filtered by the same filter, the overall mean feature value for all textures will be

$$\begin{aligned} \mu_v &= \frac{1}{I}(\mu_{v_1} + \cdots + \mu_{v_I}) \\ &= \mathbf{h}^T \frac{1}{I}(\mathbf{R}_{x_1 x_1} + \cdots + \mathbf{R}_{x_i x_i})\mathbf{h}. \end{aligned} \quad (8)$$

This expression is identical to the expression for the mean of one texture, if we replace the matrix $\mathbf{R}_{x_i x_i}$ for that texture with the average $\mathbf{R}_{x_i x_i}$ of all the textures in the group. Hence, the expression for the derivative of the feature mean for one texture is valid also for multiple textures.

2.2 Optimal texture-group filters

Optimizing filters for two-texture feature extraction with respect to the relative distance between the feature means,

$$J(\mathbf{h}) = \frac{(\mu_{v_1} - \mu_{v_2})^2}{\mu_{v_1}\mu_{v_2}}, \quad (9)$$

has proven useful [11] (the optimization approach presented here will also be valid for the criterion μ_{v_1}/μ_{v_2} proposed for two-texture optimal filter design by Singh and Mahalanobis [10]). If we let μ_{v_i} be the feature mean for texture *group* i and solve $\partial J(\mathbf{h})/\partial \mathbf{h} = 0$, it may easily be shown that the optimal solution is [2]

$$(\mathbf{R}_{x_2 x_2}{}^{-1} \mathbf{R}_{x_1 x_1})\mathbf{h} = \lambda \mathbf{h}, \quad (10)$$

where

$$\lambda = \frac{(\mathbf{h}^T \mathbf{R}_{x_1x_1} \mathbf{h})}{(\mathbf{h}^T \mathbf{R}_{x_2x_2} \mathbf{h})}. \tag{11}$$

The optimal filter coefficient vector is found as the eigenvector of $(\mathbf{R}_{x_2x_2}{}^{-1}\mathbf{R}_{x_1x_1})$ with the maximum corresponding criterion value $J(\mathbf{h})$. The proof that this is a *valid* solution to the optimization problem, i.e. that all eigenvectors $\mathbf{h}$ will satisfy Eq. (11) where λ is the corresponding eigenvalue is given in [2]. It is noted that this approach yields an exact closed form optimal solution, guaranteeing that the global optimum is attained.

3 Texture grouping

If a filter is required to separate one group of textures well from another, it is reasonable to assume that the textures in one group must share some characteristics which are less prevalent in the textures of the other group. In order to make such a grouping, some measure of the similarity is needed. It can in general not be expected that very dissimilar textures may be grouped with success. Of course, filters corresponding to all possible group pairs of the textures may be designed and the expected feature statistics computed for each texture to determine which grouping is best. However, if there are many textures, such an approach becomes exhaustive, e.g. with 16 textures to be divided into two groups, there are $2^{15} - 1 = 32767$ different group pairs. Hence, another way of determining the group pairs is desired.

In order to form two texture groups that may be successfully separated, we must ensure that the maximum of some distance measure between any two textures *within* a group is less than the minimum distance between any two textures *between* the groups. More formally, assume that textures $x_{i_1}(m,n) \cdots x_{i_I}(m,n)$ are one texture group, and $x_{j_1}(m,n) \cdots x_{j_J}(m,n)$ are another. The conditions for any group pair to yield successful filters are then

$$\max\{d_{i_k,i_l}\} < \min\{d_{i_p,j_q}\} \;\; \forall\, k,l,p < I,\; q < J \tag{12}$$

$$\wedge$$

$$\max\{d_{j_k,j_l}\} < \min\{d_{j_p,i_q}\} \;\; \forall\, k,l,p < J,\; q < I. \tag{13}$$

As similarity criterion, the criterion value for the optimal filter for the two textures is used, i.e. $d = -J$.

A disadvantage with the above conditions, is that the textures are fragmented into a lot of groups, since typically only small groups satisfy conditions (12) and (13). To be able to separate two groups, one filter is needed, and with many groups the number of group pairs will be large, hence a large number of filters must be applied. To reduce the number of filters, textures are therefore added to the groups as long as all the modeled feature means for one of the groups are above the means for the other.

4 Experiments

An example of the application of this filter bank design for the textures D4, D9, D19, D21, D24, D28, D29, D36, D37, and D38 [13] is shown in Fig. 2. There are just a few small misclassified regions, although some of the textures are visually very similar. In this particular example, the texture grouping yielded 17 texture group pairs where four were duplicates, i.e. 13 filters were designed. The filtering hierarchy is illustrated in Fig. 2(c). Note that due to the hierarchical nature of the solution, it is not necessary to compute filter responses for all image positions, e.g. it is not necessary to apply the filter separating between classes 0 and 5 to regions of the image that are already recognized as belonging to one of the classes 1 or 6. Hence, the *effective* number of filters is less than 13 in this case, which is very low compared to many filter bank approaches [2].

In an artificial mosaic of textures like the one in Fig. 2(a), the edges between textures are very discontiguous, i.e. having large high-frequency contents. Depending on the textures, the feature extraction filter may very well have high-pass characteristics. Hence it may have a boosted response at the edge instead of a smooth transition between the texture regions. In combination with the local energy function, this implies reduced edge accuracy, an effect which is evident e.g. around the second texture from the right in the top row of Fig. 2(b). Texture edges in real world images are likely to be much smoother. Hence such a boosted response will probably not occur to a great extent in real world texture mosaics.

5 Summary

In this paper an estimator for the mean value of texture features extracted with a texture filter, a quadratic nonlinearity, and a smoothing filter have been proposed. Based on the mathematical model for this estimator, an exact closed form multi-texture optimal filter bank design approach was developed. The performance of the design approach was assessed by an experiment on a complex 10-texture image. Generally, compared to using fixed filter banks the main disadvantages with an optimal approach are the need for better insight and a more complex design. The main advantages are the optimality, fewer filters and hence fewer features, and much simpler classification (thresholding with one feature is far less complex than applying a neural net or statistic classifier with mul-

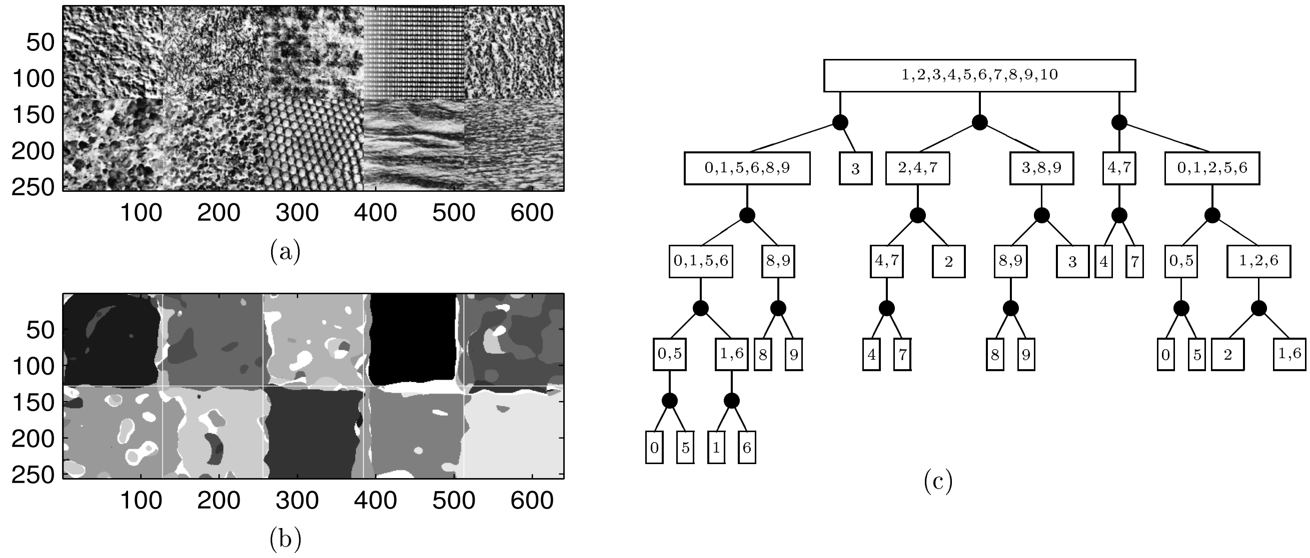

Figure 2: (a) Ten-texture image, (b) segmentation by the multi-texture approach, and (c) texture grouping hierarchy where each black disc corresponds to one filter. The numbers in (c) refer to texture number, where the textures in (a) are numbered 0 to 4 from left to right in the first row and 5 to 9 in the second.

tiple features). Furthermore, compared to the commonly used Gabor filters [3], successful results are obtained with significantly smaller, i.e. faster filters. Further discussion and results may be found in [2].

Acknowledgments

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