# TEXTURE SEGMENTATION WITH OPTIMAL LINEAR PREDICTION ERROR FILTERS

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## ABSTRACT

An approach to feature extraction for texture images using optimal linear (autoregressive) predictors is presented. The features used for classification are calculated from the prediction error using a local energy function. Experimental results are given to show the applicability of the method.

## 1. INTRODUCTION

Most natural surfaces exhibit texture. A texture recognition system will therefore be a natural part of many computer vision systems. Applications of texture analysis spans from remote sensing through medical diagnosis to document image analysis.

Several methods for texture discrimination have been proposed, and a recent overview over some of them is given by Tuceryan and Jain [1]. One important approach to texture discrimination is the use of filters – together with a filter response energy measure – as feature extractor. The most commonly used filter is the Gabor filter, however other filters have also been employed. In [2], for instance, we suggest a class of computationally efficient critically sampled filters.

The most common approach to feature extraction with filters is to use a bank of filters which is more or less determined irrespective of the actual textures to be discriminated. Thus, there is no guarantee that the bank of filters will be able to work properly for a specific set of textures.

For these reasons, the design of *optimal* filters for texture discrimination is gaining increased interest. The design methods presented by Dunn et.al. [3] and Teuner et.al. [4] are approaches to the design of optimal Gabor filters. However, the choice of a Gabor filter is a restriction of the search space. It is therefore reasonable to believe that the true optimum will be excluded in most cases. In [5], Husøy et.al. present an approach to the determination of optimality of a bank of critically sampled perfect reconstruction infinite impulse response (IIR) filters, based on first order all pass filter building blocks. This approach is restricted to a class of filters with very few free parameters (actually only two), thus leading to a very limited search space. The approach also includes critical sampling in order to be computationally effective, implying reduced edge accuracy in a segmentation application.

Jain and Karu [6] proposes an approach to optimal filter design using a multi layer perceptron neural network for the filtering and local energy operations, and the back propagation training rule for the determination of the optimal parameters. Finally Singh and Mahalanobis [7] designs filters for maximization of the ratio between the energies of the filter responses of two textures.

In this paper we propose yet another approach to optimal texture feature extraction. We assume that the textures may be modelled as auto-regressive (AR) [8] stochastic processes. The AR models are used as linear predictors, predicting the current pixel based on the surrounding pixels. One optimal predictor is determined for each texture in the image, and all the predictors are applied to the image, yielding different prediction error images. It is then reasonable to assume that a pixel belongs to the texture corresponding to the predictor which yields the minimum local prediction error energy. The AR models are determined using a least squares estimator. The complete experimental setup is illustrated in Figure 1.

Remark that the linear prediction may also be viewed as a filtering operation. The predictor error calculation is therefore often called whitening filtering or prediction error filtering.

## 2. LEAST SQUARES AR PARAMETER ESTIMATION

The error in estimating the current sample, $x_i(m,n)$, of the image with a circular non-causal model is given

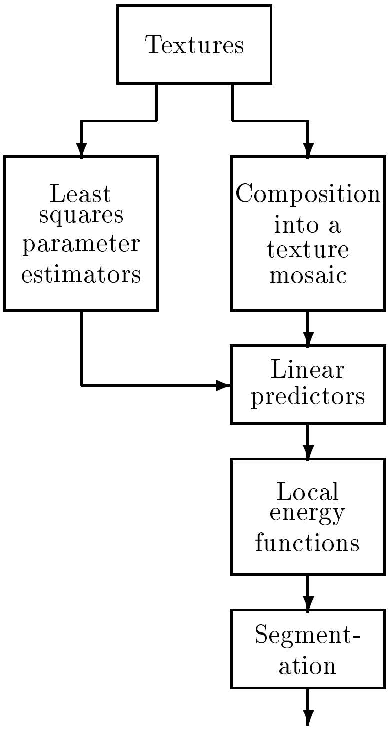


Figure 1: Experimental setup

by

$$e_i(m,n) = x_i(m,n) - \sum_{k^2+l^2 \leq r^2} x_i(m-k,n-l)\theta_i(k,l), \quad (1)$$

where $r$ is the radius of region of support of the model. If we arrange the $x_i(m-k,n-l)$ and $\theta_i(k,l)$ terms for all $(k,l)$ in the summation above in the vectors $\underline{x}_i(m,n)$ and $\underline{\theta}_i$, respectively, then Equation (1) may be written in vector form as

$$e_i(m,n) = x_i(m,n) - \underline{x}_i(m,n)^T \underline{\theta}_i. \quad (2)$$

For a model with radius $r = 2$, for instance, $\underline{\theta}_i$ may be given by

$$[\theta_i(-2,0)\ \theta_i(-1,-1)\ \theta_i(-1,0)\ \theta_i(-1,1)$$
$$\theta_i(0,-2)\ \theta_i(0,-1)\ \theta_i(0,0)\ \theta_i(0,1)\ \theta_i(0,2)$$
$$\theta_i(1,-1)\ \theta_i(1,0)\ \theta_i(1,1)\ \theta_i(2,0)]^T$$

and $\underline{x}_i(m,n)$ by

$$[x_i(m-2,n)\ x_i(m-1,n-1)\ x_i(m-1,n)$$
$$x_i(m-1,n+1)\ x_i(m,n-2)\ x_i(m,n-1)$$
$$x_i(m,n)\ x_i(m,n+1)\ x_i(m,n+2)$$
$$x_i(m+1,n-1)\ x_i(m+1,n)\ x_i(m+1,n+1)$$
$$x_i(m+2,n)]^T.$$

If we define $\underline{e}_i$ to be the vector of all $e_i(m,n)$, $\underline{\tilde{x}}_i$ to be the vector of all $x_i(m,n)$, and $\mathbf{X}_i$ to be the matrix with the $\underline{x}_i(m,n)^T$'s as the row vectors we may write the linear equation set determined by Equation (2) as

$$\underline{e}_i = \underline{\tilde{x}}_i - \mathbf{X}_i \underline{\theta}_i,$$

The mean square error can then be found by minimizing $\underline{e}_i^T \underline{e}$. Finding the $\underline{\theta}_i$ which yields the minimum squared error entails the solution of $\partial \underline{e}_i^T \underline{e}_i / \partial \underline{\theta}_i = 0$ which is found by solving (see [9])

$$\mathbf{X}_i^T \mathbf{X}_i \underline{\theta}_i = \mathbf{X}_i^T \underline{\tilde{x}}_i$$

commonly known as the normal equations. The normal equations can be solved by Cholesky decomposition or other methods [10].

## 3. FEATURE EXTRACTION AND CLASSIFICATION

Based on Equation (1) the prediction error images may be calculated. One optimal predictor is found for each texture, and one prediction error image is calculated for each predictor. To calculate the local prediction error energy, the prediction error image pixels are squared and smoothed with a Gaussian low-pass filter with transfer function

$$H_G(u,v) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2\sigma^2}(u^2+v^2)}.$$

In [2] we found

$$\sigma = \frac{1}{2}\sqrt{2}\pi f_0^{-1},$$

to be appropriate, where $f_0$ was the center frequency of a band-pass filter. In the current approach, however, we are not restricted to band-pass filters, and we are therefore not able to use the same selection scheme. It turned out that a choice of $\sigma$ corresponding to a frequency $f_0$ in the middle of the 2-D frequency range, i.e. $\sigma = 2\sqrt{2}\pi$, was an appropriate choice.

A pixel is then classified to be the texture corresponding to the predictor yielding the minimum local energy.

## 4. EXPERIMENTAL RESULTS

For our experiments, we have chosen to examine the texture calf leather (D92) versus the textures pressed cork (D4), and mica (D5), all scanned from the Brodatz [11] album. The textures are shown in Figure 2.

Experimentally we found a linear prediction neighborhood with radius 4 to be appropriate.

Figure 3 shows the feature images and the segmentation for the texture mosaic of D92 and D4. The gray-levels of the feature images have been linearly scaled to exploit the maximum dynamic range of the printer – to ensure optimal displaying conditions.

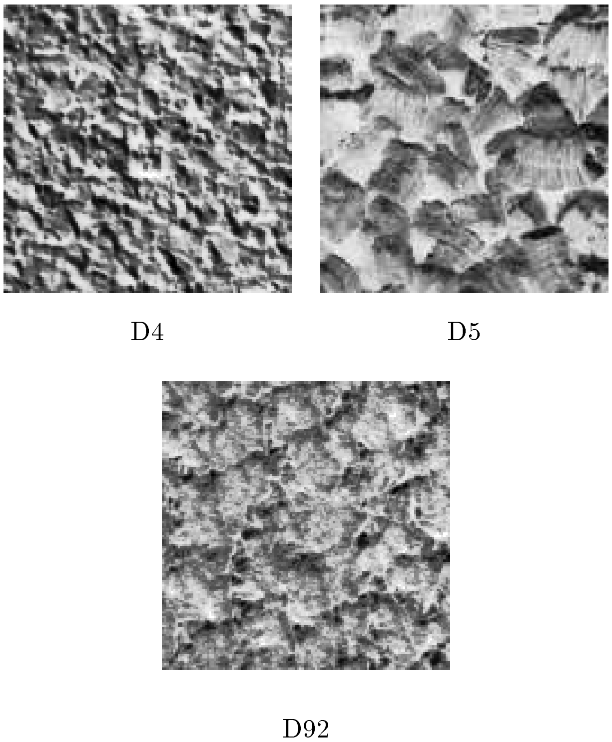


Figure 2: Textures used in our experiments.

It is somewhat difficult to visually interpret the separability using gray-level images, therefore we have chosen to demonstrate the results with the plot of the normalized means of all the feature pixels per column, see Figure 4. The solid curve represents the local prediction error energy for the optimal predictor for D92 and the dashed line represents the local prediction error energy for the optimal predictor for D4.

Remember that the segmentation is done by classifying the pixels to the textures corresponding to the predictors yielding minimum local prediction error energy. To obtain a robust segmentation, it is therefore important that the prediction error of the predictor corresponding to the actual texture must be much lower than for the other predictor. Thus, we see from Figure 4 that for this texture mosaic, the approach yields excellent results.

However, there is no guarantee that the prediction errors will have a large difference. Figure 5 shows the normalized column mean feature values for the texture mosaic of D92 and D5. As we see, the prediction error difference is far from distinct, and thus the segmentation, Figure 6, is not acceptable. It is, however, still possible to discriminate this texture mosaic with good margins using a supervised classifier, due to the fact that the texture D5 yields a much lower prediction error for both of the predictors than D92 does.

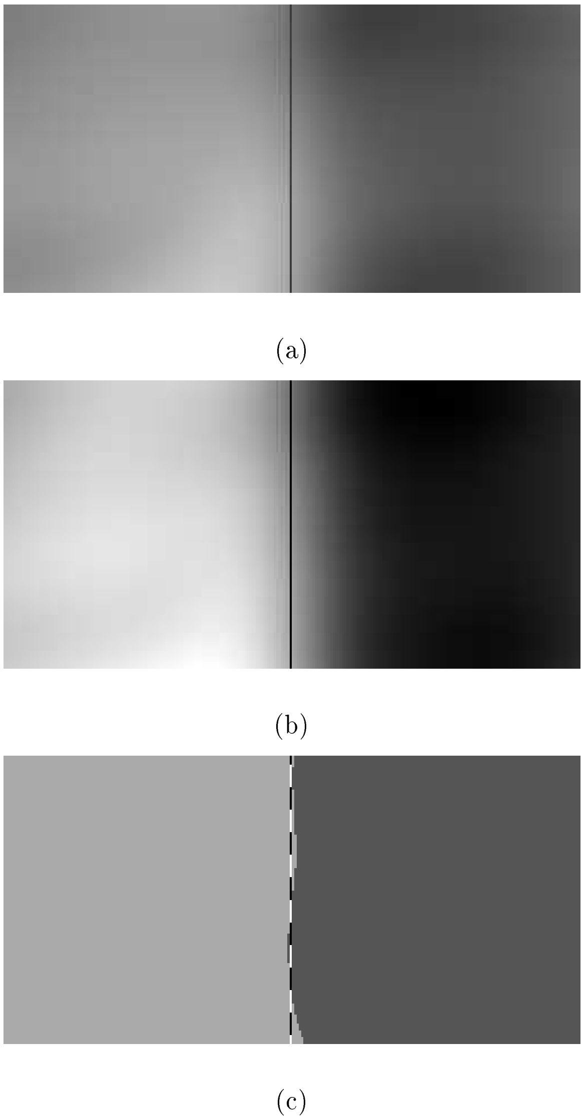


Figure 3: Local prediction error for the texture mosaic of D92 (left) and D4 (right) with the optimal predictor for D92 (a), the optimal predictor for D4 (b), and finally the segmentation (c). The vertical lines indicates the true texture boundaries.

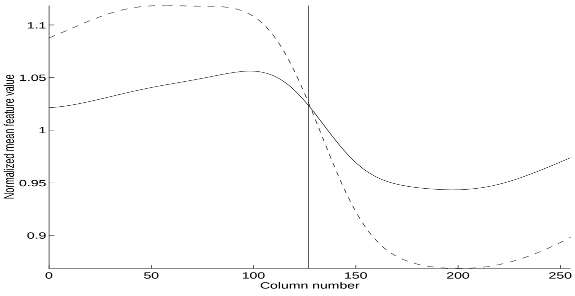


Figure 4: Normalized mean feature value per column for the texture mosaic of D92 (left) and D4 (right). The solid line corresponds to the optimal predictor for D92 and the dashed corresponds to D4. The vertical line indicates the true texture boundary.

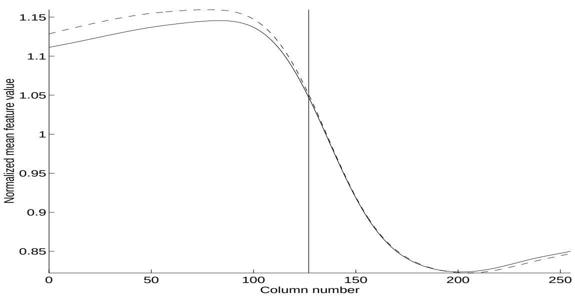


Figure 5: Normalized mean feature value per column for the texture mosaic of D92 (left) and D5 (right). The solid line corresponds to the optimal predictor for D92 and the dashed corresponds to D5. The vertical line indicates the true texture boundary.

Figure 6: Segmentation of the texture mosaic of D92 (left) and D5 (right). The vertical line indicates the true texture boundary.

## 5. CONCLUSION

In this paper, we have proposed a new approach to texture feature extraction using optimal linear predictors, where the local prediction error energies are used as the features. Experimental results indicate that these features are suitable for segmentation for some textures. The method, however, does not *guarantee* any discriminable difference in the prediction errors. As our last experiment indicates, it may still be possible to segment the image even if the prediction errors are in-discriminable, based on the textures predictability. This, on the other hand, would not be an optimal approach.

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