Combining Space-Time Block Codes and Multiplexing in Correlated MIMO channels: An Antenna Assignment strategy.

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Abstract

Multiplexing and space-time coding are competing ways of extracting capacity out of MIMO wireless systems. We address the problem of finding an optimal combination of these approaches over a MIMO array when a priori knowledge about correlation is available at the transmitter. Our approach is an optimal spatial assignment of antennas, over which to multiplex space-time coded symbol blocks, in the highly practical case when correlation is not uniform across all antenna pairs.

1 Introduction

One way to improve data rate and transmission reliability over wireless links is through the use of Multiple-Input Multiple-Output (MIMO) systems (see e.g. [4] for a recent tutorial).

Diversity-oriented transmission through space-time coding (STC) [7] and spatial multiplexing (SM) [3] are two different, so far competing, approaches to exploiting the spatial dimension offered by MIMO systems. The former uses the antennas to combat Rayleigh fading, while the latter uses spatial degrees of freedom to increase data rate by sending independent symbol streams simultaneously. The trade-offs between those approaches are only beginning to be understood [4].

While it is clear that diversity schemes yield diminishing returns when increasing the number of antennas [2], it is also known that an SM scheme with a simple (e.g. linear) receiver lag in performance because of a lack of diversity [5].

We address both of these open problems by attempting to combine the schemes. Previous work has been carried out for the case where the desired combination is to switch between STC and SM over time [6].

There, the proposed scheme exploits the fact that STC is sensitive to total channel matrix energy, while SM performance depends on the channel eigenvalue spread.

Our approach will be different, as we focus on the problem of switching between SM and STC over space. To our knowledge, this problem has not been addressed before. We propose simple algorithms allowing us to generalize the work of [6], both for the case where instantaneous full channel feedback is available to the transmitter and for the case where only long term correlation statistics are known. We show the performance gains with bit-error rate (BER) simulations using a realistic channel model.

Notation

x, x and X denote a scalar, a vector and a matrix, respectively. \( x^* \) is the complex conjugate of the scalar. For vectors and matrices, \( x^* \) denotes the complex conjugate of each element, while \( x^T \) and \( x^H \) are the transpose and the conjugate transpose. \( X^\dagger \) is the pseudo-inverse of \( X \). \( \sqrt{X} \) is the hermitian square root of \( X \).
2 Channel model

For simplicity, all signals are represented in complex baseband.

Our system consists of uniform, linear arrays of $N$ transmit and $M$ receive antennas. The channel propagation coefficient from transmit antenna $n$ to receive antenna $m$ is the complex value $h_{mn} \sim CN(0,1)$.

All channel coefficients are placed in the $M \times N$ sized channel matrix $H$, which can be seen as a collection of $N$ transmit vectors, $h_n, n \in [1,N]$.

$$H = \begin{bmatrix} h_1 & h_2 & \ldots & h_N \end{bmatrix}, \quad (1)$$

where each $M \times 1$-sized $h_n$ is given by

$$h_n = \begin{bmatrix} h_{1n} \\
   h_{2n} \\
   \vdots \\
   h_{Mn} \end{bmatrix}, \quad n \in [1,N] \quad (2)$$

The level of correlation in $H$ depends, amongst others, on the inter-element distances in the transmit and receive antenna arrays. We can model the correlated MIMO channel by

$$H = \sqrt{\mathbf{R}_t} H_0 \sqrt{\mathbf{R}_r}, \quad (3)$$

where $\mathbf{R}_t$ and $\mathbf{R}_r$ are correlation matrices for the transmit and receive side, and $H_0$ is the uncorrelated channel matrix.

The correlation coefficient between transmit antennas $i$ and $j$ is given by $\{R_t\}_{ij}$, where $i, j \in [0,N-1]$. Correlation is modelled as

$$\{R_t\}_{ij} = J_0 \left( \frac{2\pi(i-j)d}{\lambda} \right),$$

where $J_0(z)$ is the zero-order Bessel function of the first kind, $d$ denotes the inter-element distance of the array and $\lambda$ is the carrier wavelength.

In a general model, complex modulation symbols $s_k$ are arranged in a transmit matrix $S$, possibly with some STC structure. The power of each symbol is normalised, $E(|s_k|^2) = 1$.

We transmit $S$ over the wireless channel, and at the receiver noise is added. The received matrix $Y$ is

$$Y = HS + N \quad (4)$$

$S$ has dimensions $N \times L$, where $L$ is the number of time periods spanned by block $S$ (for which the channel is considered constant). The $M \times L$-sized $N$ contains independent, complex Gaussian noise entries, $n_{ml} \sim CN(0,\sigma_n^2)$.

2.1 The Alamouti scheme

The Alamouti algorithm [1] is a simple, full rate, Space-Time Block Code (STBC), for $N = 2$ transmit and $M$ receive antennas.

The Alamouti-coded symbol matrix for time interval $(2k,2k+1)$ is given by $S = S_k$, where

$$S_k = \begin{bmatrix} s_{2k} & -s_{2k+1}^* \\
   s_{2k+1}^* & s_{2k} \end{bmatrix} \quad (5)$$

With this $S$, transmission with the Alamouti algorithm may be expressed using the model in (4).

A modified version of this system is obtained if we move the STC structure from $S$ to $H$, yielding a coded channel matrix $\tilde{H}$, of dimensions $2M \times 2$. With this model, the received signal is given by

$$\tilde{y} = \tilde{H}s + n, \quad (6)$$

where the signal vector $s$ is

$$s = s_k = \begin{bmatrix} s_{2k} \\
   s_{2k+1} \end{bmatrix} \quad (7)$$

Using the notation from (2), the modified channel matrix is given by

$$\tilde{H} = \begin{bmatrix} h_1 & h_2 \\
   h_2^* & -h_1^* \end{bmatrix} \quad (8)$$

To detect the original information symbols, the Alamouti algorithm exploits the orthogonal structure of $\tilde{H}$, so that the retrieved symbols may be expressed as

$$\tilde{s} = \tilde{H}^T \tilde{y} = (|h_1|^2 + |h_2|^2)s + n^t \quad (9)$$

2.2 Spatial Multiplexing

With the Spatial Multiplexing scheme [3], independent streams of symbols are transmitted from each antenna. In a single time period, the vector $s$, given by
is transmitted, and the received symbol vector is given by
\[ y = Hs + n \]
After arrival on the receive side, one example of detection is by zero-forcing, so that the vector of retrieved symbols is
\[ \hat{s} = H^T y \]

3 Combining Alamouti and Spatial Multiplexing

To avoid allocating all degrees of freedom to any particular scheme (STC or SM), which would result in either an excess of diversity or a lack of it, we consider the problem of combining both techniques on the same array via allocation of antennas to one scheme or the other.

This text is inspired by the ideas in [6], where the combination is done in time, by switching between SM and STC schemes, based on instantaneous channel information. Although our approach, combining in space, is different, parts of the work and results from [6] still apply, and we will take some time to present them.

3.1 Combining in Time

In [6], SM and STC schemes are compared for instantaneous channel matrices, to determine which is best suited under the current conditions. We may use different modulation schemes for STC and SM, to ensure the same bit rate.

The evaluation of which scheme to choose is based on a desire to minimize the BER. If the symbols in the receive constellations are spaced far apart, wrong decisions are less likely.

Hence, at any given time, we choose the approach which offers the largest minimum, squared, Euclidean distance of the receive constellations, denoted \( d_{\text{min,STC}}^2(H) \) for STC and \( d_{\text{min,SM}}^2(H) \) for SM methods.

Bounds on these distances are derived as [6]
\[
\begin{align*}
\frac{d_{\text{min,SM}}^2(H)}{N} & \geq \sigma_{\min}(H) \frac{d_{\text{min,sm}}^2}{N} \\
\frac{d_{\text{min,STC}}^2(H)}{N} & \leq \frac{1}{N} \|H\|_{F}^2 d_{\text{min,STC}}^2 \\
\end{align*}
\]

where \( d_{\text{min,sm}}^2 \) and \( d_{\text{min,STC}}^2 \) are the minimum, squared, Euclidean distances of the transmit constellations, and \( \sigma_{\min}(H) \) is the minimum singular value of \( H \). Using a conservative approach, spatial multiplexing is only used when
\[
\sigma_{\min}(H) d_{\text{min,sm}}^2 \geq \|H\|_{F}^2 d_{\text{min,STC}}^2
\]

For a given channel matrix \( H \), a large minimum eigenvalue implies SM transmission, while a large Frobenius norm means diversity is preferable.

3.2 Combining in space

In this paper, the STC scheme considered is the Alamouti algorithm. The combining scheme is called SMAL (Spatial Multiplexing of Alamouti). We wish to spatially multiplex several Alamouti-coded blocks, containing independent groups of two symbols each, and assume that \( N = 2k \), \( k \geq 2 \) and \( M \geq N/2 \).

Assume we wish to transmit the block
\[ s = [s_0 \ s_1 \ \ldots \ s_{N-1}]^T \]
over two symbol durations. To do that, we build an \( N \times 2 \) sized symbol matrix \( S \), from \( N/2 \) Alamouti matrices, \( S_k \). The most trivial way to assign the Alamouti blocks to antennas is by selecting \( S \) in (4) such that
\[
S = \begin{bmatrix}
S_0 \\
S_1 \\
\vdots \\
S_{N/2-1}
\end{bmatrix}
\]

Alternatively, the combination may be expressed by moving the structure from \( S \) to the channel matrix, as in (6) and repeated here
\[ \tilde{y} = \tilde{H}s + n \]

More generally, the Alamouti blocks may be assigned to any antenna combination. Let us
define $P_N$, the number of non-trivially equivalent antenna assignment patterns. These patterns are labelled $p_k$, $k \in [1, P_N]$. For instance, for $N = 4$ we have $P_4 = 3$, and the patterns are shown in Figure 1. The trivial arrangement of (12) corresponds to pattern $p_1$.

For each possible pattern $p_k$, we obtain a different structure in $\tilde{H}$ and denote it $\tilde{H}(p_k)$. The general input/output model is now

$$\tilde{y} = \tilde{H}(p_k)s + n, \quad (14)$$

The three patterns in Figure 1 result in the space-time coded channel matrices $\tilde{H}(p_1)$, $\tilde{H}(p_2)$ and $\tilde{H}(p_3)$

$$\tilde{H}(p_1) = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ h_2^* & -h_1^* & h_4^* & -h_3^* \end{bmatrix},$$
$$\tilde{H}(p_2) = \begin{bmatrix} h_1 & h_3 & h_2 & h_4 \\ h_3^* & -h_1^* & h_2^* & -h_4^* \end{bmatrix},$$
$$\tilde{H}(p_3) = \begin{bmatrix} h_1 & h_4 & h_2 & h_3 \\ h_4^* & -h_1^* & h_2^* & -h_3^* \end{bmatrix}$$

where $h_n$ is the $n$-th transmit vector of $\mathbf{H}$.

The goal is to choose the best pattern, given instantaneous channel conditions or some long-term statistics. The latter reduces the feedback load and complexity.

### 3.2.1 Instantaneous pattern optimisation

One way to solve this problem is to find the optimal antenna groups over which the independent Alamouti blocks will be multiplexed. This can be done by picking the pattern $p_{k_0}$ which maximises the performance in detecting $s$ in (14).

According to (10), this is equivalent to finding the patterns $p_{k_0}$ such that

$$p_{k_0} = \arg \max_{p_k} \left( \sigma_{\text{min}}^2(\tilde{H}(p_k)) \frac{d_{\text{min}}^2}{N} \right), \quad (15)$$

We refer to this approach as the instantaneous SMAL, optimized over time.

### 3.2.2 Pattern optimisation based on correlation

In the preceding section, it was assumed that the transmitter had access to instantaneous channel information, through feedback from the receiver. An alternative solution is to find the best pattern based only on long-term statistics, such as the correlation.

To solve this problem, our approach is to base the optimisation on the 'average behaviour' of the channel.

Based on singular value decomposition theory, it is clear that (15) is equivalent to

$$p_{k_0} = \arg \max_{p_k} \left( \lambda_{\text{min}} \left( \tilde{H}(p_k)^H \tilde{H}(p_k) \right) \right), \quad (16)$$

that is; maximising the minimum eigenvalue of $(\tilde{H}(p_k)^H \tilde{H}(p_k))$. By replacing $(\tilde{H}(p_k)^H \tilde{H}(p_k))$ with its average, we obtain

$$p_{k_0} = \arg \max_{p_k} \left( \lambda_{\text{min}} \left( E(\tilde{H}(p_k)^H \tilde{H}(p_k)) \right) \right) \quad (17)$$

This criterion is used for the correlation-based SMAL.

### 4 Simulation results

We consider uniform arrays with $N = M = 4$. The BER results are averaged over 1000 independent channel realisations. At the receiver, zero-forcing detection is used.

The plots in Figures 2 and 3 both show BER against signal-to-noise ratio (SNR). The coefficient $r$ is the correlation between any two neighbouring antennas. For Figure 2, $r = 0.90$, while $r = 0.29$ in 3.

There are 5 curves in each plot, of which 3 represent BER-results for when a pattern $(p_1, p_2$ and $p_3)$ is fixed over time. The curve representing random pattern selection exhibits an average behaviour. The last curve is the results of the instantaneously optimized SMAL, using the criterion given in (15).

**Instantaneous SMAL**

We observe that in Figure 2 (high correlation),
pattern $p_1$ yields approximately equal BER-results as the optimal case, indicating that this clearly the best of the three.

In figure 3 (low correlation), all three patterns return similar performance, and the instantaneous SMAL achieves a performance gain of 2 dB over the random pattern selection at a target BER of $10^{-4}$.

Correlation-based SMAL
Figure 4 shows the minimum eigenvalues of $E(\hat{\mathbf{H}}^H(p_k)\hat{\mathbf{H}}(p_k))$ for increasing levels of correlation, one curve for each pattern. As seen from (17), the pattern yielding the largest minimum eigenvalue for a given correlation-level $r$ is chosen in the correlation-based SMAL.

At $r = 0.90$, it is clear from figure 4 that pattern $p_1$ gives the best result and will be selected. This is consistent with figure 2. Choosing $p_1$ over time yields a gain over random pattern selection of almost 5 dB at BER $10^{-3}$.

At correlation $r = 0.29$, the minimum eigenvalues of $\hat{\mathbf{R}}_r(p_k)$ are more similar, there is no pattern which is clearly better than the others. This result, too, is consistent with the BER-plot in figure 3.

5 Conclusion
We have presented a combination of STC and SM approaches; the SMAL scheme, combining Alamouti STC and SM in space. SMAL is developed in two versions, both for the case when instantaneous channel state information is known at the transmitter and the case when only long-term correlation statistics are available.

We have shown that both versions offer significant performance improvements over an approach that picks a random antenna assignment pattern.

The instantaneously optimised SMAL is seen to be especially useful for low levels of correlated fading, while the SMAL based on long-term statistics is more suitable when the correlation is strong.

Then, instantaneous and correlation-based SMAL show similar performance. The reason is that long-term correlation statistics dominates over short-term variations.

We conclude that the SMAL scheme offers substantial gain when transmission is done using the best spatial pattern, either instantaneous or over time. This is particularly interesting under conditions of heavy correlation, when the pattern optimisation offers ample gain based on long-term statistics alone.

References
Figure 1: The patterns for 4 transmit antennas.

Figure 2: BER results for all patterns and optimal case, N = 4, correlation r = 0.90

Figure 3: BER results for all patterns and optimal case, N = 4, correlation r = 0.29

Figure 4: Minimum eigenvalue of $E(\tilde{\mathbf{H}}^H(p_k)\tilde{\mathbf{H}}(p_k))$ for $k = 1, 2, 3$, and $N = 4$. 

Statistical version, minimum eigenvalues of $E(\tilde{\mathbf{H}}^H(\mathbf{p}_k)\tilde{\mathbf{H}}(\mathbf{p}_k))$ for patterns, when $N = 4$. 

Correlation coefficient between neighbouring antennas.