DESIGN OF OPTIMAL TWO-PASS ZERO-PHASE RECURSIVE FILTERS
WITH APPLICATION TO PROCESSING OF SEISMIC DATA

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ABSTRACT

In this paper we examine frequency-space filter techniques for optimal multiple attenuation of multi-component ocean bottom seismic (OBS) data. Local frequency-space domain filters can be attractive when the seabed medium has lateral velocity variations or the data are irregularly sampled along the seafloor.

We concentrate on pressure de-multiple by combining pressure recordings and filtered vertical particle velocity data. For simplicity, we give the 2-D version of the pressure de-multiple filter. We investigate two different types of numerically optimized spatial filters: (i) zero-phase non-recursive filters and (ii) two-pass zero-phase recursive filters. Recursive spatial filters are shown to work significantly better than non-recursive spatial filters.

1. INTRODUCTION

In the frequency-wavenumber domain, the upgoing pressure, $U^P$, just below a flat and homogeneous water-seabed interface is related to the recorded pressure, $P$, and the recorded vertical particle velocity, $V_z$, by [1]

$$U^P = 0.5 \left( P - F \cdot V_z \right),$$

(1)

where $F$ is the decomposition filter (also called the pressure de-multiple filter). $F$ is given by

$$F = \frac{\rho \omega}{k_{z,\alpha}} \left[ \left( 1 - \frac{k_{x}^2}{k_{\beta}^2} \right)^2 + 4k_{z,\alpha}k_{z,\beta} \right],$$

(2)

and depends on the wavenumbers $k_{z,\alpha} = \omega/\alpha$ and $k_{\beta} = \omega/\beta$ in which $\alpha$ and $\beta$ are the P- and S-wave velocities in the solid. The vertical P- and S-wavenumbers are defined as $k_{z,\alpha} = (k_{x}^2 - k_{\beta}^2)^{1/2}$ and $k_{z,\beta} = (k_{x}^2 - k_{\alpha}^2)^{1/2}$. Finally, $\omega$ is the angular frequency, $\rho$ is the density, and $k_{x}$ is the horizontal wavenumber.

The zeroth-order Taylor series of equation (2) is $\tilde{F} = \rho \omega$. This filter is commonly used by the seismic contractor industry for de-multiple of the pressure recordings (so-called PZ-summation). A better alternative is to design numerically optimized pressure de-multiple filters, $\tilde{F}$. In the frequency-space domain, equation (1) is written symbolically as

$$\tilde{U}^P = 0.5 \left( \tilde{P} - \tilde{F} \ast \tilde{V}_z \right),$$

(3)

where $\ast$ denotes spatial convolution. Osen et. al [2] designed optimal non-recursive filters, but since such filters can only accommodate zeros in the frequency-wavenumber response they have quite poor performance close to the critical wavenumber $k_x = k_{x,n}$. In this paper we design optimal recursive filters which will work better since they also provide poles. In addition, we include constraints in the optimization problem formulation to, e.g., prevent boosting of evanescent waves by the numerically optimized non-recursive and recursive spatial filters.

2. OPTIMIZATION PROBLEM FORMULATION

The discrete horizontal wavenumber response of a spatial digital filter can be written $[\tilde{F}_k = \tilde{F}(k\Delta k_x)]$

$$\tilde{F}_k = \frac{\sum_{m=-M/2}^{M/2} b_m e^{-i k \Delta k_x m \Delta x}}{\sum_{n=0}^{N} a_n e^{-i k \Delta k_x n \Delta x}} = \frac{\Theta_{km} b_m}{\Phi_{kn} a_n},$$

(4)

where repeated subscript indices indicate summation, $i^2 = -1$, and $\{a_n, b_m\}$ are the backward and forward filter coefficients with $N, M$ the respective filter orders. Without loss of generality, $M$ and $N$ are assumed to be even numbers and $a_0 \equiv 1$. Furthermore, $\Delta x$ is the spatial sampling interval and $\Delta k_x = \pi/\Delta x (K - 1)$ is the distance between the discrete horizontal wavenumbers for $K \gg 0$. The relationship between a discrete-space semi-causal input signal, $u(\ell \Delta x)$, and a discrete-space causal output signal, $v(\ell \Delta x)$, for the digital filter given in equation (4) is

$$v(\ell \Delta x) = \sum_{m=-M/2}^{M/2} b_m u((\ell - m) \Delta x) - \sum_{n=1}^{N} a_n v((\ell - n) \Delta x).$$
If \( F_k = F(k \Delta k_x) \) denotes the desired discrete horizontal wavenumber response, then the design of the spatial filters can be stated as a general weighted non-linear least-squares optimization problem:

\[
\text{minimize } \sum_{k=-(K-1)}^{K-1} W_k |\hat{\Phi}_{kn} b_m - F_k|^2, \quad (5)
\]

where \( \{W_k\} \) is a set of positive weights. In the following, the optimization problem is formulated for propagating waves only, for which the ideal decomposition filter, \( F_k \), is purely real (i.e., zero-phase) and positive. Let \( \kappa \) denote the index corresponding to a discrete horizontal wavenumber close to the critical wavenumber, \( k_x = k_\alpha = \omega/\alpha \), e.g., \( \kappa = \lfloor \omega/(\alpha \Delta k_x) \rfloor + 1 \), where \( \lfloor \cdot \rfloor \) means a floor-rounding operator.

2.1. Zero-phase FIR filters

For a zero-phase non-recursive or finite impulse response (FIR) filter, the forward filter coefficients are symmetric, i.e., \( b_m = b_{-m} \) for \( m = 0, 1, \ldots, M/2 \), and the backward filter order is \( N = 0 \). This reduces the non-linear least-squares optimization problem given in equation (5) to the following linear least-squares optimization problem:

\[
\text{optimal FIR filter;}
\text{minimize } \sum_{k=0}^{\kappa-1} W_k |\Gamma_{km} g_m - F_k|^2, \quad (6)
\]

where \( \Gamma_{km} = \cos(k \Delta k_x \cdot m \Delta x) \). The \( \{g_m\} \)'s are related to the \( \{b_m\} \)'s through \( b_m = 0.5 \cdot g_m \) for \( m = 1, 2, \ldots, M/2 \) and \( b_0 = g_0 \) [3].

2.2. Two-pass zero-phase IIR filters

Spatial FIR filters can only accommodate zeros in the horizontal wavenumber response [2]. Therefore, they will not perform particularly well close to the discrete critical wavenumber, \( \kappa \), at least for small forward filter orders. It is expected that recursive or infinite impulse response (IIR) filters will work significantly better since they also provide poles.

Spatial IIR filters with three pre-fixed and real-valued backward filter coefficients, i.e., \( N = 2 \), are used which give two complex-conjugated poles of the system function \( \hat{F}(z) \) for \( z = \exp(i k_x \Delta x) \). The two poles should be located closely to the negative and positive critical wavenumbers, \( \pm \omega/\alpha \approx \pm (\kappa - 1) \Delta k_x \). Causal and stable IIR filters require that the poles lie strictly inside the unit circle in the complex \( z \)-plane. The two backward filter coefficients \( a_1 \) and \( a_2 \) are hence given by (a \( = 1 \))

\[
a_1 = -2 \delta \cdot \cos((\kappa - 1)/(K - 1) \cdot \pi) \quad \text{and} \quad a_2 = \delta^2, \quad (7)
\]

with \( 0 < \delta \lesssim 1 \) a parameter that guarantees the stability.

Causal and stable IIR filters can not have zero phase response. However, by first filtering the input signal in the causal direction and then filtering the output signal in the anti-causal direction (so-called two-pass filtering), zero phase response is obtained with magnitude response being squared [4]. Hence, for the design of two-pass zero-phase IIR filters the square root of the ideal decomposition filter has to be taken. Now, equation (5) can be rewritten as the following linear least-squares optimization problem:

\[
\text{optimal IIR filter;}
\text{minimize } \sum_{k=0}^{\kappa-1} \tilde{W}_k |\Gamma_{km} g_m - \tilde{F}_k|^2, \quad (8)
\]

where \( \{\tilde{W}_k\} \) is a set of modified weights and \( \{\tilde{F}_k\} \) is the modified desired horizontal wavenumber response:

\[
\tilde{W}_k = \frac{W_k}{|\Phi_{kn} a_n|^2} \quad \text{and} \quad \tilde{F}_k = |\Phi_{kn} a_n| \sqrt{F_k}. \quad (9)
\]

2.3. Quadratic Programming (QP)

The linear least-squares optimization problems shown in equations (6) and (8) are unconstrained, and unique and optimal solutions are given by the pseudo-inverses. Nevertheless, it can be important that the magnitude responses of the optimal FIR and IIR filters have zero error at particular horizontal wavenumbers, or that they have magnitude responses smaller than a given function for a certain horizontal wavenumber range. The combination of a linear least-squares optimization problem with linear equality and/or inequality constraints is called Quadratic Programming (QP), and can be solved efficiently with many different algorithms [5].

We incorporate a single linear equality constraint at \( k_x = 0 \) to ensure that vertically propagating waves are handled perfectly by the optimal FIR and IIR filters. In addition, we include linear inequality constraints in the evanescent horizontal wavenumber range to force the spatial filters’ magnitude responses to be less than or equal to the magnitude response of the ideal decomposition filter. This prevents boosting of evanescent waves by the numerically optimized spatial filters.

3. MODELING AND RESULTS

Consider a simple 2-D plane-layered earth model consisting of a 100 m homogeneous water column overlaying a 700 m thick isotropic elastic layer. The bottom part of the model is a homogeneous isotropic elastic medium of infinite extent. The elastic parameters are
shown in Table 1. The data are generated using the modeling package OSIRISTM, and a zero-phase Ricker 3-loops source wavelet with a center frequency equal to 35 Hz is applied to the data. The temporal sampling interval is $\Delta t = 4$ ms and the spatial sampling interval is $\Delta x = 12.5$ m. The set of 511 receivers at the seabed is centered below a point source located 6 m beneath the water surface. Figure 3 on the last page shows the modeled data: the $P$-component is displayed in (a), the $V_z$-component in (b), and the modeled de-multipled pressure in (c). A zero-phase bandpass filter with cut-off frequencies 5-15-60-70 Hz is applied to the data.

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<td>$\rho$ (kg/m$^3$)</td>
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Table 1: Medium parameters used for the plane-layer modeling.

Figure 1 shows the magnitude of the frequency-wavenumber responses for the ideal decomposition filter (given in solid black lines), the zeroth-order Taylor series (dashed red lines), the optimal FIR filters (green solid lines with triangles), and the optimal IIR filters (blue solid lines with circles) for three different frequencies equal to 15 Hz in (a), 35 Hz in (b), and 55 Hz in (c). The forward filter order is fixed to $M = 6$. The optimal IIR filters give prominently better match to the desired magnitude response than the optimal FIR filters for all frequencies and horizontal wavenumbers, but particularly around the critical wavenumber. Note that the magnitude responses of the optimal FIR and IIR filters never exceed the magnitude response of the ideal decomposition filter in the evanescent region due to the incorporated linear inequality constraints.

Figure 2 displays the frequency-wavenumber magnitude responses for the ideal decomposition filter in (a), and the difference ($\times 4.0$) between the desired magnitude response and the magnitude responses of the optimal FIR and IIR filters in (b) and (c), respectively. The evanescent regions of the two difference sections are most naturally set to zero due to the incorporated inequality constraints.

Figure 4 shows the de-multipled pressure using the Taylor series in (a), the optimal FIR filter in (b), and the optimal IIR filter in (c). Finally, the bottom three panels are the difference plots ($\times 6.0$) between the modeled de-multipled pressure and the processed de-multipled pressure using these three different types of local spatial filters. The main caption of this figure explains the results in detail.

4. DISCUSSION

One of the main reasons for developing the compact spatial filter methodology is to accomplish wavefield separation in the common shot gather (CSG) domain. We now briefly discuss the advantages and disadvantages of wavefield decomposition in the CSG domain and how the compact optimal spatial filter methodology can help to achieve this goal.

4.1. Advantages

The CSG domain is the physical domain. The seismic data is acquired in this domain, which means that the transform over the spatial coordinates will give the correct slowness of the seismic energy recorded at the receiver array. In contrast, when the decomposition filter is done in the common receiver gather (CRG) domain, the spatial transform is taken over the source coordinates and one implicitly assumes a horizontally layered earth.

Vessel noise and other types of seismic noise (interference noise) are coherent in the CSG domain. Therefore, wavefield decomposition techniques have the potential to process the noise correctly in the CSG domain. Moreover since vessel noise and interference noise dominantly arrive as downgoing events at the receiver cable, wavefield separation techniques offer the possibility of filtering them out. It remains to be investigated at which bandwidth noise can be correctly treated by wavefield separation techniques.

4.2. Disadvantages

Disadvantages of wavefield decomposition in the CSG domain are that the seafloor properties can be laterally varying along the receiver cable, prohibiting the exact $(\omega, k_x)$-domain decomposition expression. Other disadvantages are receiver to receiver varying coupling with the seabed and the near-surface statics. In the CRG domain, such coupling and statics issues are constant for each common receiver gather. Much of these problems, however, are solved through the use of the compact spatial filter methodology presented in this article. Good results have been obtained for compact spatial filters using only 5-7 consecutive receivers, hence statics and lateral velocity variations can be dealt with, as long as the dominant wavelength of their variation is not significantly shorter than 3-4 receiver intervals. Moreover, recently, good techniques for calibration of the $V_z$-component against the $P$-component are developed, although the calibration of the $V_z$-component still represents a problem. Hence we conclude that the compact spatial filter methodology is a viable tool for decomposition of seismic data in the CSG domain.
5. CONCLUSIONS

We have presented frequency-space filter techniques for optimal de-multiple of multicomponent OBS data in the least-squares sense. We have designed two different types of numerically optimized and local spatial filters, and we have shown that recursive or IIR filters work better than non-recursive or FIR filters. Appropriate forward and backward filter orders are 6 and 2, respectively. The design of local pressure de-multiple filters can to some extent handle variations in medium parameters along the seafloor.

6. ACKNOWLEDGMENTS

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7. REFERENCES


Figure 1: Horizontal wavenumber magnitude responses for three different frequencies; (a) 15 Hz, (b) 35 Hz, and (c) 55 Hz.

Figure 2: Frequency-wavenumber magnitude responses. For the two difference plots in (b) and (c) the evanescent regions are set to zero.

Figure 3: Modeled data; (a) pressure, (b) vertical particle velocity, and (c) modeled de-multipled pressure (i.e., reference solution).
Figure 4: De-multipled pressure and difference with reference solution given in Figure 3 (c). Observe the huge improvement using an optimal IIR filter compared to zeroth-order Taylor series and an optimal FIR filter. The only small events that remain for the optimal IIR filter is a PS-wave and an SS-wave. Note that the direct wave is not completely removed by the spatial filtered de-multipled pressure, which can be expected since the optimization was only done for propagating waves in the seabed. Hence, all energy that is propagating in the water layer, but evanescent in the seabed will not be treated correctly. In fact, since the optimal IIR filter is a highly efficient dip-filter for evanescent energy on the $V_z$-component (low magnitude response for evanescent waves), such energy from the $P$-component are mainly left. Finally, note the apparent smearing of the de-multipled pressure for the optimal FIR filter. This can be related to the oscillating behavior of the optimal FIR filter around the ideal decomposition filter [see Figure 2 (b) and the green solid lines with triangles in Figure 1].