ABSTRACT

Multiple antennas at the receiver and transmitter are often used to combat the effects of fading in wireless communication systems. However, implementing multiple antennas at the mobile stations is impractical for most wireless applications due to the limited size of the mobile unit. In this paper we emulate spatial diversity using mobile relay stations, which cooperate by retransmitting the information received from a mobile station to a destination station. We propose an Alamouti based cooperative system with two relay stations and we provide an approximate formula for the average symbol error probability of this system in a Rayleigh fading environment.

1. INTRODUCTION

There is an increasing trend in the development of communication systems that allow their users to communicate “anywhere and anytime” at high data rates. Wireless networks have the potential to offer this ubiquitous high-rate communication among mobile users. A wireless network is a collection of mobile terminals that are capable of transmitting and receiving information using wireless multiple access protocols. Because the terminals in the network are mobile, communication among the terminals suffers from time-varying fading, which frequently reduces the signal level making it difficult or sometimes impossible to recover the transmitted information. In order to combat fading in wireless networks we allow cooperation among the terminals in the network [1–3].

The idea of increasing the throughput of a system using cooperation among users has been first introduced in [1, 4] for a cellular environment. The main idea is that after selecting a partner from the in-cell mobile users, each user detects a faded and noisy version of the partner’s transmitted signal and combines this information with its own information data to construct its transmitted signal. It has been shown that in a flat fading environment the code division multiple access (CDMA) cooperative system of [4] achieves a higher throughput than the regular CDMA system. Instead of detecting and regenerating the cooperative signal, a simple amplification of partner’s cooperative signal results in a similar performance improvement as it is illustrated in [5]. Shadowing effects have been considered in [3], where an approximate formula has been provided for the outage probability of a cooperative system in Rayleigh fading environment with lognormal shadowing.

In this paper we specialize the system model of [5, 6] by implementing a distributed space-time coding system based on Alamouti’s space-time coding scheme. We establish approximate formulas for the average symbol error probability of this system in a Rayleigh fading environment.

2. SYSTEM MODEL

The cooperative network analyzed here uses $K$ possibly idled terminals $R_k$, $k \in [1, K]$, to relay the information transmitted by a terminal $S$, to the destination terminal $D$. The relays can decode and regenerate the received signal (regenerative system), or they can just amplify the received signal with a gain (non-regenerative system) [5, 6]. Furthermore, we assume the relays can perform simple operations on the resulting signals, which do not require regeneration of the information symbols, like delaying and conjugating. In order to reduce the size of the terminals, it is preferable to limit future cooperative systems, regenerative or non-regenerative, to only one antenna per terminal. In order to allow the relays to receive and transmit in the same time on a single antenna, we assume two orthogonal signal subspaces for the received and the transmitted signals, e.g., two different frequency bands. Unlike the relays that can only receive one of the signal subspaces, we assume the destination terminal $D$ can receive both signal subspaces.

For simplicity we only allow a one hop (i.e., the direct path from $S$ to $D$) and $K$ two-hop transmissions from $S$ to $D$ in our network, and we analyze the case when $K = 2$ as it is depicted in Fig. 1. We show in Fig. 2 the discrete-time baseband equivalent multi-relay channel model consisting of the three subchannels between the source and the destination. We assume that the transmissions suffer from the effects of slowly time-varying flat fading in order to write
the two orthogonal signals received at the destination as
\[ y_0[n] = h_0 \sqrt{\varepsilon_0} x[n] + z_0[n], \]
\[ y_1[n] = y_1[n] + y_2[n], \]
where for \( k \in \{1, 2\} \)
\[ y_k[n] := h_{2,k} u_k[n - n_k] + z_{2,k}[n], \]
and where \( \varepsilon_0 \) is the transmitted symbol energy at terminal S, since we assume that the information bearing symbols \( x[n] \)'s are drawn from a constellation with unit energy, and \( z_{0}[n], z_{1,k}[n], z_{2,k}[n] \) are additive noises. For non-regenerative systems, \( u_k[n] \) is either \( \alpha_k r_k[n] = \alpha_k (h_{1,k} \sqrt{\varepsilon_0} x[n] + z_{1,k}[n]), \) or plus/minus its complex conjugate. For regenerative systems, \( u_k[n] \) is either an estimate of \( x[n], \) or plus/minus its complex conjugate. The effect of the slowly time-varying flat fading is captured by \( h_0, h_{1,k}, \) and \( h_{2,k} \), which we assume to be:

a1) mutually independent complex Gaussian distributed variables with zero mean and variances \( \Omega_0, \Omega_1, \) and \( \Omega_2, \) respectively.

We further assume that:
a2) the additive noises \( z_{0}[n], z_{1,k}[n], \) and \( z_{2,k}[n] \) are mutually independent complex Gaussian distributed sequences with zero mean and variances \( \Omega_0, \Omega_1, \) and \( \Omega_2, \) respectively.

With the fading realizations \( h_0, h_{1,k}, \) and \( h_{2,k} \) as in a1), we find the signal to noise ratios (SNRs) per hop
\[ \gamma_0 := \frac{\varepsilon_k |h_0|^2}{\Omega_0}, \quad \gamma_{1,k} := \frac{\varepsilon_k |h_{1,k}|^2}{\Omega_{1,k}}, \quad \text{and} \quad \gamma_{2,k} := \frac{\varepsilon_k |h_{2,k}|^2}{\Omega_{2,k}}, \]
to be independent and exponentially distributed with means
\[ \bar{\gamma}_0 = \mathbb{E} \left[ \gamma_0 \right] = \frac{\varepsilon_k |h_0|^2}{\Omega_0}, \quad \bar{\gamma}_{1,k} = \frac{\varepsilon_k |h_{1,k}|^2}{\Omega_{1,k}}, \quad \text{and} \quad \bar{\gamma}_{2,k} = \frac{\varepsilon_k |h_{2,k}|^2}{\Omega_{2,k}}, \]
respectively, where \( \varepsilon_k \) is the average radiated energy per symbol at \( R_k. \)

For a non-regenerative system, an automatic gain control (AGC) front-end is required at the relay in order to prevent \( r_k[n] \) from saturating the relay amplifier. Besides adjusting the input power to the amplifier, the AGC facilitates control on the relay's output power. Specifically, if we want to constrain the average radiated energy per symbol at the 4th relay to be \( \varepsilon_k, \) a good choice is to adopt an AGC that employs
\[ \alpha_k = \frac{h_{1,k}^*}{|h_{1,k}|} \sqrt{\frac{\varepsilon_k}{\varepsilon_0 |h_{1,k}|^2 + \Omega_{1,k}}}, \quad k \in \{1, 2\}. \]

### 3. An Alamouti-Based Distributed Space-Time Diversity System

In this section, we consider a specific implementation of the more general system described above. More specifically, we implement a system based on the Alamouti's space-time coding scheme. We then compare this Alamouti-based scheme with the distributed diversity system of [5].

#### 3.1. The proposed system

In the non-regenerative distributed space-time diversity (DSTD) system, terminal S broadcasts a block of two symbols \( x[i] := [x[2i], x[2i - 1]]^T, \) which is received by the two relays and by terminal D. In the same time slot \( i, \) relay \( R_1 \) transmits the block \( u_1[i - 1] := [\alpha_1 r_1[2i - 2], -\alpha_1^* r_1[2i - 3]]^T, \) and relay \( R_2 \) transmits the block \( u_2[i - 1] := [\alpha_2 r_2[2i - 3], \alpha_2^* r_2[2i - 2]]^T, \) which corresponds to the Alamouti space-time coding scheme (see [7]) with the two transmit antennas distributed over two different terminals. An Alamouti decoder recovers \( x[i - 1] \) at the destination terminal D, as described in [7]. Processing of \( x[i] \) at terminal D is delayed by 2 symbols until the relays have transmitted \( u_1[i] \) and \( u_2[i]. \) If the power of the noise in \( y_1[n] \) and the power of the noise in \( y_2[n] \) are the same (i.e., \( |h_{2,1}|^2 \varepsilon_1 N_{1,1}/(\varepsilon_0 |h_{1,1}|^2 + N_{1,1}) + N_{2,1} = |h_{2,2}|^2 \varepsilon_2 N_{1,2}/(\varepsilon_0 |h_{1,2}|^2 + N_{1,2}) + N_{2,2}, \) the Alamouti decoder is equivalent with an MRC decoder. However, the noise powers cannot be equal since they depend on different channel coefficients. Our subsequent analysis assumes...
equal noise powers, and consequently, we only obtain a tight lower bound on the average symbol error probability (ASEP) of this non-regenerative DSTD system. The ASEP depends on the overall SNR at the destination terminal D. If we assume the same noise power in $y_1[n]$ and $y_2[n]$, we can write the overall SNR as $\gamma = \gamma_0 + \gamma_1 + \gamma_2$, where $\gamma_k$, $k \in \{1, 2\}$, is the SNR in (2). With the relay gain as in (4), it turns out $\gamma_k = \gamma_1 + \gamma_2 + \gamma_k + 1$, $k \in \{1, 2\}$. In order to find the performance of our system with an MRC receiver, we first derive the moment generating functions (MGFs) of $\{\gamma_k\}_{k=1}^2$, which are

$$M_{\gamma_k}(s) = \frac{s^2\gamma_k - 4\gamma_{pk}}{s^2\gamma_k - 4\gamma_{pk} + 2s^{\gamma_{pk}} - 2s^{\gamma_{pk}} + 1} \left(1 - \frac{s^2\gamma_k - 4\gamma_{pk}}{s^2\gamma_k - 4\gamma_{pk} + 2s^{\gamma_{pk}} - 2s^{\gamma_{pk}} + 1}\right),$$

for $\mathcal{R}\{s\} < \gamma_{pk}$ and $\gamma_{pk} > 2\sqrt{\gamma_{pk}}$, where $\gamma_{pk} := \frac{\gamma_1 + \gamma_2}{\gamma_1 + \gamma_2 + 1}$, $\gamma_{pk} := \frac{\gamma_1 + \gamma_2}{\gamma_1 + \gamma_2 + 1}$, and $\gamma_{pk} := \frac{\gamma_{pk}}{\gamma_{pk} + 1}$, $k \in \{1, 2\}$, and where $E_1(\cdot)$ is the exponential integral function defined as in [8, Eq. 5.1.1]. The proof of (5) is given in Appendix A.

Second, we use the MGF-based approach of [9, Eq. 9.21] to obtain the following closed-form expression that approximates the ASEP of the non-regenerative DSTD system employing $M$-QAM modulation:

$$P_e^{(m)} = \frac{4\sqrt{M-1}}{\pi} \left[\int_{0}^{\pi/2} \frac{\prod_{k=1}^{2} M_{\gamma_k}(\frac{-g_{QAM}}{\sin^2 \varphi})}{1 + \frac{g_{QAM}}{\sin^2 \varphi}} d\varphi \right] \left[\int_{0}^{\pi/4} \frac{\prod_{k=1}^{2} M_{\gamma_k}(\frac{-g_{QAM}}{\sin^2 \varphi})}{1 + \frac{g_{QAM}}{\sin^2 \varphi}} d\varphi \right],$$

where $g_{QAM} := 3/(2(M-1))$.

For the regenerative DSTD system, the communication scheme remains the same except for the relays, which now detect the information symbols received from terminal S. After the relays have regenerated the information symbols at time slot $i-1$, $R_1$ transmits $\hat{x}[2i-2]$, $\hat{x}[2i-2]^T$, and $R_2$ transmits $\hat{x}[2i-3]$, $\hat{x}[2i-3]^T$, where $\hat{x}[n]$ and $\hat{x}[n]$ are ML estimates of $x[n]$. We assume that $\{h_1, k\}_{k=1}^2$ are not available at the destination station D and the MRC receiver at D assumes perfect recovery of $x[n]$ at the relays. If $\{h_1, k\}_{k=1}^2$ and $\{N_1, k\}_{k=1}^2$ are known at the destination station D an overall ML receiver can be used to recover $x[n]$ at D (see [5]), but this case is not considered here.

In order to find a closed form expression for the ASEP of the regenerative DSTD system we approximate the S to D nonlinear subchannel that includes the regenerative relay $R_k$ (i.e., the subchannel k) with an equivalent Rayleigh fading channel with the same individual ASEP. Hence, we end up with 3 Rayleigh fading subchannels between S and D, and we can use the approximation in [9, p.275] to obtain the overall ASEP of the system

$$P_e^{(r)} = \left(\frac{M}{M-1}\right)^2 \prod_{k=0}^{3} P_e^{(r)},$$

where $P_e^{(r)}$ is the individual ASEP for subchannel $k$ (subchannel 0 is the direct path from S to D).

In order to find $P_e^{(r)}$ for the two-hop system that uses only subchannel $k$ and undergoes two decoding processes, one at $R_k$ and one at the destination D. For BPSK $P_e^{(r)} = P_{1,k} + P_{2,k} - 2P_{1,k}P_{2,k}$, where $P_{1,k}$ is the ASEP for transmissions between S and $R_k$, and $P_{2,k}$ is the ASEP for transmissions between $R_k$ and D (see [10, Eq.(11.4.12)]). We can extend the result in [10, Eq.(11.4.12)] by computing an exact expression of $P_e^{(r)}$ for an $M$-QAM modulation (see Appendix B for an example) or we can use the following approximation:

$$P_e^{(r)} \approx P_{1,k} + P_{2,k} - P_{1,k}P_{2,k} - \frac{P_{1,k}P_{2,k}}{M-1}, k \in \{1, 2\}$$

The last term in (8) compensates for the case when an error-free transmission is achieved between S and D even though errors occur between S and $R_k$ and between $R_k$ and D (i.e., two consecutive errors that cancel out).

3.2. The Distributed-Diversity System of [5]

The cooperative design of [5] requires the assignment of orthogonal signal subspaces for every transmitting terminal in the network, e.g., a different frequency band is allocated to each terminal. Hence, when $K = 2$, we need to employ three frequency bands. In this case, the increase in bandwidth requirements is 1.5-fold compared with the Alamouti-based DSTD system. Therefore, $M^{1.5}$-QAM modulation has to be selected if bandwidth constraints are to be met.

The major drawback of the cooperative system of [5] is that the bandwidth increases with the number of relays. However, implementing a generalized orthogonal space-time block coding scheme at the relays only comes with a 2-fold bandwidth expansion. The price paid is an increase in delay at terminal D. This delay is at least equal to the number of relays times the symbol period.

4. PERFORMANCE COMPARISON

To compare the performance of the two systems discussed in this summary, we have selected balanced channels, i.e., $\Omega_0 = \Omega_{1,k} = \Omega_{2,k} = 1$, $k \in \{1, 2\}$, equal power allocation among relays, and equal power noises, i.e., $N_0 = N_{1,k} = N_{2,k}, k \in \{1, 2\}$. In Fig. 3, we plot the ASEP of the two systems versus the $E_s/N_0$, where $E_s := \sum_{k=0}^{3} E_k = 3 \epsilon_0$. To keep the same bandwidth constraints, we select a 16-QAM modulation for the
5. CONCLUSIONS

We have proposed an Alamouti based cooperative system with two relay stations, and derived an approximate formula for the average symbol error probability of this system in a Rayleigh fading environment. Simulation results show that the proposed system outperforms an existing cooperative design based on FDMA, and hence yields an attractive approach to introduce cooperation among terminals in a wireless network.

Appendix A

PROOF OF (5)

In order to prove (5) we need to find the MGF of $\Gamma(X, Y) := XY/(X + Y + 1)$, where $X$ and $Y$ are two independent and exponentially distributed random variables with mean $\bar{X}$ and $\bar{Y}$. From [11] we know the cumulative distribution function (CDF) of $\Gamma(X, Y)$ to be for $\gamma > 0$

$$F_{\Gamma}(\gamma) = 1 - \frac{2\sqrt{\gamma^2 + \gamma}}{\sqrt{\bar{p}}} e^{-\gamma} K_1\left(\frac{2\sqrt{\gamma^2 + \gamma}}{\sqrt{\bar{p}}}\right),$$

(9)

where $\sigma := \bar{X} + \bar{Y}$, $p := \bar{X}\bar{Y}$, and $K_\nu(\cdot)$ denotes the modified Bessel function of the second kind and order $\nu$.

We take the derivative with respect to $\gamma$ in (9) to obtain the probability density function (PDF) of $\Gamma(X, Y)$ and we use [8, Eq. 9.6.26] for the derivative of $K_1(\gamma)$ in order to establish the following PDF for $\Gamma(X, Y)$:

$$f_{\Gamma}(\gamma) = \frac{4\gamma + 2}{p} e^{-\gamma} K_0\left(\frac{2\gamma}{\sqrt{\bar{p}}}\sqrt{\gamma^2 + \gamma}\right)$$

$$+ \frac{2\sigma\sqrt{\gamma^2 + \gamma}}{p\sqrt{\bar{p}}} e^{-\gamma} K_1\left(\frac{2\gamma}{\sqrt{\bar{p}}}\sqrt{\gamma^2 + \gamma}\right), \quad \gamma > 0.$$  

(10)

We use the definition of the moment generating function along with (10) to write:

$$M_{\Gamma}(s) = \frac{2}{p} \int_0^\infty \left((2\gamma + 1)e^{-\gamma} e^{s\gamma} K_0\left(\frac{2\gamma}{\sqrt{\bar{p}}}\sqrt{\gamma^2 + \gamma}\right)\right.$$  

$$+ \frac{\sigma}{\sqrt{\bar{p}}} \sqrt{\gamma^2 + \gamma} e^{-\gamma} e^{s\gamma} K_1\left(\frac{2\gamma}{\sqrt{\bar{p}}}\sqrt{\gamma^2 + \gamma}\right) d\gamma. $$

(11)
Using the change of variable $\gamma \to \gamma - 1/2$ and after some manipulations we obtain:

$$M_\Gamma(s) = \frac{2}{p} e^{\frac{s}{2}} \left[ 2J_0(s) + \frac{\sigma}{\sqrt{p}} J_1(s) \right],$$

(12)

where

$$J_0(s) = \int_{1/2}^{\infty} e^{-\alpha \gamma} K_0 \left( \beta \sqrt{\gamma^2 - (1/2)^2} \right) d\gamma,$$

(13)

and

$$J_1(s) = \int_{1/2}^{\infty} \sqrt{\gamma^2 - (1/2)^2} e^{-\alpha \gamma} K_1 \left( \beta \sqrt{\gamma^2 - (1/2)^2} \right) d\gamma,$$

(14)

and where $\alpha := (\sigma - ps)/p$, $\beta := 2/\sqrt{p}$.

In order to simplify the integrand in (13), we write

$$J_0(s) = -\frac{\partial}{\partial \alpha} \left[ \int_{1/2}^{\infty} e^{-\alpha \gamma} K_0 \left( \beta \sqrt{\gamma^2 - (1/2)^2} \right) d\gamma \right].$$

(15)

To solve the integral in (15) we use [12, Eq. 6.646] to obtain

$$J_0(s) = \frac{1}{2} e^{-\frac{\alpha^2}{2\beta^2}} E_1 \left( \frac{\alpha - \sqrt{\alpha^2 - \beta^2}}{2} \right)$$

$$- e^{-\frac{\alpha^2}{2\beta^2}} E_1 \left( \frac{\alpha + \sqrt{\alpha^2 - \beta^2}}{2} \right),$$

and after differentiating with respect to $\alpha$, and then letting $\alpha = (\sigma - ps)/p$ we find

$$J_0(s) = \frac{p}{4\beta^3} e^{-\frac{4\rho}{\beta^2}} \left[ \delta(\delta + 2p) e^{\frac{4\rho}{\beta^2}} E_1 \left( \frac{\delta - \rho}{2p} \right) \right]$$

$$+ \delta(\delta - 2p) e^{\frac{4\rho}{\beta^2}} E_1 \left( \frac{\delta + \rho}{2p} \right) - 4\rho p,$$

(16)

for $\Re \{ s \} < \sigma/p + 2/\sqrt{p}$, where $\rho := \sqrt{\delta^2 - 4p}$ and $\delta := \sigma - ps$.

Similar to (15), we write $J_1(s)$ in (14) as

$$J_1(s) = -\frac{\partial}{\partial \beta} \left[ \int_{1/2}^{\infty} e^{-\alpha \gamma} K_0 \left( \beta \sqrt{\gamma^2 - (1/2)^2} \right) d\gamma \right].$$

(17)

We note that the above integral is the same as the integral in (15). Consequently, we use [12, Eq. 6.646] again to find the integral in (17), and after differentiating with respect to $\beta$, and then letting $\beta = 2/\sqrt{p}$ we obtain:

$$J_1(s) = \frac{p\sqrt{p}}{2p^3} e^{-\frac{\delta}{\beta^2}} \left[ -(\delta + 2p) e^{\frac{4\rho}{\beta^2}} E_1 \left( \frac{\delta - \rho}{2p} \right) \right]$$

$$- (\delta - 2p) e^{\frac{4\rho}{\beta^2}} E_1 \left( \frac{\delta + \rho}{2p} \right) + \rho \delta,$$

(18)

for $\Re \{ s \} < \sigma/p + 2/\sqrt{p}$. Now, if we substitute (16) and (18) into (12), we obtain the MGF of $\Gamma(X, Y)$, and consequently, after replacing $X$ with $\gamma_{1,k}$ and $Y$ with $\gamma_{2,k}$ we obtain (5).

### Appendix B

**$P_{e,k}$ for a 4-QAM Modulation**

Because an error occurs on subchannel $k$ if an error occurs on either hop unless two consecutive errors cancel out, we can write

$$P_{e,k} = P_{1,k} + P_{2,k} - P_{1,k} P_{2,k} - P_{e,k},$$

(19)

where the formula for $P_{1,k}$ and $P_{2,k}$ is given in [9, Eq. (8.107)], and $P_{e,k}$ is the probability that an error on the S to $R_k$ transmission is eliminated by a consecutive reverse error on the $R_k$ to D transmission. For a 4-QAM modulation

$$P_{e,k} = 2(p_{1,k} - p'_{1,k})(p_{2,k} - p'_{2,k}) + p'_{1,k} p'_{2,k},$$

(20)

where

$$p_{i,k} := \frac{1}{2} \left[ 1 - \sqrt{\frac{\gamma_{i,k}}{2 + \gamma_{i,k}}} \right], \text{ and }$$

$$p'_{i,k} := \frac{1}{4} \left[ 1 - \sqrt{\frac{\gamma_{i,k}}{2 + \gamma_{i,k}}} \right] \left( \frac{4}{\pi} \tan \sqrt{\frac{2 + \gamma_{i,k}}{\gamma_{i,k}}} \right),$$

$i \in \{1, 2\}$, and where atan(·) is the inverse tangent function.

### 6. REFERENCES


