MIXED-PHASE INVERSE FILTERING OF SEISMIC DATA

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ABSTRACT

We describe an algorithm for mixed-phase inverse filtering of seismic data. Inverse filtering is applied to seismic data to remove the effect of the source wavelet to obtain an estimate of the Earth’s reflectivity. Solving the Yule-Walker equations gives the inverse filter which corresponds to a minimum-phase source wavelet. When the source wavelet is mixed phase, however, this minimum-phase inverse filter produces a poor result. By solving the “extended” Yule-Walker equations with lag \( \alpha \) on the diagonals of the autocorrelation function matrix, it is possible to obtain a suitable anti-causal all-pass filter which can be convolved with the minimum-phase inverse filter to give a mixed-phase inverse filter. For each lag \( \alpha \) tested, the resulting mixed-phase inverse filter is applied to the seismic data, and the best overall mixed-phase inverse filter is chosen to be the one for which the deconvolved seismic data has the largest \( L^5 \)-norm. After best \( \alpha \) has been selected, a genetic algorithm can be used to find an optimum all-pass filter of reduced order \( \beta \leq \alpha \). The optimum all-pass filter is finally convolved with the minimum-phase inverse filter to produce an optimal mixed-phase inverse filter. This mixed-phase spiking deconvolution algorithm has been implemented as a socket tool in the commercial seismic data processing package named ProMAX.

1. INTRODUCTION

A simple model for a recorded seismic signal is the convolution of the source wavelet with the impulse responses of the hydrophones and the recording system (in sum the effective source wavelet), folded with the Earth’s impulse response. Spiking deconvolution attempts to remove the impact of the effective source wavelet to reveal the reflectivity of the Earth. Standard spiking deconvolution assumes that the Earth’s impulse response is a white random process, and that the effective source wavelet solely constitutes the redundant part of the seismic signal. Spiking deconvolution is, as such, the design and application of an inverse filter to remove the band-limitation introduced by convolution of the effective source wavelet with the reflectivity of the Earth.

If the effective source wavelet was known, spiking deconvolution could be carried out using its inverse filter (i.e., deterministic deconvolution). However, since the effective source wavelet is commonly unknown, the calculation of the inverse filter has to be based on an estimate of the source wavelet’s autocorrelation function (ACF). This ACF can be estimated from the recorded seismic data because of the white random process reflectivity model assumption. An optimal inverse filter can be found, using Wiener filter theory, if the effective source wavelet has minimum phase (i.e., statistical deconvolution).

Nevertheless, when the effective source wavelet is far from minimum phase, violation of the underlying assumption of minimum phase generally leads to poor spiking deconvolution. In such cases it is reasonable to believe that mixed-phase spiking deconvolution will work better. Porsani and Ursin have published one possible mixed-phase spiking deconvolution algorithm [1, 2, 3]. ProMAX\(^1\) is the main seismic data processing package used by Statoil and other oil companies. Current version of ProMAX offers minimum-phase spiking deconvolution. To be able to test the benefits of mixed-phase spiking deconvolution we have implemented Porsani and Ursin’s method as a socket tool in ProMAX. This new tool has so far been successfully tested on a synthetic source wavelet with mixed phase, and on a marine seismic dataset from the North Sea. For the synthetic dataset (Section 3.1), mixed-phase deconvolution gave much better spiking than standard minimum-phase deconvolution since the chosen wavelet was strongly mixed phase. For the North Sea dataset (Section 3.2), mixed-phase deconvolution also lead to good spiking, but the estimated mixed-phase inverse filter was almost minimum phase.

2. THEORY

Seismic data \( x(n) \) are assumed to consist of a (mixed-phase) real-valued wavelet \( p(n) \) convolved with a white

\(^1\)Trademark of Landmark Graphics Corporation.
reflectivity series \( r(n) \) plus white noise \( w(n) \) as
\[
x(n) = \sum_{k=0}^{N} p(k)r(n-k) + w(n). \tag{1}
\]

The z-transform of the mixed-phase wavelet \( p(n) \) for \( n = 0, 1, \ldots, N \) is written as
\[
P(z) = \sum_{n=0}^{N} p(n)z^{-n}, \tag{2}
\]
where we have used the sign convention as defined in the electrical-engineering community. We assume that \( P(z) \) has \( \alpha \) zeros outside the unit circle [corresponding to a maximum-phase wavelet \( B(z^{-1}) \)], no zeros on the unit circle, and \( N-\alpha \) zeros inside the unit circle [agreeing to a minimum-phase wavelet \( A(z) \)]. We then have,
\[
P(z) = A(z)B(z^{-1})z^{-\alpha}, \tag{3}
\]
where \( a(n) = \{1, a(2), \ldots, a(N-\alpha)\} \) is causal (the first coefficient corresponds to \( n = 0 \)) and minimum-phase with
\[
A(z) = \sum_{n=0}^{N-\alpha} a(n)z^{-n}, \quad a(0) = 1, \tag{4}
\]
and \( \bar{b}(n) = b(-n) = \{b(\alpha), b(\alpha-1), \ldots, 1\} \) is anti-causal (the last coefficient corresponds to \( n = 0 \)) and maximum-phase with
\[
B(z^{-1}) = \sum_{n=0}^{\alpha} b(n)z^{n}, \quad b(0) = 1. \tag{5}
\]
In the time domain, the z-transform in equation 3 reads
\[
p(n) = a(n) * \bar{b}(n) * \delta(n-\alpha), \tag{6}
\]
where * denotes convolution and \( \delta(n) \) is the Kronecker-delta. The causal and minimum-phase wavelet with the same amplitude spectrum as \( p(n) \) is
\[
\tilde{p}(n) = a(n) * b(n), \tag{7}
\]
with z-transform
\[
\tilde{P}(z) = A(z)B(z) \tag{8}
\]
since both \( a(n) \) and \( b(n) \) are causal and minimum-phase time series.

The main purpose in spiking deconvolution is to compute the inverse filter \( H(z) \) of the wavelet \( P(z) \) such that
\[
P(z)H(z) = 1. \tag{9}
\]
However, since the causal wavelet \( P(z) \) is mixed-phase the inverse filter \( H(z) = 1/P(z) \) cannot simultaneously be stable and causal. But the inverse filter \( \hat{H}(z) \) of the causal and minimum-phase wavelet \( \hat{P}(z) \) is stable and causal,
\[
\hat{H}(z) = \frac{1}{A(z)} \frac{1}{B(z)}. \tag{10}
\]
The inverse filter of the mixed-phase wavelet \( P(z) \) can be written as
\[
H(z) = \frac{1}{A(z)} \frac{1}{B(z^{-1})z^{-\alpha}} = \hat{H}(z) \frac{B(z)}{B(z^{-1})z^{-\alpha}} = \hat{H}(z)F(z^{-1}). \tag{11}
\]
This shows that the mixed-phase inverse filter is equal to the minimum-phase inverse filter \( \hat{H}(z) \) convolved with an anti-causal all-pass filter \( F(z^{-1}) \). Thus, \( H(z) \) becomes non-causal.

### 2.1. Basic assumptions

We assume that the mixed-phase wavelet \( p(n) \) is unknown, but that an estimate of its ACF \( R_p(n) \) is available. If this is not the case, \( R_p(n) \) may be estimated directly from the seismic data by making the following assumptions:

1. The white reflectivity series \( r(n) \) with variance \( \sigma_r^2 \), and the white noise \( w(n) \) with variance \( \sigma_w^2 \) are uncorrelated so the that ACF of the seismic data is (see equation 1)
   \[
   R_x(n) = R_p(n) * R_r(n) + R_w(n) = R_p(n) * \sigma_r^2 \delta(n) + \sigma_w^2 \delta(n).
   \]

2. The signal-to-noise ratio is large, i.e.,
   \[
   \sigma_r^2 \gg \sigma_w^2.
   \]
   With these assumptions, we may use the estimate
   \[
   R_p(n) \approx \sigma_r^{-2} R_x(n), \tag{12}
   \]
where the scaling factor \( \sigma_r^{-2} \) is not important.

### 2.2. Minimum-phase spiking deconvolution

To find the minimum-phase inverse filter \( \hat{h}(n) \) and its associated minimum-phase wavelet \( \tilde{p}(n) \) we use optimum FIR linear prediction filter theory [4]. The minimum-phase inverse filter \( \hat{h}(n) \) obeys the famous Yule-Walker (YW) equations:
\[
\begin{bmatrix}
R_{\tilde{p}}(0) & \cdots & R_{\tilde{p}}(-N) \\
R_{\tilde{p}}(1) & \cdots & R_{\tilde{p}}(-1-N) \\
\vdots & \ddots & \vdots \\
R_{\tilde{p}}(N) & \cdots & R_{\tilde{p}}(0)
\end{bmatrix}
\begin{bmatrix}
\hat{h}(0) \\
\hat{h}(1) \\
\vdots \\
\hat{h}(N)
\end{bmatrix} =
\begin{bmatrix}
\sigma_r^2 \\
\delta \\
\vdots \\
0
\end{bmatrix},
\]
where \( R_{\tilde{p}}(n) \) denotes the ACF of the minimum-phase wavelet, with \( R_{\tilde{p}}(n) > R_{\tilde{p}}(n+1) \) for all \( n \geq 0 \) and
on (or close to) the unit-circle \( R_p \leq 1 \) is introduced in the YW equations. This is called a filter and solving the YW equations once more. Computing the ACF of the minimum-phase inverse (the matrix are identical) and symmetric (the matrix and its transpose are identical). To solve YW equations efficiently (i.e., find \( h_N = (0) \hat{h}(1) \ldots \hat{h}(N) \)\(^T\)) we can use the Levinson-Durbin algorithm [4].

2.2.1. Minimum-phase wavelet, \( \hat{p}(n) \)

The minimum-phase wavelet \( \hat{p}(n) \) can be found by computing the ACF of the minimum-phase inverse filter and solving the YW equations once more.

2.2.2. Pre-whitening, \( \epsilon \)

After the Fourier-transform, we have that

\[
\mathcal{H}(\omega) = \frac{1}{P(\omega)},
\]

which becomes numerically unstable if \( P(z) \) has zeros on (or close to) the unit-circle \( z = \exp(j\omega) \). To ensure numerical stability, an artificial level of white noise is introduced in the YW equations. This is called pre-whitening, and is achieved by adding a constant \( 0 \leq \epsilon < 1 \) to the zero-lag of the normalized ACF, i.e., \( R_p(0) = 1 + \epsilon \). For example, \( \epsilon = 0.01 \) corresponds to 1.0 \% additive white noise.

2.3. Mixed-phase spiking deconvolution

We have shown how the mixed-phase inverse filter can be decomposed as the minimum-phase inverse filter convolved with an anti-causal all-pass filter (see equation 11). Porsani and Ursin [1] have shown that suitable all-pass filters can be computed by solving the “extended” YW equations (EWY) with different lags \( \alpha \) on the diagonals of the ACF matrix, and then choose the all-pass filter which performs most superior when applied to the seismic data. Solving the EWY equations with lag \( \alpha \) on the diagonals of the ACF matrix, gives the infinite-length series \( c_\alpha(n) \):

\[
\begin{bmatrix}
R_p(\alpha) & R_p(\alpha - N) & \ldots & R_p(\alpha - N) \\
R_p(\alpha + 1) & R_p(\alpha - (N - 1)) & \ldots & R_p(\alpha - (N - 1)) \\
\vdots & \ddots & \ddots & \vdots \\
R_p(\alpha + N) & \ldots & R_p(\alpha + N) & \ldots \\
\end{bmatrix}
\begin{bmatrix}
c_\alpha(0) \\
c_\alpha(1) \\
\vdots \\
c_\alpha(N) \\
\end{bmatrix} = 
\begin{bmatrix}
s^2_{\alpha,N} \\
\sigma^2_\alpha \\
\sigma^2_\alpha \\
0 \\
0 \\
\end{bmatrix},
\]

where \( c_\alpha(0) = 1 \) can be used without loss of generality. Remark, the \((N + 1) \times (N + 1)\) matrix to the left of the EWY equations is Toeplitz, but non-symmetric (the matrix and its transpose are not identical). To solve the EWY equations efficiently (i.e., find \( c_\alpha = [c_\alpha(0) \ldots c_\alpha(N)]^T \)) we can use a Levinson-Durbin-type algorithm which we call the Porsani-Urlych algorithm [5] (see Appendix A).

After obtaining the infinite-length series \( c_\alpha(n) \), for a given integer delay \( \alpha \), we compute the finite-length series \( b_\alpha(n) [1] \)

\[
b_\alpha(n) = \hat{p}(n) \ast c_\alpha(n).
\]

It can be shown that \( b_\alpha(n) = 0 \) for \( n > \alpha \). Factorizing the \( z \)-transform of \( b_\alpha(n) \) gives (\( b_\alpha(0) = 1 \))

\[
B_\alpha(z) = \sum_{n=0}^{\alpha} b_\alpha(n)z^{-n} = \prod_{k=1}^{\alpha} (1 - z^{-1}r_k),
\]

where there may be complex roots \( r_k \), \( k = 1, 2, \ldots, \alpha \), which always occur in complex-conjugated pairs since \( b_\alpha(n) \) is real-valued. The roots of \( B_\alpha(z) \) are all inside the unit circle because \( b_\alpha(n) \) is minimum-phase, so that \( |r_k| < 1 \). They are also the roots of \( \hat{P}(z) \) which are closest to the unit circle and, therefore, the ones which are most important for the mixed-phase inverse filter.

In [1], \( \alpha \) was fixed and \( B_\alpha(z) \) was used directly for \( B(z) \) in equation 11. In [2], on the other hand, the phase of the mixed-phase inverse filter was varied systematically by letting only \( \beta \leq \alpha \) of the zeros of \( B_\alpha(z) \) be inside the unit circle while neglecting the other \( \alpha - \beta \) roots. This is done by forming

\[
B_\beta(z) = \prod_{k=1}^{\beta} (1 - z^{-1}r_k),
\]

and applying the modified mixed-phase inverse filter

\[
H_\beta(z) = \hat{H}(z) \frac{B_\beta(z)z^\beta}{B_\beta(z^{-1})}
\]

to the seismic data. For \( \beta = 0 \), the modified mixed-phase inverse filter becomes identical to the minimum-phase inverse filter. By systematically allowing all possible combinations of the roots \( r_k \), to be inside the unit circle, we obtain, for a given value of \( \alpha \), at most \( 2^\alpha \) different all-pass filters. There may be less than \( 2^\alpha \) all-pass filters because a pair of complex-conjugated roots is treated as one root (they are both either inside or outside the unit circle). Alternatively, a genetic algorithm (GA) can be used to obtain the “best” minimum-phase series \( b_\beta \) when \( \alpha \) is large [3]. Such a scheme is used in this paper.

2.3.1. Methodology

From the ACF of the seismic data we compute a minimum-phase inverse filter using YW equations. The minimum-phase wavelet is then found by computing the ACF of the minimum-phase inverse filter and solving the YW equations once more. Then, for a given...
1 ≤ α ≤ \text{max}_n$, we find the infinite-length series $c_\alpha(n)$ using the EYW equations followed by estimating the finite-length series $b_\alpha(n)$ (see equation 13). For each $\alpha$ we find a mixed-phase inverse filter $h_\alpha(n)$ (see equation 11 or equation 16 with $\alpha = \beta$) which is applied to the seismic data, to give the filtered output:

$$\hat{x}_\alpha(n) = x(n) * h_\alpha(n) \approx r(n).$$  (17)

To find an optimal $\alpha$ we calculate the “spikiness” of the deconvolved result by computing the $L^m$-norm for $m > 2$ of the filtered output

$$\Phi'(\alpha, m) = \left\{ \sum_n |\hat{x}_\alpha(n)|^m \right\}^{1/m}.  \tag{18}$$

This $L^m$-norm is normalized with respect to the value for minimum-phase spiking deconvolution (i.e., $\alpha = 0$) giving the normalized filter performance criterion of

$$\Phi(\alpha, m) = \frac{\Phi'(\alpha, m)}{\Phi'(0, m)}.$$  (19)

When this normalized objective function is maximum (for $m$ given), we obtain an estimate of $\alpha$, the number of zeros inside the unit circle of the minimum-phase series $b_\alpha(n)$. Experience has shown that the optimal value of $\alpha$ is not very sensitive to the choice of $m$ as long as $m > 2$. In practice $m = 5$ is often used [1]. After optimal $\alpha$ is found, an optimal minimum-phase series $b_\beta(n)$ for $\beta \leq \alpha$ of the anti-causal all-pass filter is finally obtained using a GA. In the z-domain, once a suitable $B_\beta(z)$ has been computed, we achieve an optimal mixed-phase inverse filter $H_\beta(z)$ using equation 16. Similarly, the optimal mixed-phase wavelet $P_\beta(z)$ is given by:

$$P_\beta(z) = \frac{1}{H_\beta(z)} = \hat{P}(z) \frac{B_\beta(z^{-1})z^{-\beta}}{B_\beta(z)}.  \tag{20}$$

3. RESULTS

3.1. Synthetic data example

Minimum-phase and mixed-phase spiking deconvolution have been applied to the synthetic source wavelet shown in Figure 1 (a).

The deconvolution operator length is 2000 ms (i.e., with 4 ms sampling distance the order of the YW equations is $N = 500$), the taper length is 0.0 ms (i.e., no tapering of the seismic data at each end before estimating its ACF), and the deconvolution operator white noise level is 0.0 % (i.e., no stabilization of the matrix inversion). Furthermore, the delay or order of the all-pass filter $\alpha$ has minimum value 1 and maximum value 50. The number of models per generation in the GA is 300 and the maximum number of generations is 100.

The result after spiking deconvolution using a minimum-phase inverse filter is shown in (b), and the result after optimal mixed-phase spiking deconvolution is shown in (c). The impulse responses of the two inverse filters are shown in (d) and (e), with the minimum-phase inverse filter at (d) and the mixed-phase inverse filter at (e). Note that the mixed-phase inverse filter is non-causal (i.e., two-sided). Using these two inverse filters, the corresponding wavelets were reconstructed. These are shown in (f) and (g), respectively, and they can be compared with the original source wavelet shown in (a). It is clear that the mixed-phase algorithm provides a much better inverse filter and a much better wavelet estimate then the minimum-phase algorithm.

We have included this example mainly to demonstrate the effectiveness of the algorithm to the inversion of a wavelet in a controlled situation where the wavelet is known and can be compared to the reconstructed wavelets.

3.2. Deconvolution of a North Sea dataset

The mixed-phase spiking deconvolution method has also been tested on a marine seismic dataset from the North Sea. The dataset consists of 1001 CDP gathers with 72 traces each. The trace length is equal to 3000 ms and the sampling distance is 4 ms. Minimum-phase and mixed-phase spiking deconvolution has been tested both prestack and poststack. The CDP gather closest to the well location has been used to design the inverse filters.

The deconvolution operator length is 120 ms (i.e., $N = 30$), and the deconvolution operator white noise level is 0.1 %. Moreover, the deconvolution design window are chosen from 1200 to 2000 ms. These main parameters were chosen equal for all four test cases. In the prestack mixed-phase deconvolution case, the all-pass filter length $\alpha$ was fixed to 30, and 10 prestack traces were used to design the all-pass operator. In the poststack mixed-phase decon case, the all-pass filter order $\alpha$ was fixed to 25, and only the stacked trace of the CDP gather closest to the well location was used to design the all-pass operator. The parameters for the GA were selected equal for prestack and poststack mixed-phase deconvolution tests with 500 number of models per generation and maximum 50 number of generations.

Prestack minimum-phase and mixed-phase spiking deconvolution results are shown, after stacking, in Figure 2 (a) and (b), respectively. It is evident that mixed-phase deconvolution provides improved resolution, i.e., it gives sharper, better defined, and more continuous reflectors (compare the dashed circles).

Poststack minimum-phase and mixed-phase spiking deconvolution results are given in Figure 3 (a)
and (b), respectively. In general, the poststack deconvolution results are more noisy compared to the prestack deconvolution results. This can be explained by less stationary wavelets due to uncertainties in the stacking velocities, etc. Again, the minimum-phase and mixed-phase deconvolution results are different, but the differences are smaller than for the prestack deconvolution case. For example, mixed-phase spiking deconvolution shows a thin layer around 1535 ms (compare the solid circles) and a better defined reflector at 1615 ms (compare the dashed circles).

4. CONCLUSIONS

We have implemented mixed-phase spiking deconvolution in ProMAX. This new tool has been successfully applied to the inversion of a synthetic source wavelet, and it was tested on a marine seismic dataset from the North Sea with promising results. It is evident that mixed-phase spiking deconvolution can provide improved resolution compared to standard minimum-phase spiking deconvolution of seismic data. The new algorithm should be tested extensively to verify to which extent seismic processing can benefit from relaxing the assumption of a minimum-phase effective source wavelet in spiking deconvolution.

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6. REFERENCES


A. THE PORSANI-ULRYCH ALGORITHM

We want to efficiently solve extended YW equations for augmented lag $\alpha$ of the non-symmetric Toeplitz matrix

$$
\begin{bmatrix}
R_p(\alpha) & R_p(\alpha+1) & \cdots & R_p(\alpha-N) \\
R_p(\alpha+1) & R_p(\alpha+2) & \cdots & R_p(\alpha+N) \\
\vdots & \vdots & \ddots & \vdots \\
R_p(\alpha+N) & \cdots & \cdots & R_p(\alpha+1)
\end{bmatrix} \begin{bmatrix}
\alpha_0 \cr \alpha_1 \cr \vdots \cr \alpha_N
\end{bmatrix} = \begin{bmatrix}
1 \\
\alpha_0(1) \\
\alpha_0(2) \\
\vdots \\
\alpha_0(N)
\end{bmatrix},
$$

where we have found the optimal solution of the corresponding YW equations (i.e., $\alpha = 0$)

$$
\hat{h}_N = [\hat{h}(1) \ldots \hat{h}(N)]^T,
$$

and the associated linear prediction error variance $\sigma^2_N$.

We denote the column vectors $\alpha r_N = [R_p(\alpha) R_p(\alpha+1) \ldots R_p(\alpha+N)]^T$, $\alpha c_N = [\alpha_0(1) \ldots \alpha_0(N)]^T$, and $\alpha c_N = [\alpha_0(N) \alpha_0(N-1) \ldots 1]^T$. The Porsani-Ulrych algorithm can be formulated as follows [1, 5]:

Initialzation

$$
0 c_N = \hat{h}_N; \\
-1 c_N = \frac{1}{\alpha_0(N)} \alpha c_N; \\
\sigma^2_{0,N} = \sigma^2_N.
$$

For $k = 1, 2, \ldots, N$

$$
\sigma^2_{k,N} = \frac{k r_N^T k^{-1} c_N}{\alpha_0(N)}; \\
k g_N = k^{-1} c_N - \frac{\alpha_0(N)}{\alpha_0(N)} k^{-2} c_N; \\
\begin{bmatrix}
k c_N \\
\vdots \\
0
\end{bmatrix} = \begin{bmatrix}
k^{-1} c_N \\
\vdots \\
0
\end{bmatrix} + \frac{\sigma^2_{k,N}}{\sigma^2_{k-1,N}} \begin{bmatrix}
0 \\
\vdots \\
k g_N
\end{bmatrix}.
$$

End do
Figure 1: Synthetic source wavelet inversion example.

Figure 2: Prestack deconvolution of a North Sea dataset shown after stacking. Mixed-phase deconvolution provides improved resolution, i.e., it gives sharper, better defined, and more continuous reflectors. The dashed circles indicate two areas with large differences between the two methods.

Figure 3: Poststack deconvolution of a North Sea dataset. For example, mixed-phase spiking deconvolution shows a thin layer around 1535 ms (compare the solid circles). Moreover, mixed-phase spiking deconvolution gives a better defined reflector at 1615 ms (compare the dashed circles).