ERRATUM: FIBRATIONS ON GENERALIZED KUMMER VARIETIES

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1. The Fourier-Mukai transform of the structure sheaf of a finite subscheme

Under the thesis defense, prof. Manfred Lehn pointed out the following mistake: It is claimed that, whenever $Z \subset A$ is a finite subscheme of an abelian variety, then the Fourier-Mukai transform of its structure sheaf is

$$\widehat{\mathscr{O}_Z} \cong \bigoplus_i \mathscr{P}_{a_i}, \tag{*}$$

where $[Z] = \sum_i [a_i]$ as zero-cycles on A. By the invertibility of the Fourier-Mukai transform, this cannot possibly hold for non-reduced Z, as the right hand side only remembers Z as a cycle, and forgets its scheme structure.

The correct statement is the following: Choose any flag

$$Z_1 \subset Z_2 \subset \cdots \subset Z_n = Z,$$

where Z_i is a finite subscheme of length i. For each i there is a point $a_i \in A$ and a short exact sequence

$$0 \to k(a_i) \to \mathscr{O}_{Z_i} \to \mathscr{O}_{Z_{i-1}} \to 0.$$

Let $\mathscr{F}_i$ denote the Fourier-Mukai transform of $\mathscr{O}_{Z_i}$. Then there are induced short exact sequences

$$0 \to \mathscr{P}_{a_i} \to \mathscr{F}_i \to \mathscr{F}_{i-1} \to 0.$$

Thus there exists a *cofiltration*

$$\widehat{\mathscr{O}_Z} = \mathscr{F}_n \overset{\pi_n}{\twoheadrightarrow} \cdots \overset{\pi_2}{\twoheadrightarrow} \mathscr{F}_1 \overset{\pi_1}{\twoheadrightarrow} 0 \tag{**}$$

where the kernel of π_i is isomorphic to $\mathscr{P}_{a_i}$, and $[Z] = \sum_i [a_i]$ as zero-cycles. Note that this is in fact a Jordan-Hölder cofiltration of $\widehat{\mathscr{O}_Z}$, which thus is semistable. In addition, each $\mathscr{F}_i$ is locally free.

1.1. **Reparation.** At page 38 in the dissertation, the isomorphism (*) is used to conclude that $\det \widehat{\mathscr{O}_Z} \cong \mathscr{P}_{\sigma(Z)}$, where $\sigma(Z)$ denotes the sum $\sum_i a_i$ with respect to the group law. For this it is, however, sufficient that (*) holds when interpreted as an equality in the Grothendieck group, which is true, and follows from the existence of the cofiltration (**).

At page 53, the isomorphism (*) is applied in "Step 3" in the proof of a result due to Maciocia (note that the mistake is mine, and not Maciocia's). The argument must be rephrased as follows to go through with the cofiltration (**) in place of (*):

Firstly, with notation as in the proof on page 53, we have

$$c_1(\mathscr{F}) = 0 \quad \text{and} \quad \chi(\mathscr{F}) = 0.$$

This is (4.3.4) without dualizing, and is contained in the argument on page 53–54. This argument works without using (*), as long as we know that the dual of $\widehat{\mathscr{O}_Z}$ is semi-stable with $c_1 = 0$ and $\chi = 0$. This follows from noting that the dual of the cofiltration (**) is a Jordan-Hölder filtration of $\widehat{\mathscr{O}_Z}^\vee$ with factor modules $\mathscr{P}_{-a_i}$ (using that the $\mathscr{F}_i$ are locally free).

Secondly, as $\mathscr{F}$ is a quotient of $\widehat{\mathscr{O}_Z}$ with the same reduced Hilbert polynomial, it is also semi-stable. Hence it has a Jordan-Hölder cofiltration, and the kernels appearing have to be among the $\mathscr{P}_{a_i}$ appearing in the cofiltration (**). It follows that $\mathscr{F}$ satisfies WIT_2.

Now, the quotient $R^1\widehat{S}(\mathscr{E}) \twoheadrightarrow \mathscr{F}$ induces a quotient $R^2S(R^1\widehat{S}(\mathscr{E})) \twoheadrightarrow \widehat{\mathscr{F}}$. But $R^2S(R^1\widehat{S}(\mathscr{E}))$ vanishes by Lemma 4.7, hence so does $\mathscr{F}$. This shows that $R^1\widehat{S}(\mathscr{E})$ is torsion, which gives a contradiction as before, and concludes Step 3 in the proof.

2. Minor corrections

- Page 72: Replace last sentence in the proof of Lemma 4.26 with: "As both sheaves are stable, with coinciding reduced Hilbert polynomials, any nonzero map between them is an isomorphism, and we have the result."